ÉRETTSÉGI VIZSGA • 2021. május 4.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
$y = 79$	2 points	
Total:	**2 points**	

2.		
The number of faces is 6,	1 point	
the number of edges is 12,	1 point	
the number of vertices is 8.	1 point	
Total:	**3 points**	

3.		
$(9 \cdot 5 =)$ 45	2 points	
Total:	**2 points**	

4.		
$\left(\frac{6}{4} \cdot 7 =\right)$ 10.5 (dl)	2 points	
Total:	**2 points**	

5.		
$x\ (= 0 + 1 + 2 + \ldots + 8) = 36$	2 points	
Total:	**2 points**	

6.		
$(\sqrt{25^2 - 24^2} =)$ 7 (metres)	2 points	
Total:	**2 points**	

7.		
The common difference of the sequence is $(3.5 - 2 =)$ 1.5.	1 point	
The equation $2 + (n - 1) \cdot 1{,}5 = 80$ is to be solved.	1 point	
Here $n - 1 = 52$,	1 point	
and so 80 is the 53rd term of the sequence.	1 point	
Total:	**4 points**	

8.		
The monthly numbers of customers form a geometric sequence. The first term is 3400, the common ratio is 1.07.	1 point	*These 2 points are also due if the correct reasoning is reflected only by the solution.*
The 13^{th} term of the sequence is to be calculated.	1 point	
In January, 2020 a total $3400 \cdot 1.07^{12} \approx$	1 point	
≈ 7700 customers visited the site (with appropriate rounding).	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**4 points**	

9.		
A: false B: true C: false	2 points	*Award 1 point for two correct answers, 0 points for one correct answer.*
Total:	**2 points**	

10.		
$\frac{6}{25} = 0.24$	2 points	
Total:	**2 points**	

11.		
For example 360°.	2 points	
Total:	**2 points**	

12.		
A total of $6 \cdot 75$ points must be scored over the six games.	1 point	$\frac{77+60+83+73+90+x}{6} = 75$
The number of points they have to score on the sixth game must then be $6 \cdot 75 - (77 + 60 + 83 + 73 + 90) =$	1 point	
$= 67$.	1 point	
Total:	**3 points**	

II. A

13. a)		
The zeroes are: $x = -1$ and $x = 3$.	2 points	
The maximum is at $x = 1$,	1 point	
the maximum value is $f(1) = 2$.	1 point	
The range is [–3; 2].	2 points	
Total:	**6 points**	

13. b)		
$m = -1$	2 points	
$b = 3$	1 point	
Total:	**3 points**	

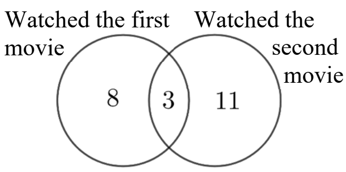
13. c)		
The solutions of the inequality are: $-2 \le x < 0$,	2 points	
and $2 < x \le 6$.	2 points	
Total:	**4 points**	

14. a)		
Multiply both side of the equation by the common denominator of the fractions: $6(x-2) + 6(x-3) = 5(x-3)(x-2)$.	1 point	
Distribute and rearrange: $5x^2 - 37x + 60 = 0$.	2 points	
The solutions of the quadratic equation: $x_1 = 5$ and $x_2 = 2.4$.	2 points	
Check by substitution or by reference to equivalent steps.	1 point	
Total:	**6 points**	

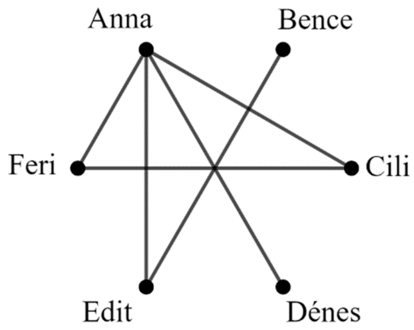
14. b)		
(Use the identities of powers:) $7^2 \cdot 7^x - 7 \cdot 7^x = 2058$.	1 point	
$42 \cdot 7^x = 2058$	1 point	
$7^x = 49$ $(= 7^2)$	1 point	
(As the exponential function is a one-to-one mapping:) $x = 2$.	1 point	
Check by substitution or by reference to equivalent steps.	1 point	
Total:	**5 points**	

15. a) **Solution 1**		
(Without considering the order) there are $\binom{13}{2}$ ways to select 2 girls out of 13 (this is the number of favourable cases).	1 point	*If the order is considered the number of favourable cases is* $13 \cdot 12$,
There are $\binom{32}{2}$ ways to select 2 out of the 32 students (this is the total number of cases).	1 point	*the total number of cases is* $32 \cdot 31$.
The probability: $\dfrac{\binom{13}{2}}{\binom{32}{2}} =$	1 point	$\dfrac{13 \cdot 12}{32 \cdot 31} =$
$= \dfrac{78}{496} \approx 0.157.$	1 point	
Total:	**4 points**	

15. a) **Solution 2**		
The probability of first selecting a girl is $\dfrac{13}{32}$.	1 point	
The probability of selecting a girl for the second time, too (assuming we have already selected a girl first), is $\dfrac{12}{31}$.	1 point	
The probability: $\dfrac{13}{32} \cdot \dfrac{12}{31} =$	1 point	
$= \dfrac{156}{992} \approx 0.157.$	1 point	
Total:	**4 points**	

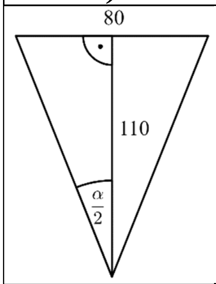
15. b)		
The number of students who watched the first movie but not the second is $11 - 3 = 8$.	1 point*	
The number of students who watched the second movie but not the first is $14 - 3 = 11$.	1 point*	
The number of students who watched at least one movie out of the first two is $8 + 11 + 3 = 22$.	1 point*	
The number of students who watched the third movie only is $32 - 22 = 10$.	1 point	
Total:	**4 points**	

*Note: The three points marked by * may also be given for applying the inclusion-exclusion principle: 11 + 14 – 3 = 22 students watched at least one of the first two movies.*

15. c)		
	2 points	
The maximum number of “friends” acquaintances (pairs) is $\binom{6}{2}=15$. Six of these are already present.	1 point	*Award this point if the candidate gives the correct answer by drawing the missing edges, thereby creating a complete graph.*
There are a further (15 – 6 =) 9 pairs that are not yet friends.	1 point	
Total:	**4 points**	

II. B

16. a)		
The radius of both the base circle of the cone and the cylinder is 40 cm (4 dm).	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The volume of the tank (cone and cylinder combined): $V=\frac{40^2\cdot\pi\cdot110}{3}+40^2\cdot\pi\cdot120\approx$	2 points	$V_{cylinder}\approx 603\ 186\ cm^3$ $V_{cone}\approx 184\ 307\ cm^3$
$\approx 787\ 493\ cm^3$ ($\approx 787\ dm^3$),	1 point	
i.e. the maximum holding capacity of the tank is 787 litres.	1 point	
Total:	**5 points**	

16. b)		
 Correct diagram, let α be the vertical angle.	1 point	*This point is also due if the correct answer is given without a diagram.*
$\tan\frac{\alpha}{2} = \frac{40}{110} \approx 0.3636$	1 point	
$\frac{\alpha}{2} \approx 20°$,	1 point	
the vertical angle of the cone is about 40°.	1 point	
Total:	**4 points**	

16. c)		
The probability that a tank is faultless is 0.92.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The probability that all 10 tanks are faultless is $0.92^{10} \approx 0.434$.	1 point	
The probability that exactly 1 out of 10 tanks is faulty: $\binom{10}{1} \cdot 0.92^9 \cdot 0.08^1 \approx 0.378$.	2 points	
The final probability is the sum of the above, about 0.812.	1 point	
Total:	**5 points**	

16. d) Solution 1		
At company M the (absolute) difference between the respective monthly wages and the average salary is consistently higher for each worker than that at company A.	2 points	
So, the standard deviation of the monthly wages is higher at company M.	1 point	
Total:	**3 points**	

16. d) Solution 2		
The standard deviation of the wages at company M is $\sqrt{\frac{120^2+3\cdot 40^2}{4}}$, that is, about 69.28 (thousand Ft).	1 point	
The standard deviation of the wages at company A is $\sqrt{\frac{60^2+3\cdot 20^2}{4}}$, that is, about 34.64 (thousand Ft).	1 point	
So, the standard deviation of the monthly wages is higher at company M.	1 point	
Total:	**3 points**	

17. a) Solution 1		
Let h be the thickness of the sheet (in cm). The volume of one sheet (in cm^3) is then $V = 21\cdot 29.7\cdot h$.	1 point	
Density is the product of mass and volume: $0.8=\frac{5}{V}=\frac{5}{21\cdot 29.7\cdot h}$,	1 point	
$h=\frac{5}{21\cdot 29.7\cdot 0.8}\approx 0.010$ cm,	1 point	
this is about 0.1 millimetre.	1 point	
Total:	**4 points**	

17. a) Solution 2		
The volume of a single sheet is the ratio of mass and density: $V=\frac{5}{0.8}=6.25\,(\text{cm}^3)$.	1 point	
Let h be the thickness of the sheet (in cm). The volume of a single sheet is then $6.25 = 21\cdot 29.7\cdot h$.	1 point	
$h=\frac{6.25}{21\cdot 29.7}\approx 0.010$ cm,	1 point	
this is about 0.1 millimetre.	1 point	
Total:	**4 points**	

17. b)		
The longer side of the magnified image will be 297 mm.	1 point	*The ratio of similarity is* $\frac{297}{150}=1.98$.
The shorter side will be $\frac{2}{3}\cdot 297=198$ mm.	1 point	*The length of the shorter side is* $1.98\cdot 100 = 198$ mm.
The width of the strips (parallel to the longer side) will be $\frac{210-198}{2}=6$ mm.	2 points	
Total:	**4 points**	

17. c)		
The shorter side of the magnified image will be 210 mm.	1 point	*The ratio of similarity is* $\frac{210}{100}=2.1$.
The longer side will be $\frac{3}{2}\cdot 210=315$ mm.	1 point	*The length of the longer side is* $2.1\cdot 150 = 315$ mm.
The part missing from the magnified image will altogether be $210\times(315-297)=210\times 18$ mm.	1 point	
This is $\frac{18}{315}\cdot 100\approx$	1 point	$\frac{210\cdot 18}{210\cdot 315}\cdot 100$
≈ 5.7% of the area of the magnified image (and this ratio is also the same on the original image).	1 point	
Total:	**5 points**	

17. d)		
Balázs paid $51\cdot 49=2499$ Ft for the 51 photos.	1 point	
As Hajni ordered fewer photos, she paid 59 Ft per photo, so the minimum number of Hajni's photos would be $\frac{2499}{59}\approx 42.4$.	1 point	
Hajni ordered a minimum of 43, maximum 50 photos.	2 points	
Total:	**4 points**	

18. a)		
Rearranging the equation of the circle: $(x-1)^2+(y-2)^2=20$,	2 points	
The coordinates of the centre of the circle are (1; 2).	1 point	
The radius of the circle is $r=\sqrt{20}$ (≈ 4.47).	1 point	
Total:	**4 points**	

18. b)		
Plug $x = 3$ into the equation of circle k: $y^2-4y-12=0$.	1 point	$(y-2)^2=16$
The positive solution of this equation is $y = 6$ (the negative is $y = -2$).	2 points	
(As point A is on circle k the tangent line is perpendicular to the radius drawn to the point of tangency, so) one normal vector of the tangent line is $\overrightarrow{KA}(2; 4)$.	2 points	
The equation of the tangent is $2x + 4y = 30$.	2 points	$x + 2y = 15$
Total:	**7 points**	

18. c) **Solution 1**		
Colouring with 4 colours will yield 4! = 24 possible options.	1 point	
Colouring with 3 colours gives 4 different possibilities to select these colours. (Let, for example, the selected colours be red, yellow and blue.)	1 point	
Region *A* may be filled in 3 different colours, while *B* may be filled in two colours. (Let, for example, these be red for *A* and yellow for *B*.)	1 point	*One of the three colours must be used twice. This means 3 different options. The other two colours must be used on opposite regions, this gives 2 options.*
Regions *C* and *D* may be coloured in 2 different ways (if *C* is red, then *D* is blue, if *C* is blue, then *D* is yellow).	1 point	*There are 2 different options to colour the remaining two regions with the remaining two colours.*
Three colours will, therefore, give $4 \cdot 3 \cdot 2 \cdot 2 = 48$ possible options.	1 point	
The total number of option is then (24 + 48 =) 72.	1 point	
Total:	**6 points**	

18. c) **Solution 2**		
Region *A* may be filled in 4 different colours, while *B* may be filled in 3 colours,	1 point	
this is $4 \cdot 3 = 12$ options for the first two regions.	1 point	
(Let, for example, *A* be red and *B* yellow.) There are two possibilities now: if region *D* is of the same colour as region *B* then *C* can be of two different colours (blue or green in the example).	1 point	
If *B* and *D* are of different colours, then *C* and *D* may be filled in 4 different ways. (If *D* is blue then *C* is red or green, if *D* is green then *C* is red or blue.)	1 point	
To a particular colouring of *A* and *B* there belongs (2 + 4 =) 6 different options to fill *C* and *D*.	1 point	
The total number of options is then $12 \cdot 6 = 72$.	1 point	
Total:	**6 points**	