ÉRETTSÉGI VIZSGA • 2022. október 18.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

minden vizsgázó számára

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI HIVATAL

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
$A = \{2; 3; 5; 7; 11\}$	1 point	
$B = \{1; 2; 4; 5; 7; 8\}$	1 point	
$A \cap B = \{2; 5; 7\}$	1 point	
$B \setminus A = \{1; 4; 8\}$	1 point	
Total:	**4 points**	

2.		
$(4^3 =)\ 64$	2 points	
Total:	**2 points**	

3.		
$n = 10$	2 points	
Total:	**2 points**	

4.		
$(0.35 \cdot 520 =)\ 182$ (kcal)	2 points	
Total:	**2 points**	

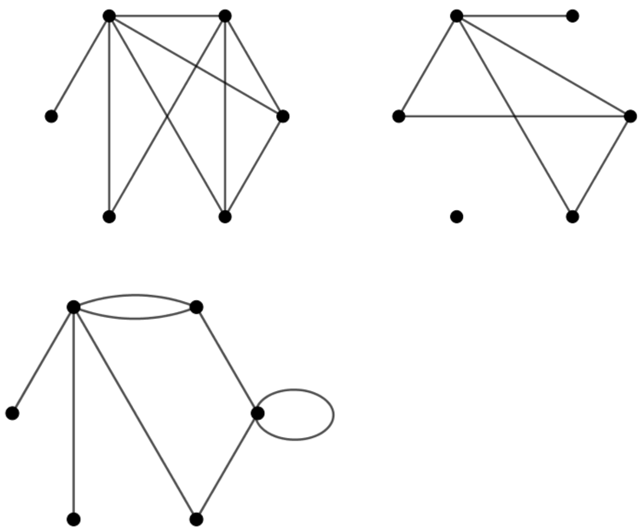
5.		
The range: $[-4; 5]$.	2 points	$-4 \le y \le 5$
The maximum is assumed at –1.	1 point	
Total:	**3 points**	

6.		
$\left(\frac{8 \cdot 5}{2} =\right) 20$	2 points	
Total:	**2 points**	

7.		
$(x = \log 30 \approx)\ 1.477$	2 points	
Total:	**2 points**	

Note: Award a maximum of 1 point if the candidate does not round, or rounds incorrectly.

8.		
$\frac{3}{5} = 0.6$	2 points	
Total:	**2 points**	

9.		
A proper graph, e.g. 	2 points	
Total:	**2 points**	

10. Solution 1		
$\beta = (180° - 30° - 100° =)\ 50°$	1 point	
(Use the Law of Sines:) $\frac{a}{6} = \frac{\sin 30°}{\sin 50°}$,	1 point	
$a \approx 3.92$ (cm).	1 point	
Total:	**3 points**	

10. Solution 2		
The height that belongs to side c is $m_c = 6 \cdot \sin 30° = 3$ (cm).	1 point	
(The angle between the height and side a is 40°, so) in the right triangle defined by this height $a = \frac{3}{\cos 40°}$,	1 point	
$a \approx 3.92$ (cm).	1 point	
Total:	**3 points**	

11.		
The mean: $\frac{43+40+42+39+40+36}{6} = 40$,	1 point	
the standard deviation: $\sqrt{\frac{3^2+0^2+2^2+(-1)^2+0^2+(-4)^2}{6}} =$	1 point	*Award this point if the candidate obtains the correct answer using a calculator.*
$= \sqrt{5} \approx 2.24$.	1 point	
Total:	**3 points**	

12. **Solution 1**		
There are a total of 36 different outcomes.	1 point	
The number of favourable cases is 4, these are: 2-3, 3-2, 1-6, 6-1.	1 point	
The probability is: $\frac{4}{36}=\frac{1}{9}$ (≈ 0.111).	1 point	
Total:	**3 points**	

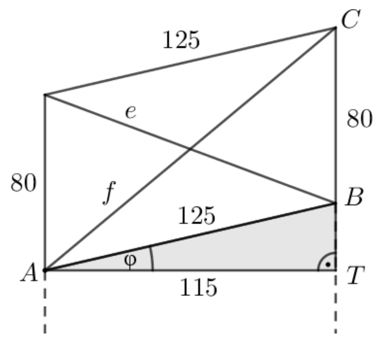
12. **Solution 2**		
The positive divisors of 6 are: 1, 2, 3, 6, and so the probability of throwing one of these first is $\frac{4}{6}$.	1 point	
As the first number determines what the other should be (e.g. throwing 1 first means the other number must be 6), there is a $\frac{1}{6}$ probability that the second number will be an appropriate pair.	1 point	
The final probability is the product of the above: $\frac{4}{6}\cdot\frac{1}{6}=\frac{4}{36}$.	1 point	
Total:	**3 points**	

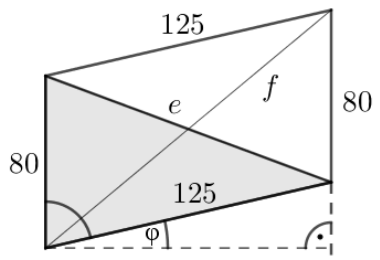
II. A

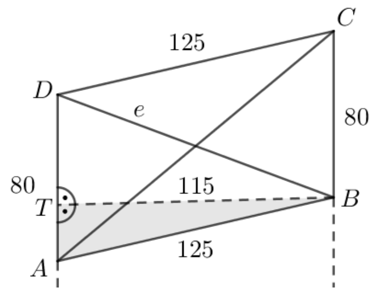
13. a)		
$\frac{3x}{6}+\frac{2x-2}{6}=8$	1 point	$3x + 2(x - 1) = 8 \cdot 6$
$5x - 2 = 48$	1 point	
$x = 10$	1 point	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**4 points**	

13. b)		
Let x be the smaller number. $x^2+(x+1)^2=10\,513$	1 point	
$x^2+x^2+2x+1=10\,513$	1 point	
$2x^2+2x-10\,512=0$	1 point	
$x_1 = 72$, $x_2 = -73$	2 points	
The numbers may be 72 and 73,	1 point	
or –73 and –72.	1 point	
Check: $72^2+73^2=10\,513$, $(-73)^2+(-72)^2=10\,513$.	1 point	*Award this point if the candidate only finds and checks one solution.*
Total:	**8 points**	

Note: Award 1 point for each solution if the candidate finds these by trial and error. Award a further 1 point for checking. Award full score if the candidate also correctly explains why there aren't any further solutions.

14. a)		
 In triangle ABT $\cos\varphi=\frac{115}{125}=0.92$.	2 points	
Rounded correctly, $\varphi = 23°$, indeed.	1 point	
Total:	**3 points**	

14. b) **Solution 1**		
 The angle of the parallelogram next to φ is $90° - 23° = 67°$.	1 point	
With the Law of Cosines (applied to the grey triangle): $e^2 = 125^2 + 80^2 - 2 \cdot 125 \cdot 80 \cdot \cos 67°$,	1 point	
$e \approx 119$ cm.	2 points	
Total:	**4 points**	

14. b) **Solution 2**		
 (Apply the Pythagorean theorem) in the right triangle ABT: $AT = \sqrt{125^2 - 115^2} \approx 49$ (cm).	2 points	$AT = 125 \cdot \sin 23° \approx$ ≈ 49 (cm)
Then $TD = 80 - 49 = 31$ (cm),	1 point	
(apply the Pythagorean theorem in the right triangle BDT:) $e = \sqrt{31^2 + 115^2} \approx 119$ cm.	1 point	
Total:	**4 points**	

14. c) **Solution 1**		
$A = 80 \cdot 115 =$	1 point	$A = 125 \cdot 80 \cdot \sin(90° - φ)$ $= = 125 \cdot 80 \cdot \sin 67° \approx$
$= 9200$ cm^2	1 point	≈ 9205 cm^2
As 1 m^2 = 10 000 cm^2, the statement is true.	1 point	
Total:	**3 points**	

14. c) **Solution 2**		
One side of the parallelogram is 0.8 m, the height that corresponds to this side is 1.15 m.	1 point	
$A = 0.8 \cdot 1.15 =$	1 point	
$= 0.92$ m^2, the statement is true.	1 point	
Total:	**3 points**	

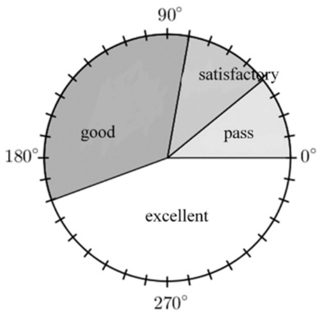
15. a)		
(After the first half year) the revenue grows by a factor of 1.05 through 18 months.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
$300\,000 \cdot 1.05^{18} \approx$	1 point	
≈ 720 000 Ft in sales revenue in the 24th month.	1 point	
The total revenue in the first half year is $(6 \cdot 300\,000 =)$ 1 800 000 (Ft).	1 point	
Monthly revenues throughout the next 18 month form consecutive terms of a geometric sequence. The first term of the sequence is $300\,000 \cdot 1.05 = 315\,000$, the common ratio is 1.05. The total revenue is the sum of the first 18 terms.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
$S_{18} = 315\,000 \cdot \frac{1.05^{18} - 1}{1.05 - 1} \approx$	1 point	
≈ 8 861 701 (Ft)	1 point	
Altogether 1 800 000 + 8 861 701 (= 10 661 701),	1 point	
rounded to the nearest ten thousand, the total revenue in the first two years is 10 660 000 Ft.	1 point	
Total:	**9 points**	

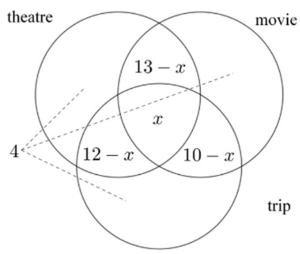
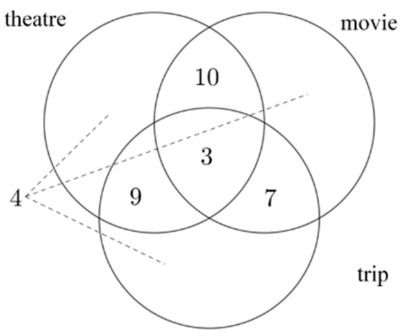
Notes:
1. Deduct a maximum of 1 point for all rounding errors throughout the candidate's work.
2. Deduct a maximum of 1 point for omitting the unit of measurement.

15. b)		
If András is driving then Cili sits next to him and the others can sit in the back in 3! = 6 different ways.	1 point	
If Dóra is driving then András, Cili and the third person can sit in 4 different ways in the back (_AC, _CA, AC_, CA_).	1 point	
Either way, Balázs and Endre can take the remaining seats in 2 different ways,	1 point	
that gives (4·2 =) 8 possibilities.	1 point	
Altogether, there are (6 + 8 =) 14 different seating arrangements.	1 point	
Total:	**5 points**	

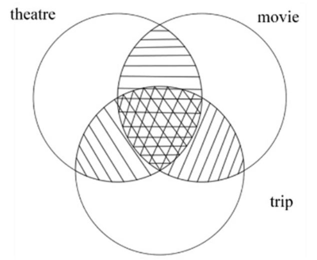
Note: Award full score if the candidate answers correctly by systematically listing all possible seating arrangements.

II. B

16. a)		
The missing grades are: 4, 4, 2, 3, respectively.	2 points	*Award 1 point in case of 1 mistake, 0 points for more than 1 mistakes.*
(Out of 9 grades one is a 2, one is a 3, three are 4-s and four are 5-s.) The central angles for each grade are: 2-s: 40°, 3-s: 40°, 4-s: 120°, 5-s: 160°.	1 point	
	2 points	*Award 1 point for the correct central angles, and 1 point for labelling the diagram.*
Total:	**5 points**	

16. b) Solution 1		
Let the number of students participating in all three events be x. In this case, $13 - x$ students went to the theatre and to the movie but not the trip. $12 - x$ students went to the theatre and the trip but not to the movie, and $10 - x$ students went to the trip and the movie but not to the theatre.	2 points	
As per the text: $4 + (13 - x) + (12 - x) + (10 - x) + x = 33$.	2 points	
$39 - 2x = 33$	1 point	
$x = 3$ (i.e. 3 students participated in all three events.)	1 point	
Check:  $10 + 3 + 9 + 7 + 4 = 33$	1 point	
Total:	**7 points**	

Note: Award a maximum of 4 points if the candidate gives their answer based on a correct Venn-diagram but fails to explain their workings.

16. b) Solution 2		
(33 – 4 =) 29 students participated in at least two of the events.	1 point	
The sum 13 + 12 + 10 is more than this, insofar as those students participating in all three events had been counted not once, but three times,	2 points	
and so their number is (13 + 12 + 10 – 29) : 2,	2 points	
which means there were 3 students participating in all three events.	1 point	
Check.	1 point	
Total:	**7 points**	

16. c) Solution 1		
(The number of chairs in consecutive rows of the theatre hall form an arithmetic sequence.) The difference between the number of chairs in rows ten and six is 8,	1 point	
so the common difference is (8 : 4 =) 2.	1 point	
The first term of the sequence is (26 – 5·2 =) 16.	1 point	
$S_{15}=\frac{2\cdot16+14\cdot2}{2}\cdot15=$	1 point	$a_{15}=44$, $S_{15}=\frac{16+44}{2}\cdot15=$
= 450, which is the total number of seats in the hall.	1 point	
Total:	**5 points**	

16. c) Solution 2		
(The number of chairs in consecutive rows of the theatre hall form an arithmetic sequence.) Being such: $a_8=\frac{a_6+a_{10}}{2}=30$ on the one hand,	2 points	
and $S_{15}=15\cdot a_8$ on the other.	2 points	
Therefore, there are 450 seats in the hall.	1 point	
Total:	**5 points**	

17. a) **Solution 1**		
The radius of one cake is 10 cm, its volume is $V = 10^2\pi \cdot 25 =$	1 point	
$= 2500\pi$ (≈ 7854 cm^3).	1 point	
The radius of the core rods is 0.1 cm, its length is *h* cm. The volume of the rod is then: $V_{rod} = 0.1^2 \cdot \pi \cdot h$ (cm^3).	1 point	
(These volumes are equal, therefore) $0.1^2 \cdot \pi \cdot h = 2500 \cdot \pi$.	1 point	
$h = 250\,000$ cm,	1 point	
i.e. the length of one such core rod is 2500 metres.	1 point	
Total:	**6 points**	

17. a) **Solution 2**		
As the diameter of the base circle of the core rod is one hundredth of the diameter of the cake, its area will be one tenth of a thousandth of the base area of the cake. (The two circles are similar, and the ratio of the areas of similar figures is the square of the ratio of similarity.)	3 points	
(Because of the conservation of volume) the height (length) of the core rod will be $25 \cdot 10\,000 = 250\,000$ cm,	2 points	
that is 2500 metres.	1 point	
Total:	**6 points**	

17. b)		
Let the number of women be $3x$, the number of men $2x$.	1 point	*Let w be the number of women, and m be the number of men. In this case* $\frac{w}{m} = \frac{3}{2}$,
According to the text: $\frac{3x+5}{2x+6} = \frac{4}{3}$.	1 point	*also:* $\frac{w+5}{m+6} = \frac{4}{3}$.
$9x + 15 = 8x + 24$	1 point	$3 \cdot 1.5m + 15 = 4m + 24$
$x = 9$	1 point	*so* $m = 18$,
Currently, there are $3 \cdot 9 = 27$ women, and $2 \cdot 9 = 18$ men working at the company.	1 point	*and* $w = 27$.
Check, based on the text: $27 : 18 = 3 : 2$, $(27 + 5) : (18 + 6) = 4 : 3$.	1 point	
Total:	**6 points**	

17. c)		
The probability that the tip of a particular pencil will not break on falling is 0.8.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The probability that none of the tips will break is $0.8^{12} \approx 0.069$.	1 point	
The probability that exactly one tip will break is $\binom{12}{1} \cdot 0.2^1 \cdot 0.8^{11} \approx 0.206$.	2 points	
The final probability is about 0.069 + 0.206 = 0.275.	1 point	
Total:	**5 points**	

18. a) Solution 1		
There are (36 – 24 =) 12 blue polygons on the table.	2 points	*There are* (36 – 27 =) 9 *quadrilaterals on the table.*
As there are 5 blue quadrilaterals, there must be (12 – 5 =) 7 blue triangles.	1 point	*As there are 5 blue quadrilaterals, there must be* (9 – 5 =) 4 *red quadrilaterals.*
Also, there are (27 – 7 =) 20 red triangles on the table.	1 point	*Also, there are* (24 – 4=) 20 *red triangles on the table.*
Total:	**4 points**	

Note: There are 4 red and 5 blue quadrilaterals, as well as 20 red and 7 blue triangles on the table.

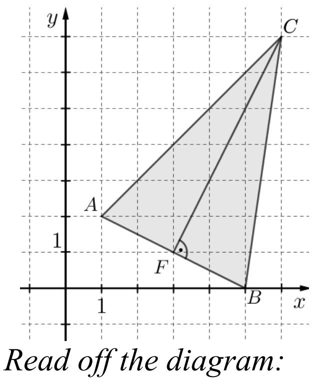
18. a) Solution 2		
Arrange the unknowns into a table:	1 point	

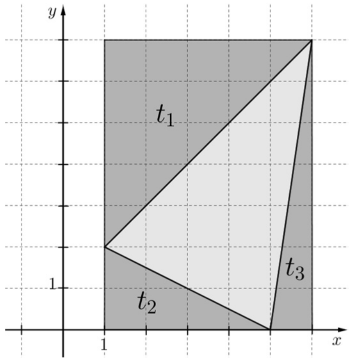
	triangle	quadrilateral
red	x	y
blue	z	5

According to the text: $x + y + z = 31$.	1 point	
As $x + y = 24$, there must be $z = 7$ blue triangles.	1 point	
As $x + z = 27$, there must be $x = 20$ red triangles on the table.	1 point	
Total:	**4 points**	

18. b)		
(If the order of selection is irrelevant, then) there are $\binom{36}{2}$ different ways to select two polygons.	1 point	*If the order of selection is relevant then there are* $36\cdot 35$ *different ways to select.*
The number of favourable cases is $\binom{27}{2}$.	1 point	*The number of favourable cases is* $27\cdot 26$.
The probability is $\frac{\binom{27}{2}}{\binom{36}{2}}=$	1 point	*The probability is* $\frac{27\cdot 26}{36\cdot 35}=$
$=\frac{351}{630}\left(=\frac{39}{70}\right)\approx 0.557.$	1 point	
Total:	**4 points**	

18. c)		
$\lvert AC\rvert=\sqrt{(6-1)^2+(7-2)^2}=\sqrt{50}$ and $\lvert BC\rvert=\sqrt{(5-6)^2+(0-7)^2}=\sqrt{50}$,	2 points	
as sides AC and BC are equal in length, the triangle is isosceles, indeed.	1 point	
Total:	**3 points**	

18. d) Solution 1		
The midpoint of side AB is $F\left(\frac{1+5}{2};\frac{2+0}{2}\right)=(3;1)$.	2 points	*Read off the diagram:* $F(3;\,1)$.
The length of side AB is $\lvert\overrightarrow{AB}\rvert=\sqrt{4^2+2^2}=\sqrt{20}$ (≈ 4.47).	1 point	
The height FC is $\lvert\overrightarrow{FC}\rvert=\sqrt{3^2+6^2}=\sqrt{45}$ (≈ 6.71).	1 point	
The area of triangle ABC is $A=\frac{\sqrt{20}\cdot\sqrt{45}}{2}=$	1 point	
= 15 (area units).	1 point	
Total:	**6 points**	

18. d) Solution 2		
(Inscribe the triangle into a rectangle. Subtract the areas of the three right triangles from the area of the rectangle.) 	1 point	
The area of the rectangle is $(5 \cdot 7 =)$ 35,	1 point	
the areas of the right triangles are: $t_1 = \frac{5 \cdot 5}{2} = 12,5$, $t_2 = \frac{2 \cdot 4}{2} = 4$, $t_3 = \frac{1 \cdot 7}{2} = 3,5$ (area units).	2 points	*Award 1 point in case of 1 mistake, 0 points for more than 1 mistakes.*
The area of triangle *ABC* is 35 – (12,5 + 4 + 3.5) =	1 point	
= 15 (area units).	1 point	
Total:	**6 points**	

18. d) Solution 3		
The length of side *AB* is $\sqrt{4^2 + 2^2} = \sqrt{20}\ (= 2\sqrt{5})$.	1 point	
Use Heron's Formula to calculate the area of the triangle. Sides *BC* and *AC* are $\sqrt{50}$ long, so the semi-perimeter is: $s = \frac{\sqrt{50} + \sqrt{50} + \sqrt{20}}{2} = \sqrt{50} + \sqrt{5}$ (≈ 9.31).	2 points	
$T = \sqrt{(\sqrt{50} + \sqrt{5})(\sqrt{50} - \sqrt{5}) \cdot \sqrt{5} \cdot \sqrt{5}} =$	2 points	$\approx \sqrt{9.31 \cdot 4.83 \cdot 2.24 \cdot 2.24}$
$(= \sqrt{45 \cdot 5}) = 15$ (area units).	1 point	
Total:	**6 points**	

Note: Award full score if the candidate correctly calculates using approximated values.