ÉRETTSÉGI VIZSGA • 2023. május 9.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

minden vizsgázó számára

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI HIVATAL

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **remark** in brackets, the solution should be considered complete without that remark as well.

6. Deduct points for **missing units** only if the missing unit is part of the answer or a unit exchange (without parentheses).
7. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. The score given for the solution of a problem, or part of a problem, **may never be negative**.
10. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
11. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
12. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
13. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
14. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
15. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
$B \setminus A = \{c; d; f\}$	2 points	*Award 1 point in case of 1 error (missing or incorrect element), 0 points for more errors.*
Total:	**2 points**	

2.		
There are $(10 \cdot 9 \cdot 8 =)$ 720 possibilities.	2 points	
Total:	**2 points**	

3.		
$\left(\frac{308\,000}{275\,000} = 1.12\right)$ Zita's salary has been raised by 12%.	2 points	
Total:	**2 points**	

4. Solution 1		
$\overrightarrow{AF} = \frac{1}{2}\mathbf{b}$, $\overrightarrow{AG} = \frac{1}{2}\mathbf{c}$	1 point	
$\overrightarrow{FG} = (\overrightarrow{AG} - \overrightarrow{AF} =)\ \frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{b}$	2 points	
Total:	**3 points**	

4. Solution 2		
$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = \mathbf{c} - \mathbf{b}$	1 point	
Midsegment *FG* is parallel to side *BC* of the triangle and half as long.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
($\overrightarrow{FG}$ and $\overrightarrow{BC}$ point in the same direction, therefore) $\overrightarrow{FG} = \frac{\mathbf{c} - \mathbf{b}}{2}$.	1 point	
Total:	**3 points**	

5.		
The median of the 5 positive numbers given is 3,	1 point	*A possible solution:*
the range is 7.	1 point	1, 2, 3, 7, 8.
Total:	**2 points**	

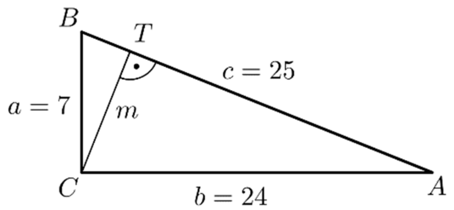
6.		
(32 + 8 + 2 + 1 =) 43	2 points	
Total:	**2 points**	

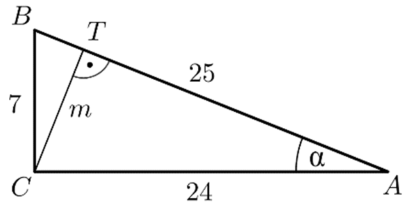
7.		
$\log_2(2x) = \log_2 2 + \log_2 x =$	1 point	$x = 32$
$(= 1 + 5) = 6$	1 point	$\log_2 64 = 6$
Total:	**2 points**	

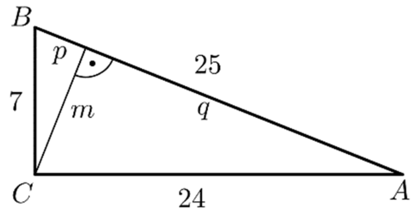
8.		
–3; –2; –1; 0; 1; 2	2 points	*Award 1 point in case of 1 missing or incorrect value, 0 points for more errors.*
Total:	**2 points**	

Note: Award 1 point for $-4 < x \le 2$.

9.		
There are $\left(\binom{16}{2} =\right)$ 120 possibilities.	2 points	*Award 1 point for* $\binom{16}{2}$.
Total:	**2 points**	

10. Solution 1		
 Let m be the height in question. Express the area of the triangle in two different ways: $A = \frac{a \cdot b}{2} = \frac{c \cdot m}{2}$.	2 points	*Let T be the point where the height intersects the hypotenuse. Triangles BCT and BAC are then similar, so the ratios of their corresponding sides are equal:* $\frac{a}{c} = \frac{m}{b}$.
$m = \frac{7 \cdot 24}{25} =$	1 point	
$= 6.72$.	1 point	
Total:	**4 points**	

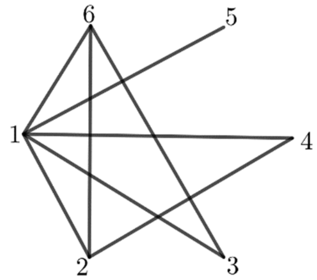
10. Solution 2		
 In the right triangle ABC $\sin\alpha = \frac{7}{25}$. ($\alpha \approx 16.26°$)	2 points	
In the right triangle ACT $m = 24 \cdot \sin\alpha =$	1 point	
$= 6.72$.	1 point	
Total:	**4 points**	

10. Solution 3		
 Apply the leg-theorem: $7 = \sqrt{p \cdot 25}$,	1 point	
$p = 1.96$.	1 point	
$q = 25 - 1.96 = 23.04$	1 point	
Apply the height-theorem: $m = \sqrt{pq} = \sqrt{1.96 \cdot 23.04} = 6.72$.	1 point	
Total:	**4 points**	

11.		
a) E.g. **n**(5; –1).	1 point	
b) $5x - y = 13$	2 points	
Total:	**3 points**	

12.		
The minimum value is (–2): *f*, *h*	2 points	*Award 1 point in case of one correct answer or two correct and an incorrect answers. Award 0 points otherwise.*
Has at least two zeroes: *g*, *h*	2 points	*Award 1 point in case of one correct answer or two correct and an incorrect answers. Award 0 points otherwise.*
Total:	**4 points**	

II. A

13. a)		
	3 points	*Deduce 1 point for each missing or incorrectly drawn edge (but no more than a total 3 points).*
Total:	**3 points**	

13. b)		
Statement I is true,	1 point	
for example: 6 has 4 positive divisors: 1, 2, 3 and 6.	1 point	
Statement II is false,	1 point	
a counterexample: 4 and 6 (4 is not a divisor of 6, and yet, 4 and 6 are not relative primes).	1 point	
Total:	**4 points**	

13. c)		
Of the numbers 1, 2, 3, 4, 5 and 6 only 5 is not a divisor of 24 (there are 5 favourable cases).	1 point	
In case of event B there are $5 \cdot 5$ favourable cases,	1 point	
while the total number of cases is $6 \cdot 6$.	1 point	
$P(A)=\frac{5}{6}$ and $P(B)=\frac{25}{36}$.	1 point	
The probability of event A is higher.	1 point	
Total:	**5 points**	

14. a)		
The average of the four results obtained by Norbi and Emma: $\frac{1.9+2.0+1.8+2.3}{4}=2$ (m/s^2).	1 point	*Award these 2 points if the candidate calculates the correct value of the standard deviation using a calculator.*
The standard deviation of the four numbers: $\sqrt{\frac{(1.9-2)^2+(2.0-2)^2+(1.8-2)^2+(2.3-2)^2}{4}}\approx$	1 point	
≈ 0.187 (m/s^2).	1 point	
Total:	**3 points**	

14. b)		
The sum of the four results obtained by Norbi and Emma is 8.	1 point	
The sum of the other 20 results is $20 \cdot 1.9 = 38$.	1 point	
The average of the 24 results: $\frac{8+38}{24}\approx$	1 point	
≈ 1.92 (m/s^2) rounded correctly.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**4 points**	

14. c)		
The distance between the ball and the ground after 0.5 seconds is $h(0.5) = 6 \cdot 0.5 - 5 \cdot 0.5^2 =$	1 point	
$= 1.75$ (metres).	1 point	
Total:	**2 points**	

14. d)		
The equation $6t - 5t^2 = 1$ is to be solved.	1 point	
The solutions of the quadratic equation are $t = 0.2$ and $t = 1$. The ball will be at a height of 1 m above the ground 0.2 and 1 second after being shot upwards.	2 points	
Total:	**3 points**	

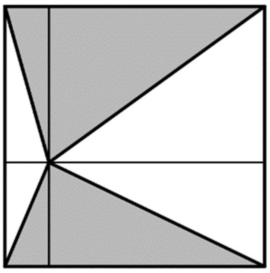
Note: Award full score if the candidate correctly gives both values of t *but in their answer, only refers to the smaller (t = 0.2 s) value.*

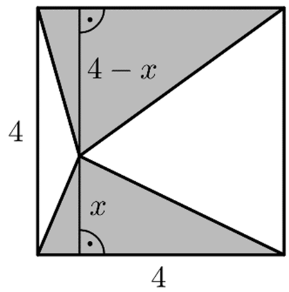
15. a)		
One leg of the coloured triangle is 4 cm long, the angle between this leg and the hypotenuse is 30°.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The length of the other leg: $4 \cdot \tan 30° = 4 \cdot \frac{\sqrt{3}}{3} \approx 2.31$ cm.	2 points	*The length of the hypotenuse:* $\frac{4}{\cos 30°} \approx 4.62$ cm.
The area of the triangle: $A = \frac{4 \cdot 2.31}{2} = 4.62\ \text{cm}^2$.	1 point	$A = \frac{4 \cdot 4.62 \cdot \sin 30°}{2}$
Total:	**4 points**	

15. b) Solution 1		
Suppose, the first triangle is coloured blue. The two adjoining triangles may either be coloured the same (yellow-yellow or green-green) or they may be coloured different (yellow-green, green-yellow).	1 point	
In all of these four cases the fourth triangle may only be coloured in a single way (green, yellow, blue, blue, respectively).	1 point	
There are three colour choices for the first triangle to start with.	1 point	
There are a total 4 · 3 = 12 different options to colour the whole square.	1 point	
Total:	**4 points**	

15. b) Solution 2		
After the colouring is done, there must be exactly two triangles of the same colour such that they only share a common vertex.	1 point	
These two triangles may be positioned in two different ways and coloured in three different colours.	1 point	
The remaining two triangles may be coloured in two different ways with the remaining two colours.	1 point	
This gives a total 2 · 3 · 2 = 12 possible ways to colour the square.	1 point	
Total:	**4 points**	

Notes:
1. Award full score if the candidate produces an organised list of all possible colouring and hence gives the correct answer.
2. Award a maximum of 2 points if the candidate also considers the 6 cases in which not all three colours are used.

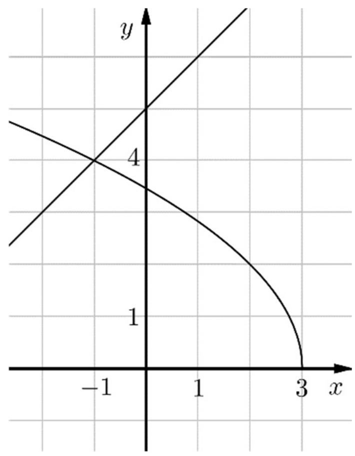
15. c) **Solution 1**		
Draw parallels to the sides of the square across the given point. 	1 point	
The diagonals bisect the area of all four rectangles obtained this way.	2 points	
The sum of the areas of the two grey triangles is therefore really equal to the sum of the areas of the other two triangles.	1 point	
Total:	**4 points**	

15. c) **Solution 2**		
Let the height that belongs to the 4 cm side of one of the grey triangles be x (cm). In this case the height that belongs to the 4 cm side of the other grey triangle is $4 - x$ (cm).	1 point	
The grey area: $\frac{4\cdot x}{2}+\frac{4\cdot(4-x)}{2}=\frac{4x+16-4x}{2}=8\,\text{cm}^2$,	2 points	
this is half of the area of the full square of side 4 cm, and so the combined area of the grey triangles is really equal to that of the other two triangles.	1 point	
Total:	**4 points**	

Note: Award a maximum of 2 points if the candidate calculates the areas of the triangles using a concrete value for their heights.

II. B

16. a) **Solution 1**		
Square the whole equation: $4\cdot(3-x) = x^2 + 10x + 25$.	2 points	
Rearrange to zero: $x^2 + 14x + 13 = 0$.	1 point	
The roots of the equation: $x = -1$ and $x = -13$.	1 point	
Check by substitution: $x = -13$ is incorrect,	1 point	*(Considering the domain and range of the square root function)* $-5 \le x \le 3$,
$x = -1$ is a correct solution.	1 point	*only equivalent steps were made within this interval, and so $x = -1$ is a solutions, but $x = -13$ is not.*
Total:	**6 points**	

16. a) **Solution 2**		
Correct diagram of the graph of the function $x \mapsto 2\cdot\sqrt{3-x}$ $(x \le 3)$,	3 points	
and the graph of the function $x \mapsto x+5$ in the same coordinate system.	1 point	
According to the diagram, the only solution is $x = -1$.	1 point	
Check by substitution: $2\cdot\sqrt{3-(-1)} = 4$ and $-1 + 5 = 4$.	1 point	
Total:	**6 points**	

16. b)		
$\frac{x(x-1)}{(x+1)(x-1)}+\frac{x^2}{(x+1)(x-1)}=2$	2 points	*Award these 2 points if the candidate correctly multiplies both sides of the equation by* (x^2-1).
$x^2-x+x^2=2x^2-2$	1 point	
$x=2$	1 point	
Check by substitution.	1 point	*Only equivalent steps were taken where* $\lvert x\rvert \neq 1$.
Total:	**5 points**	

16. c) Solution 1		
If the sum of the first six terms of a sequence is equal to the sum of the first seven terms then the seventh term must be 0.	2 points	$\frac{2\cdot 18+5d}{2}\cdot 6=$ $=\frac{2\cdot 18+6d}{2}\cdot 7$
Let d be the common difference of the sequence. Now $18 + 6d = 0$,	1 point	$108 + 15d = 126 + 21d$
and so $d = -3$.	1 point	
The sum of the first 13 terms of the sequence: $S_{13}=\frac{2\cdot 18+12\cdot(-3)}{2}\cdot 13=0$ indeed.	1 point	
$a_{13} = 18 + 12\cdot(-3) = -18$	1 point	
Total:	**6 points**	

Note: Award 2 points if the candidate assumes (without proving it) that $S_{13} = 0$ and hence proves that $a_{13} = -18$.

16. c) Solution 2		
If the sum of the first six terms of a sequence is equal to the sum of the first seven terms then the seventh term must be 0.	2 points	
According to the properties of the arithmetic sequence: $a_7=0=\frac{a_6+a_8}{2}=\frac{a_5+a_9}{2}=...=\frac{a_1+a_{13}}{2}$,	1 point	
$a_6=-a_8$, $a_5=-a_9$, … , $a_1=-a_{13}$.	1 point	
So $S_{13}=a_1+a_2+...+a_{12}+a_{13}=0$.	1 point	
As $a_1=-a_{13}$, $a_{13}=-18$.	1 point	
Total:	**6 points**	

17. a)		
The annual production values form a geometric sequence with a common ratio of 1.05,	1 point	*These 2 points are also due if the correct reasoning is reflected only by the solution.*
and a first term of $500 \cdot 1.05 = 525$ (million Ft).	1 point	
The sum of the first 20 terms of the sequence is $S_{20} = 525 \cdot \frac{1.05^{20} - 1}{1.05 - 1} \approx$	1 point	
$\approx$ 17 360 (million forints).	1 point	
Total:	**4 points**	

Note: Award full score if the candidate gives the correct answer by listing the terms of the sequence using reasonable and correct rounding.

17. b)		
(The production of factory *A* in the n^{th} year after 2018 is $500 \cdot 1.05^n$, the production of factory *B* in the same year is $400 \cdot 1.06^n$.) The correct values:	3 points	
The difference between the production values in the given years: 525 – 424 = 101, 551.3 – 449.4 = 101.9 and 578.8 – 476.4 = 102.4 (million forints).	2 points	
As the difference between the production values is increasing (in the period in question), the statement is, in fact, false.	1 point	
Total:	**6 points**	

Table of correct values (17. b):

	2018	2019	2020	2021
factory *A* (million Ft)	500	**525**	**551.3**	**578.8**
factory *B* (million Ft)	400	**424**	**449.4**	**476.4**

17. c)		
(Let n be the number of years passed since 2018.) The equation $500 \cdot 1.05^n = 400 \cdot 1.06^n$ is to be solved.	1 point	
Rearranged: $1.25 \cdot 1.05^n = 1.06^n$.	1 point	
Divide both sides by 1.05^n: $1.25 = \left(\frac{1.06}{1.05}\right)^n \approx 1.0095^n$.	1 point	$\log 1.25 + n \cdot \log 1.05 =$ $= n \cdot \log 1.06$
Take the base 10 logarithm of both sides and rearrange: $n = \frac{\log 1.25}{\log 1.0095} \approx$	2 points	$n = \frac{\log 1.25}{\log 1.06 - \log 1.05} \approx$
≈ 23.6.	1 point	≈ 23.5
It will be in the 24th year after 2018, i.e. in 2042, that the volume of production of factory B first catches up with that of factory A.	1 point	
Total:	**7 points**	

Notes:
1. Award full score if the candidate gives the correct answer by listing the terms of the sequences using reasonable and correct rounding.
2. Award full score if the candidate gives the correct solution by solving an inequality instead of an equation.

18. a)		
A regular hexagon may be divided into six congruent regular triangles, the height of each of which is $5 \cdot \frac{\sqrt{3}}{2}$ cm.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The area of the hexagon: $A = 6 \cdot \frac{5^2 \cdot \sqrt{3}}{4}$ (≈ 64.95 cm^2).	2 points	
The volume of the box: $V_{box} = 6 \cdot \frac{5^2 \cdot \sqrt{3}}{4} \cdot 3$ (≈ 195 cm^3).	1 point	
The radius of a single chocolate ball is 1.4 cm,	1 point	
the combined volume of all chocolate balls is therefore: $V_{total} = 6 \cdot \frac{4}{3} \cdot 1.4^3 \cdot \pi$ (≈ 69 cm^3).	1 point	
The combined volume of the six chocolate balls is $\frac{69}{195} \cdot 100 \approx 35.4$ percent of the volume of the box.	1 point	
Total:	**7 points**	

18. b)		
At least five golden balls means either 5 or 6 of them.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The probability of exactly 6 golden balls is: $\left(\frac{2}{3}\right)^6 \approx 0.088$.	1 point	
The (binomial) probability of exactly 5 golden balls is: $\binom{6}{5} \cdot \left(\frac{2}{3}\right)^5 \cdot \frac{1}{3} \approx 0.263$.	2 points	
The total probability is the sum of the above: about 0.351.	1 point	
Total:	**5 points**	

18. c)		
The resulting solid of revolution consists of two congruent truncated cones sharing a common base circle (of diameter *FC*).	1 point	*These two points are also due if the correct reasoning is reflected only by the solution.*
The base radius of each truncated cone is 5 cm, the top radius is 2.5 cm.	1 point	
The height of each truncated cone is: $\sqrt{5^2 - 2.5^2} \approx 4.33$ cm.	1 point	
The volume of the resulting solid is: $V = 2 \cdot \frac{4.33 \cdot \pi}{3} \cdot (5^2 + 5 \cdot 2.5 + 2.5^2) \approx 396.8\ \text{cm}^3$.	2 points	
Total:	**5 points**	