ÉRETTSÉGI VIZSGA • 2023. október 17.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

minden vizsgázó számára

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI HIVATAL

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **remark** in brackets, the solution should be considered complete without that remark as well.

6. Deduct points for **missing units** only if the missing unit is part of the answer or a unit exchange (without parentheses).
7. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. The score given for the solution of a problem, or part of a problem, **may never be negative**.
10. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
11. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
12. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
13. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
14. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
15. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
$1848 = 2^3 \cdot 3 \cdot 7 \cdot 11$	2 points	
Total:	**2 points**	

2.		
$\left(\frac{5 \cdot 8}{4} =\right) 10$	2 points	
Total:	**2 points**	

3.		
The length of the hypotenuse (with the Pythagorean Theorem): $\sqrt{10^2 + 24^2} =$	1 point	
$= 26$ (cm).	1 point	
(Let the measure of the angle be α) $\tan \alpha = \frac{10}{24}$ (≈ 0.417)	1 point	
$\alpha \approx 22.62°$	1 point	
Total:	**4 points**	

4.		
C	2 points	*Not to be divided.*
Total:	**2 points**	

5.		
($6 \cdot 6300 - 5 \cdot 7500 =$) 300 Ft more.	2 points	
Total:	**2 points**	

6.		
The mean: $\left(\frac{9+5+6+9+6+6+8}{7} = \frac{49}{7} =\right)$ 7 °C	1 point	
The range: 4 °C	1 point	
The median: 6 °C	1 point	
Total:	**3 points**	

7.		
5	2 points	
Total:	**2 points**	

Note: Award 1 point for stating that there are 15 marbles in the box.

8.		
$(a+1)(a-1)=a^2-1$	1 point	
$(a+4)^2=a^2+8a+16$	1 point	
The final form is: $2a^2+8a+15$.	1 point	
Total:	**3 points**	

9.		
The fuel weighs 60 000·0.85 = 51 000 kg =	1 point	
= 51 t.	1 point	
The full car weighs (51 + 23.8 =) 74.8 tons.	1 point	
Total:	**3 points**	

10.		
The centre is (2; 4),	1 point	
the radius is 5.	1 point	
Total:	**2 points**	

11.		
The zero is (x =) 9.	2 points	
Total:	**2 points**	

12. Solution 1		
Favourable cases are: HTT, THT and TTH (3 cases).	1 point	
The total number of cases is (2^3 =) 8.	1 point	
The probability: $\frac{3}{8}$ (= 0.375).	1 point	
Total:	**3 points**	

12. Solution 2		
(With binomial distribution:) The probability is $\binom{3}{1}\cdot\frac{1}{2}\cdot\left(\frac{1}{2}\right)^2=$	2 points	
$=\frac{3}{8}$ (= 0.375).	1 point	
Total:	**3 points**	

Note: Award 1 point if the candidate misses the binomial coefficient in their answer and gives the solution as $\frac{1}{8}$.

II. A

13. a)		
$(f(3.5) = (3.5-3)^2 + 2 =)\ 2.25$	2 points	
Total:	**2 points**	

13. b) Solution 1		
$(x-3)^2 + 2 = 6$	1 point	
Rearranged: $x^2 - 6x + 5 = 0$.	1 point	
$x = 1$ or $x = 5$.	2 points	
Total:	**4 points**	

13. b) Solution 2		
$(x-3)^2 + 2 = 6$, that is $(x-3)^2 = 4$.	1 point	
So $x - 3 = 2$ or $x - 3 = -2$.	2 points	
Hence $x = 5$ or $x = 1$.	1 point	*Award this point only if the candidate gives both solutions.*
Total:	**4 points**	

Note: In case the candidate provides a graphical solution, award 2 points for the correct graph of the function f, 1 point for correctly reading the correct solutions off the graph and 1 point for checking the answers.

13. c)		
B	2 points	*Not to be divided.*
Total:	**2 points**	

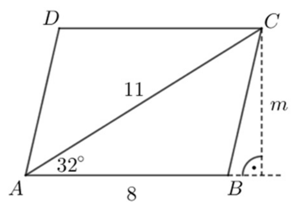
13. d) Solution 1		
Rearranged: $x^2 - 6x + 8 \le 0$.	1 point	
The roots of the equation $x^2 - 6x + 8 = 0$ are 2 and 4.	1 point	
The value of the expression on the left is negative in between these two roots (i.e. the solution over the set of real numbers would be $2 \le x \le 4$).	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The integer solutions are 2, 3 and 4.	1 point	
Total:	**4 points**	

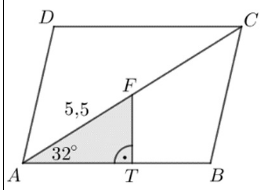
13. d) **Solution 2**		
Complete the square and rearranged: $(x-3)^2 \le 1$.	1 point	
$\lvert x-3 \rvert \le 1$	1 point	
Possible integer values of $x - 3$ are –1, 0 and 1,	1 point	$-1 \le x - 3 \le 1$
possible values of x are therefore 2, 3 and 4.	1 point	
Total:	**4 points**	

Notes:
1. Award a maximum of 3 points if the candidate solves the problem over the set of real numbers.
2. Award 2 points if the candidate correctly lists the solutions (i.e. by trial and error). Award a further 2 points for proving that there are no other solutions.

14. a)		
Law of Cosines for triangle *CAB*: $BC^2 = 11^2 + 8^2 - 2 \cdot 11 \cdot 8 \cdot \cos 32^\circ$ (≈ 35.74).	2 points	
$BC \approx 6$ cm.	1 point	
Total:	**3 points**	

14. b) **Solution 1**		
The area is the double of the area of triangle *CAB*.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
$T = 2 \cdot \dfrac{11 \cdot 8 \cdot \sin 32^\circ}{2} \approx$	1 point	*Heron's formula:* $T \approx 2\sqrt{12.5 \cdot 1.5 \cdot 4.5 \cdot 6.5}$
≈ 46.6 cm^2	1 point	≈ 46.8 cm^2
Total:	**3 points**	

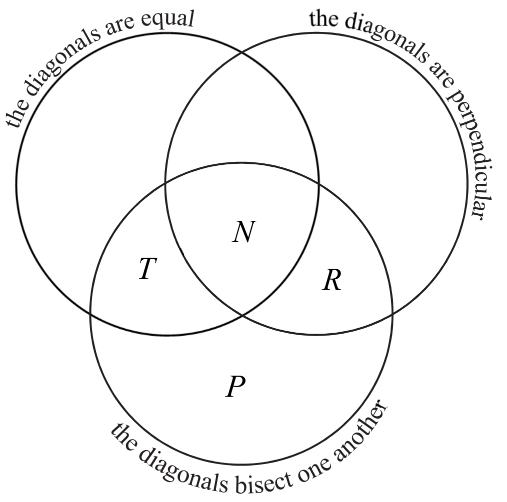
14. b) **Solution 2**		
(Determine the height m of the parallelogram that belongs to side *AB*.) $\sin 32^\circ = \dfrac{m}{11}$	1 point	
$m \approx 5.83$ cm.	1 point	
$T = 8 \cdot m \approx 46.6$ cm^2	1 point	
Total:	**3 points**	

14. c)		
 In the right triangle AFT: $\cos 32° = \frac{AT}{5.5}$,	1 point	*The length of segment FT is half the height of the parallelogram:* $FT = 2.915$ cm.
$AT \approx 4.66$ cm.	1 point	*The Pythagorean Theorem:* $AT = \sqrt{5.5^2 - 2.915^2} \approx$ ≈ 4.66 cm.
The other segment: $TB = 8 - AT = 3.34$ cm.	1 point	
Total:	**3 points**	

14. d) Solution 1		
One of the colours must be used twice. This colour can be selected in 3 different ways.	1 point	
This colour must be used on two opposite regions. Such pair of regions may be selected in 2 different ways.	1 point	
The remaining two regions will be coloured in the remaining 2 colours which can be done in 2 different ways.	1 point	
There are a total $3 \cdot 2 \cdot 2 = 12$ possibilities.	1 point	
Total:	**4 points**	

14. d) Solution 2		
Suppose, we colour the top region red. If the regions on the left and right are coloured differently (one is yellow and the other is blue), then the bottom region can only be red. This is 2 options.	1 point	
If (the top region is red and) both the left and right regions are of the same colour (both yellow or both blue), then the bottom region must be of the third colour. This is also 2 options.	1 point	
Similarly, there are $(2 + 2 =)$ 4 options if the top region is yellow or blue.	1 point	
This makes a total $3 \cdot 4 = 12$ possibilities.	1 point	
Total:	**4 points**	

Note: Award full score if the candidate gives the correct answer by listing all possible correct colourings in an organised manner.

<table>
<tr><td colspan="3">15. a)</td></tr>
<tr><td>
</td><td>7 points</td><td>Award 1 point for the correct placement of N.
Award 2 points each for the correct placement of T, R and P.
If one aspect is missed on placing T, R or P, award only 1 point for that quadrilateral.</td></tr>
<tr><td align="right">Total:</td><td>7 points</td><td></td></tr>
</table>

15. b)		
Statement I is false.	1 point	
Correct reasoning (e.g. a counterexample).	1 point	*For example:* $A = \{1; 2\}$, $B = \{1; 3\}$.
Statement II is correct.	1 point	
The elements: 16, 25, 36, 49, 64 and 81.	1 point	
Total:	**4 points**	

II. B

16. a) Solution 1		
Assuming Janka got n fives: $\frac{3+3+4+5n}{3+n}=4.5$	2 points	*Assuming she had m grades:* $\frac{3+3+4+5(m-3)}{m}=4.5$
$10+5n=13.5+4.5n$ $0.5n=3.5$	1 point	$5m-5=4.5m$
$n=7$, so Janka got 7 fives.	1 point	*$m=10$, so Janka got* $(10-3=)$ 7 *fives*.
Total:	**4 points**	

16. a) Solution 2		
Assuming Janka got 7 fives,	1 point	
her average could have been $\frac{3+3+4+7\cdot 5}{10}=4.5$	2 points	
Correct reasoning to show that there can be no other solution (e.g. fewer fives would give an average below 4.5, while more 5-s would give a higher average). Janka must have got 7 fives.	1 point	
Total:	**4 points**	

16. b) Solution 1		
In the first year, Janka got $12\cdot 1000 = 12\,000$ Ft. In the second year, $12\cdot 2000 = 24\,000$ Ft and so on, until the last year: $12\cdot 12000 = 144\,000$ Ft.	1 point	
The annual amounts form consecutive terms of an arithmetic sequence, both the first term and the common difference of which is 12 000. The sum of the first 12 terms is to be determined.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
This sum is $S_{12}=\frac{2\cdot 12\,000+(12-1)\cdot 12\,000}{2}\cdot 12=$	1 point	$S_{12}=\frac{12\,000+144\,000}{2}\cdot 12$
$= 936\,000$ Ft.	1 point	
Total:	**4 points**	

16. b) Solution 2		
Money received over 12 years: $12\cdot 1000+12\cdot 2000+\ldots+12\cdot 12\,000=$	1 point	
$=12\cdot(1000+2000+\ldots+12\,000)=$	1 point	*Award these points if the candidate obtains the correct answer using a calculator.*
$=12\cdot 1000\cdot(1+2+\ldots+12)=12\cdot 1000\cdot 78=$	1 point	
$= 936\,000$ Ft.	1 point	
Total:	**4 points**	

16. c)		
(Let a_n be the n[th] term of the geometric sequence.) The sum of the first nine terms: $a_1 \cdot \frac{3^9 - 1}{3 - 1} = 59\,046.$	1 point	
$a_1 \cdot 9841 = 59\,046$	1 point	
$a_1 = 6.$	1 point	
$a_9 = a_1 \cdot q^8 = 6 \cdot 3^8 = 39\,366$	1 point	
Total:	**4 points**	

16. d)		
$50\,000 \cdot \left(1 + \frac{p}{100}\right)^3 = 59\,046$	2 points	$50\,000 \cdot x^3 = 59\,046$
$\left(1 + \frac{p}{100}\right)^3 = 1.18092$	1 point	$x^3 = 1.18092$
$1 + \frac{p}{100} = \sqrt[3]{1.8092} \approx 1.057$	1 point	$x \approx 1.057$
So p is approximately 5.7.	1 point	
Total:	**5 points**	

17. a) **Solution 1**		
The bicycle car can be placed in 7 different positions.	1 point	
The buffet car can then be placed in 6 different positions (which also determines the placement of the second-class cars).	1 point	
This means $6 \cdot 7 = 42$ different possible arrangements.	1 point	
Total:	**3 points**	

17. a) **Solution 2**		
The positions of the second-class cars can be selected in $\binom{7}{5} = 21$ different ways.	1 point	
The other two cars can be placed on the remaining two positions in 2 different ways.	1 point	
This means a total $21 \cdot 2 = 42$ different arrangements.	1 point	
Total:	**3 points**	

17. a) **Solution 3**		
Seven cars may be placed in 7! (= 5040) different ways.	1 point	*This is a case of permutation with repeat for 7 items, 5 of which are identical.*
The second-class cars are not distinguished. They can be arranged in 5! (= 120) different ways.	1 point	
There are $\frac{7!}{5!} = 42$ different possible arrangements.	1 point	
Total:	**3 points**	

17. b)		
The full price is 3040 : 0.95 =	1 point	
= 3200 (Ft).	1 point	
Total:	**2 points**	

17. c)		
A ticket bought from the machine costs $280 \cdot 0.95 = 266$ Ft.	1 point	
(Assume that Ábel travelled n times.) $2140 < n \cdot 266$ $8.05 < n$	1 point	$2140 : 266 \approx 8.05$
The monthly ticket costs more than 8 but less than 9 discounted single tickets.	1 point	*Award this point if the candidate obtains the correct answer using a calculator.*
So, Ábel must have travelled 9 times.	1 point	
Total:	**4 points**	

17. d)		
Let x be the full price of a train ticket (in Ft-s) and let y be the price of a complementary intercity ticket. The 20% discounted ticket costs $0.8x$, the 50% discounted ticket costs $0.5x$, and the 90% discounted ticket costs $0.1x$.	1 point	*Award this point if the candidate obtains the correct answer using a calculator.*
(Write an equation for the purchase of each family. This equation system is to be solved.) $2x + 0.8x + 0.5x + 4y = 7960$, which simplifies to $3.3x + 4y = 7960$.	1 point	
$5 \cdot 0.1x + 5y = 1975$, which simplifies to $0.5x + 5y = 1975$.	1 point	
From the second equation $y = 395 - 0.1x$,	1 point*	*From the first equation* $y = 1990 - 0.825x$.
Substitute this into the first equation: $3.3x + 1580 - 0.4x = 7960$,	1 point*	*Substitute this into the second equation:* $9950 - 3.625x = 1975$.
So, (the full price of the train ticket is) $x = 2200$ (Ft).	1 point	
(The price of the intercity ticket:) $y = 175$ (Ft).	1 point	
Check: (The 20% discount ticket costs 1760 Ft, the 50% costs 1100 Ft, the 90% costs 220 Ft.) The Kiss family bought tickets for $2 \cdot 2200 + 1760 + 1100 + 4 \cdot 175 = 7960$ Ft, the Nagy family bought tickets for $5 \cdot 220 + 5 \cdot 175 = 1975$ Ft.	1 point	
Total:	**8 points**	

*Note: The 2 points marked * may also be given for the following reasoning:*

Subtract 4 times the second equation from 5 times the first: $14.5x = 31\,900$.	2 points	

18. a)		
The radius of the larger cylinder is 21 cm, the radius of the smaller one is 9 cm (the height of the solid is $m = 7$ cm).	1 point	
The volume of the larger cylinder: $21^2 \cdot \pi \cdot 7 \approx 9698$ cm^3.	1 point	
The volume of the smaller cylinder: $9^2 \cdot \pi \cdot 7 \approx 1781$ cm^3.	1 point	
The volume of the sponge is the difference of the above, 7917 cm^3.	1 point	
Total:	**4 points**	

18. b)		
The surface are of a single cushion consists of two circular rings, the outer lateral surface, and the inner lateral surface.	1 point*	*Award this point if the correct reasoning is reflected only by the solution.*
The area of the circular ring is the difference of the areas of the circles: $21^2 \cdot \pi - 9^2 \cdot \pi \approx 1131$ cm^2.	1 point*	
The area of the inner lateral surface: $2 \cdot 9 \cdot \pi \cdot 7 \approx 396$ cm^2.	1 point*	
The area of the outer lateral surface: $2 \cdot 21 \cdot \pi \cdot 7 \approx 924$ cm^2.	1 point*	
The total surface area of a single cushion: $2 \cdot 1131 + 396 + 924 = 3582$ cm^2.	1 point*	
The total surface area of 30 cushions: $30 \cdot 3582 = 107\,460$ cm^2 =	1 point	
= 10.746 m^2.	1 point	
(Rounded correctly:) 11 m^2 of cloth is needed.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**8 points**	

*The 5 points marked * may also be given for the following reasoning:*

The total surface area may be obtained by subtracting the surface area of the smaller cylinder from that of the larger one and then adding the double of the inner lateral surface.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The surface area of the larger cylinder: $2 \cdot 21^2 \cdot \pi + 2 \cdot 21 \cdot \pi \cdot 7 \approx 3695$ cm^2.	1 point	
The surface area of the smaller cylinder: $2 \cdot 9^2 \cdot \pi + 2 \cdot 9 \cdot \pi \cdot 7 \approx 905$ cm^2.	1 point	
The area of the inner lateral surface: $2 \cdot 9 \cdot \pi \cdot 7 \approx 396$ cm^2.	1 point	
The total surface area of a single cushion: $3695 - 905 + 2 \cdot 396 = 3582$ cm^2.	1 point	

18. c)		
The probability that a pillow is not defected is 0.97.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The probability that none of the 30 pillows will be defected is $0.97^{30} \approx 0.401$.	1 point	

The probability that exactly one of the pillows will be defected: $\binom{30}{1}\cdot 0.03\cdot 0.97^{29}\approx 0.372$.	2 points	
The total probability: 0.401 + 0.372 = 0.773.	1 point	
Total:	**5 points**	