ÉRETTSÉGI VIZSGA • 2020. május 5.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
8184	2 points	
Total:	**2 points**	

Note: Award 1 point for substituting the data into the appropriate formula.

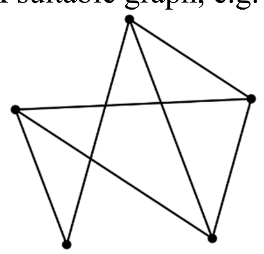
2.		
20 (°C)	2 points	*Not to be divided.*
Total:	**2 points**	

3.		
Set B contains 1 element that is not an element of set A,	1 point	*Award these 2 points for a Venn-diagram that fits all the given conditions.*
and another 2 elements that are elements of set A.	1 point	
Therefore, the cardinality of set B is 3.	1 point	
Total:	**3 points**	

Note: Award maximum score if the candidate correctly answers the question using a concrete example.

4.		
(8! =) 40320	2 points	
Total:	**2 points**	

Note: Award 1 point for 8! .

5.		
A suitable graph, e.g.  (The degrees of the vertices are 2, 3, 3, 3, 3.)	2 points	*A graph that is not a simple graph is also acceptable.*
Total:	**2 points**	

6.		
$\frac{3}{10}$	2 points	
Total:	**2 points**	

7.		
A) false B) true C) true	2 points	*Award 1 point for two correct answers, 0 points for one correct answer.*
Total:	**2 points**	

8.		
15 + 28 + 34 = 77% of all participants spends no more than 3 hours using computers.	1 point	
$1200 \cdot 0.77 =$	1 point	
= 924 (is the number of people spending no more than 3 hours per day using computers.)	1 point	
Total:	**3 points**	

9.		
$2x - 5y = 3$	2 points	
Total:	**2 points**	

10.		
$a_4 = a_1 + 3d = 72$ és $a_6 = a_1 + 5d = 64$	1 point	*The difference of the sixth and fourth terms is 2d = –8.*
Solving the system by subtracting the equations from one another or expressing a variable from one of the equations and substituting it into the other.	1 point	
$d = -4$	1 point	
$a_1 = 84$	1 point	
Total:	**4 points**	

Note: Award maximum score if the candidate answers the question by correctly listing the first six terms of the progression.

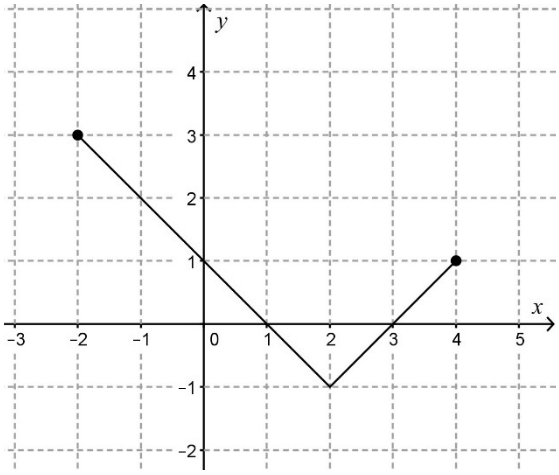
11.		
$x = \frac{3\pi}{4}$	2 points	
Total:	**2 points**	

Note: Award 1 point if the candidate gives the (correct) answer in degrees, or does not restrict the (correct) solution to the given interval.

12.		
Triangles *ABC* and *ADE* are similar (as the pairs of corresponding angles are congruent).	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$\frac{x}{1.5} = \frac{x+7}{5}$	1 point	
$5x = 1.5x + 10.5$	1 point	
The length of segment *AC* is: $x = 3$.	1 point	
Total:	**4 points**	

II. A

13. a)		
$f(-1) = 2$	2 points	*Award 1 point for* $f(-1) = \lvert -1-2 \rvert - 1$
Total:	**2 points**	

13. b)		
	3 points	*The graph drawn is the graph of* $x \mapsto \lvert x \rvert$ *shifted (1 point), the minimum is at (2; –1) (1 point), and the domain is restricted to the given interval (1 point).*
The function is (strictly) monotone decreasing if $(-2 \le)\, x \le 2$,	1 point	*Open intervals and other correct notations are also acceptable.*
(strictly) monotone increasing if $2 \le x\, (\le 4)$.	1 point	
The function has a (global) maximum $x = -2$, the maximum value is 3,	1 point	*Award 1 point only, if the candidate gives the coordinates of the points corresponding to the extremes, rather than giving their places and values.*
the (local and global) minimum is at $x = 2$, the minimum value is -1.	1 point	
The zeros of the function are at $x = 1$ and $x = 3$.	1 point	
The range of the function is [–1; 3].	2 points	*The correct answer is acceptable in any other form, too.*
Total:	**10 points**	

14. a)		
The common denominator is $2(x + 2)$.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
(Expanding the fractions and multiplying by the common denominator) $x+2+(x-2)\cdot 2=2x+1$.	1 point	
Rearranging the equation: $3x - 2 = 2x + 1$.	1 point	
$x = 3$	1 point	
Checking by substitution or by reference to equivalent steps and the condition $x \neq -2$.	1 point	
Total:	**5 points**	

14. b)		
$\log_3 81 = 4$	1 point	$\log_3 81(x^2-1)=5$
$\log_3(x^2-1)=1$	1 point	$\log_3 81(x^2-1)=\log_3 243$
$x^2-1=3$	1 point	
The solutions of the equation: $x_1=2$, $x_2=-2$.	2 points	
Checking by substitution or by reference to equivalent steps and the condition $x^2-1>0$.	1 point	
Total:	**6 points**	

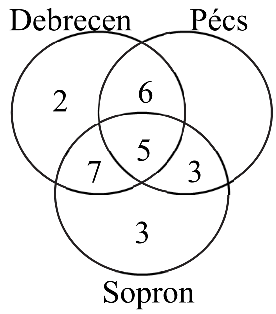
15. a)		
Let x be the number of tickets sold to sector *D*, zone 1. The total number of tickets sold to zone 1 is then $69 + 96 + 85 + x = 250 + x$.	1 point	*The total number of tickets sold to zone 1 is,* $4 \cdot 82 = 328$,
$\frac{250 + x}{4} = 82$	1 point	*250 of which were sold to sectors A, B, and C.*
The number of tickets sold to sector *D*, zone 1 is (x =) 78.	1 point	
Total:	**3 points**	

15. b)		
A total of 595 tickets were sold to sectors *A* and *B*.	1 point	
The other (1102 – 595 =) 507 tickets were sold to sectors *C* and *D*.	1 point	
The probability of a randomly selected spectator holding a ticket to sectors *C* or *D* is $\frac{507}{1102}$ (≈ 0.46).	1 point	
Total:	**3 points**	

15. c)		
Let y be the number of tickets sold to sector *C* zone 2, and let z be the number of tickets sold to sector *C*, zone 3. In this case $85 + y + z = 295$,	1 point	
and also, based on ticket prices, $85 \cdot 3200 + y \cdot 2900 + z \cdot 1500 = 752\,200$.	1 point	
Solving the equation system by substitution (or any other method):	1 point	
$y = 118$, $z = 92$.	2 points	
Check against the text of the original problem.	1 point	
118 tickets were sold to sector *C*, zone 2, while 92 tickets were sold to sector *C*, zone 3.	1 point	
Total:	**7 points**	

II. B

16. a)

Starting with the 5 students who would visit any of the three places, one can find the number of those willing to visit exactly two towns: 6 students to Pécs or Debrecen, 7 students to Debrecen or Sopron, 3 students to Pécs or Sopron.	2 points	*Venn-diagram based on the text:* 
There are 2 students who would go to Debrecen only,	1 point	
and 3, who would go to Sopron only.	1 point	
As there are 30 students in the class, the number of those willing to visit Pécs only is 4.	1 point	*There are 2 + 7 + 3 students who would not like to go to Pécs.*
The total number of students who would like to visit Pécs is (5 + 6 + 3 + 4 =) 18.	1 point	
Total:	**6 points**	

16. b)

By connecting the centres a regular triangle of side 3 cm is obtained.	1 point	*Award this point for a diagram, too, as long as it contains the same information.*
The area of the regular triangle is $A_{triangle} = \frac{3^2 \cdot \sqrt{3}}{4}$ (≈ 3.90 cm^2).	1 point	
The rest of the area (outside this triangle) consists of three congruent segments, each with a 3-cm radius and a 60° central angle.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The area of a sector of radius 3 cm and central angle 60° is $A_{sector} = \frac{3^2 \cdot \pi \cdot 60}{360}$ (≈ 4.71 cm^2).	1 point	
$A_{segment} = A_{sector} - A_{triangle}$ (≈ 4.71 – 3.90 ≈ 0.81 cm^2)	1 point	*The total area is:* $A = 3 \cdot A_{sector} - 2 \cdot A_{triangle}$
The total area is: $A = A_{triangle} + 3 \cdot A_{segment}$ ≈ 6.33 cm^2.	1 point	
Total:	**6 points**	

16. c)		
There are $\binom{30}{3}$ (= 4060) different ways to select three students out of 30.	1 point	
Select two students out of those 20 who marked Debrecen as a possible destination. This can be done in $\binom{20}{2}$ (= 190) different ways.	1 point	*These 2 points are also due if the correct reasoning is reflected only by the solution.*
The third student will be selected from the remaining 10, which can be done in 10 different ways.	1 point	
The number of favourable cases: $\binom{20}{2}\cdot 10$ (= 1900).	1 point	
The probability: $\frac{1900}{4060}\approx 0.47$.	1 point	
Total:	**5 points**	

17. a) **Solution 1**		
(Let γ be the interior angle at vertex *C* of triangle *ABC*. Apply the law of sines) $\frac{37}{41}=\frac{\sin\gamma}{\sin 60°}$.	1 point	
(As $\sin\gamma\approx 0.7815$) $\gamma_1\approx 51.4°$.	1 point	
$\gamma_2\approx 128.6°$, but this angle is not a possible solution (as 60° + 128.6° > 180°).	1 point	
The measure of the interior angle at vertex *B* of the triangle is approximately 68.6°.	1 point	
(Applying the law of sines:) $\frac{AC}{41}=\frac{\sin 68.6°}{\sin 60°}$.	1 point	*With the law of cosines:* $AC^2=37^2+41^2-2\cdot 37\cdot 41\cdot\cos 68.6°$
$AC\approx 44$ (units).	1 point	
The perimeter of the triangle is approximately 122 (units).	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**7 points**	

17. a) **Solution 2**		
(Let x be the length of side AC of triangle ABC. Apply the law of cosines.) $41^2 = x^2 + 37^2 - 2 \cdot x \cdot 37 \cdot \cos 60°$.	2 points	
Rearranged: $x^2 - 37x - 312 = 0$.	1 point	
$x_1 \approx -7$	1 point	
This may not be the length of a side in a triangle.	1 point	
$x_2 \approx 44$ (units)	1 point	
The perimeter of the triangle is approximately 122 (units).	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**7 points**	

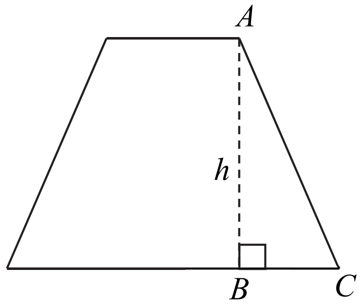
17. b)		
$\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AC} + \overrightarrow{CE}$	1 point	$\overrightarrow{BE} = \overrightarrow{BC} + \overrightarrow{CE}$ *This point is also due if the correct reasoning is reflected only by the solution.*
$\overrightarrow{BA} = -\overrightarrow{AB}$	1 point	$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$
$\overrightarrow{CE} = 2 \cdot \overrightarrow{CD}$	1 point	
Therefore $\overrightarrow{BE} = -\overrightarrow{AB} + \overrightarrow{AC} + 2 \cdot \overrightarrow{CD}$.	1 point	
Total:	**4 points**	

17. c)		
There is one possible route from point A to point B,	1 point	
two different routes to points C and D.	1 point	
From point A to point E: 1 + 2 = 3 routes,	1 point	
to point F: 1 + 2 + 2 = 5 routes.	1 point	
There are routes leading to point G from D, E, or F.	1 point	
Therefore, the number of different possible routes from point A to point G is 2 + 3 + 5 = 10.	1 point	
Total:	**6 points**	

Note: There are ten different possible routes: ABCDEG, ABCDFG, ABCDG, ABCFG, ABEG, ACDEG, ACDFG, ACDG, ACFG, AFG. In case the candidate lists different possible routes, use the table and note below to determine the number of points awarded.

Number of correctly listed routes	*Points awarded*
10	*6*
8-9	*5*
6-7	*4*
4-5	*3*
2-3	*2*
1	*1*

Deduct 1 point for each incorrect route or for any route listed multiple times (while keeping in mind that the total score given may not be negative!)

18. a)		
The shape of the tea in the can is a truncated cone, such that the diameter of the top circle is the arithmetic mean of the diameters of the base and top circles of the teapot,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
that is: 13 cm.	1 point	
(Data needed to calculate the volume of tea in the pot:) radius of base circle: 9 cm, radius of top circle: 6.5 cm,	1 point	*Award this point only if the candidate calculated both radii correctly from the diameters.*
the length of the slant height is 7 cm.	1 point	
(the cross section along the axis, where h is the height of the tea:)  In this diagram the distance BC is equal to the difference of the radii: 2.5 cm.	1 point	
(Apply the Pythagorean Theorem:) $h=\sqrt{AC^2-BC^2}=\sqrt{7^2-2.5^2}$ (≈ 6.54 cm).	1 point	
The volume of tea is approximately $\frac{6.54\cdot\pi}{3}\cdot(9^2+6.5^2+9\cdot 6.5)\approx$	1 point	
≈ 1245 (cm^3).	1 point	
The amount of tea in the can is approximately 12.5 dl.	1 point	*A different answer may be accepted as long as it is rounded reasonably and correctly.*
Total:	**9 points**	

Note: Award 5 points if the candidate correctly calculates the volume of the teapot (approximately 18 dl).

18. b)		
A quarter of an hour is 15 minutes.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$T_{\text{tea}}(15) = 23 + 56 \cdot 0.96^{15} \approx$	1 point	
the temperature of the tea is ≈ 53.4 (°C).	1 point	
Total:	**3 points**	

18. c)		
The exponential equation $37 = 23 + 56 \cdot 0.96^{t}$ is to be solved.	1 point	
Rearrange: $0.25 = 0.96^{t}$	1 point	
$\log 0.25 = \log 0.96^{t}$	1 point	$t = \log_{0.96} 0.25$
$\log 0.25 = t \cdot \log 0.96$	1 point	
It will take $t \approx 34$ minutes for the tea to cool to 37°C.	1 point	
Total:	**5 points**	

Note: Award the maximum score if the candidate (rounding reasonably and correctly) calculates the temperature of the tea minute-by-minute and gives the correct answer henceforth.