

**ÉRETTSÉGI VIZSGA • 2019. október 15.**

**MATEMATIKA  
ANGOL NYELVEN**

**KÖZÉPSZINTŰ  
ÍRÁSBELI VIZSGA**

**JAVÍTÁSI-ÉRTÉKELÉSI  
ÚTMUTATÓ**

**EMBERI ERŐFORRÁSOK MINISZTERIUMA**

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## Instructions to examiners

### Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
  - correct calculation: *checkmark*
  - principal error: *double underline*
  - calculation error or other, not principal, error: *single underline*
  - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
  - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
  - unintelligible part: *question mark and/or wave*
6. Do not assess anything written **in pencil**, except for diagrams

### Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**:  
addition, subtraction, multiplication, division, calculating powers and roots,  $n!$ ,  $\binom{n}{k}$ , replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers  $\pi$  and  $e$ , finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

**I.**

<b>1.</b>		
A proper graph.	2 points	<i>A graph that is not a simple graph may also be accepted.</i>
<b>Total:</b>	<b>2 points</b>	

<b>2.</b>		
{}, {x}, {y}, {z}, {x, y}, {x, z}, {y, z}, {x, y, z}	3 points	
<b>Total:</b>	<b>3 points</b>	

*Note: Deduce 1 point (a maximum of 3) for each missing or incorrect answer.*

<b>3.</b>		
12	2 points	<i>Accept <math>b^{12}</math> as an answer.</i>
<b>Total:</b>	<b>2 points</b>	

<b>4.</b>		
By 35 percent.	2 points	
<b>Total:</b>	<b>2 points</b>	

<b>5.</b>		
A correct example, such as 25.	2 points	<i>Award 1 point only, if the candidate gives an answer that is relatively prime to 6 but not a composite number.</i>
<b>Total:</b>	<b>2 points</b>	

<b>6.</b>		
A, C	2 points	<i>Award 1 point for one correct or two correct and an incorrect answers, 0 point in all other cases.</i>
<b>Total:</b>	<b>2 points</b>	

<b>7.</b>		
24	2 points	
<b>Total:</b>	<b>2 points</b>	

<b>8.</b>		
(Apply the Law of Cosines to find the length of side AC: $\sqrt{2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cdot (-0.5)} = \sqrt{19} \approx 4.36$ (units).	2 points	
<b>Total:</b>	<b>2 points</b>	

<b>9.</b>		
The gradient of the line is $-0.4$ .	2 points	
<b>Total:</b>	<b>2 points</b>	

<b>10. Solution 1</b>		
19 litres = $19\,000\text{ cm}^3$	1 point	
The base area of the tank is $1000\text{ cm}^2$ .	1 point	
$19\,000 = 1000 \cdot m$ , from where $m = 19\text{ cm}$ . This is how high the water fills the tank.	1 point	
The water level will be $(25 - 19 =) 6$ centimetres below the top of the tank.	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>10. Solution 2</b>		
The base area of the tank is $1000\text{ cm}^2$ ,	1 point	$10\text{ dm}^2$
pouring 1 litre = $1000\text{ cm}^3$ of water into the tank will raise the water level by 1 cm.	2 points	1 litre = $1\text{ dm}^3$ raises the water level by 0.1 dm.
After pouring 19 litres of water into the tank the water level will be $(25 - 19 =) 6\text{ cm}$ below the top of the tank.	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>11. Solution 1</b>		
There are $3 \cdot 4 = 12$ different ways to select a pair of numbers from the two sets (this is the total number of cases).	1 point	
The product will be negative if one of the selected numbers is positive and the other is negative.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
The appropriate pairs: $(-13; 1)$ , $(-13; 4)$ , $(-5; 1)$ , $(-5; 4)$ and $(29; -17)$ , there are 5 favourable cases.	1 point	
The probability: $\frac{5}{12}$ ( $\approx 0.417$ ).	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>11. Solution 2</b>		
The product will be negative if one of the selected numbers is positive and the other is negative.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
The probability that a negative number is selected from set $A$ and a positive from set $B$ is $\frac{2}{3} \cdot \frac{2}{4}$ .	1 point	
The probability that a positive number is selected from set $A$ and a negative from set $B$ is $\frac{1}{3} \cdot \frac{1}{4}$ .	1 point	
The final probability is the sum of the above: $\frac{5}{12}$ .	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>12.</b>		
The mean of the grades is 4.	1 point	
The standard deviation: $\left( \sqrt{\frac{1 \cdot 2^2 + 2 \cdot 1^2 + 1 \cdot 0^2 + 4 \cdot 1^2}{8}} \right) \frac{\sqrt{5}}{2} = \sqrt{1.25} \approx 1.12.$	2 points	
<b>Total:</b>	<b>3 points</b>	

## II. A

<b>13. a)</b>		
$f\left(-\frac{3}{4}\right) = \left(-\frac{1}{2}\right) \cdot \left(-\frac{3}{4}\right) + 4 =$	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
$= \frac{35}{8} = 4.375$	1 point	
<b>Total:</b>	<b>2 points</b>	

<b>13. b)</b>		
The graph is part of a line of $-0.5$ gradient,	1 point	
the $y$ -intercept is at 4.	1 point	
The candidate correctly applies the restricted domain.	1 point	
The range of the function is $[2; 5]$ .	2 points	<i>The correct answer is acceptable in any other form, too.</i>
<b>Total:</b>	<b>5 points</b>	

<b>13. c) Solution 1</b>		
The equation $x^2 - 4x + 3 = -\frac{3}{4}$ is to be solved.	1 point	
The solutions are: $x_1 = 1.5; x_2 = 2.5$ .	2 points	<i>The discriminant of the equation <math>x^2 - 4x + 3,75 = 0</math> is positive (<math>D = 1</math>).</i>
There are two numbers to which the function $g$ assigns $\left(-\frac{3}{4}\right)$ .	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>13. c) Solution 2</b>		
As $x^2 - 4x + 3 = (x - 2)^2 - 1$ ,	1 point	
the graph of function $g$ is an upright parabola with its vertex at $(2; -1)$ .	1 point	<i>Award these 2 points for correctly drawing the graph function <math>g</math> and the line <math>y = -\frac{3}{4}</math>.</i>
The function assumes any value greater than $-1$ at two different places.	1 point	
There are two numbers to which the function $g$ assigns $\left(-\frac{3}{4}\right)$ .	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>14. a) Solution 1</b>		
$1.5 \text{ seconds} = 1.5 \cdot \frac{1}{3600} \text{ hours.}$	1 point	
Driving at 120 km/h, the car covered a distance of $120 \cdot 1.5 \cdot \frac{1}{3600} =$	1 point	
$= 0.05 \text{ kilometres during this time.}$	1 point	
That is 50 metres.	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>14. a) Solution 2</b>		
$120 \text{ km/h} = 33 \frac{1}{3} \text{ m/s}$	2 points	
Driving at this speed for 1.5 seconds, the car will cover a distance of $1.5 \cdot 33 \frac{1}{3} =$	1 point	
$= 50 \text{ metres.}$	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>14. a) Solution 3</b>		
An hour is 3600 seconds, 1.5 seconds is one 2400 <sup>th</sup> part of this.	2 points	
Driving at 120 km/h for this long the car will cover $120\,000 : 2400 =$	1 point	
$= 50$ metres.	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>14. b)</b>		
Driving at an average speed of 120 km/h, a car will cover a distance of 100 km in $\frac{100}{120}$ hours, while driving at 130 km/h the same distance will take $\frac{100}{130}$ hours.	2 points	
At the lower speed it will take 50 minutes, at the higher speed it will take about 46 minutes.	1 point	$\frac{100}{120} - \frac{100}{130} \approx 0.064$ hours
It will take about 4 minutes less to travel 100 km at the higher average speed.	1 point	$0.064 \cdot 60 = 3.84$ minutes
<b>Total:</b>	<b>4 points</b>	

<b>14. c)</b>		
The central angle that corresponds to one accident in the pie chart is $360 : 1178 \approx 0.3056^\circ$ .	1 point	$440 : 1178 \approx 0.3735$
The total is 440 times this,	1 point	$0.3735 \cdot 360$
that is $\approx 134.5^\circ$ .	1 point	
<b>Total:</b>	<b>3 points</b>	

<b>15. a) Solution 1</b>		
(Let $a_1$ be the first term of the arithmetic sequence and let $d$ be its difference.) The following equation system is to be solved: $\left. \begin{array}{l} 2a_1 + 2d = 8 \\ 3a_1 + 9d = 9 \end{array} \right\}$	2 points	
Express $a_1$ (or $d$ ) from one of the equations: $a_1 = \frac{8 - 2d}{2} = 4 - d$ .	1 point*	$d = 4 - a_1$
Substitute it into the other equation: $12 - 3d + 9d = 9$ , that is $6d = -3$ .	1 point*	$3a_1 + 36 - 9a_1 = 9$ $-6a_1 = -27$
The solution of the system is $a_1 = 4.5$ , $d = -0.5$ .	1 point	
The sum of the first ten terms: $S_{10} = \frac{2 \cdot 4.5 + 9 \cdot (-0.5)}{2} \cdot 10 = 22.5$ .	2 points	$4.5 + 4 + 3.5 + \dots + 0 =$ $= 22.5$
<b>Total:</b>	<b>7 points</b>	



Note: The 2 points marked by \* may also be given for the following reasoning:

Multiply the first equation by 3, the second by 2: $\left. \begin{array}{l} 6a_1 + 6d = 24 \\ 6a_1 + 18d = 18 \end{array} \right\}$	1 point	
Subtract the two equations: $-12d = 6.$	1 point	

**15. a) Solution 2**

As per the properties of arithmetic sequences: $a_2 = \frac{a_1 + a_3}{2} = 4$ and $a_4 = \frac{a_3 + a_4 + a_5}{3} = 3.$	2 points	
The common difference is: $d = \frac{a_4 - a_2}{2} = -0.5,$	1 point	
the first term: $a_1 = a_2 - d = 4.5.$	1 point	
The tenth term: $a_{10} = a_1 + 9d = 0.$	1 point	
The sum of the first ten terms: $S_{10} = \frac{4.5 + 0}{2} \cdot 10 = 22.5.$	2 points	
<b>Total:</b>	<b>7 points</b>	

**15. b)**

Let $c$ be the length of the hypotenuse of the triangle. The two legs are then $a = c - 8$ and $b = c - 9.$	1 point	<i>Let <math>b</math> be the length of the shorter leg. The longer leg is then <math>a = b + 1</math>, the hypotenuse is <math>c = b + 9.</math></i>
Apply the Pythagorean Theorem: $(c - 8)^2 + (c - 9)^2 = c^2.$	1 point	$(b + 1)^2 + b^2 = (b + 9)^2$
Execute the operations: $c^2 - 16c + 64 + c^2 - 18c + 81 = c^2.$	1 point	$b^2 + 2b + 1 + b^2 =$ $= b^2 + 18b + 81$
$c^2 - 34c + 145 = 0$	1 point	$b^2 - 16b - 80 = 0$
The solutions are $c_1 = 5$ and $c_2 = 29.$	1 point	$b_1 = -4, b_2 = 20$
If $c = 5$ were true, the legs would have negative lengths which is impossible.	1 point	$b = -4$ is not a correct solution.
If $c = 29$ then $a = 21$ and $b = 20$ (and this really is a correct solution).	1 point	If $b = 20$ then $a = 21$ and $c = 29.$
<b>Total:</b>	<b>7 points</b>	

Note: Award 1 point if the candidate correctly states the length of the sides of the right triangle but provides no proof. Award a further 1 point if the candidate proves that these are, in fact, the sides of a right triangle.

## II. B

<b>16. a) Solution 1</b>		
There are $4! = 24$ different ways to arrange the four cards in order (total number of cases).	1 point	
Any number could be the first (4 options), but the second should be one of the two numbers with opposite parity. The last two numbers are determined by the first two.	1 point*	1234, 1432, 3214, 3412, 2143, 2341, 4123, 4321
The number of favourable cases is therefore $4 \cdot 2 = 8$ .	1 point	
The probability: $\frac{8}{24} \left( = \frac{1}{3} \right)$ .	1 point	
<b>Total:</b>	<b>4 points</b>	

*Note: Award the point marked by \* if the candidate correctly determines the number of unfavourable cases. Any case is incorrect where two even numbers occupy slots 1 and 2, or 2 and 3, or 3 and 4, or 1 and 4. There are  $2 \cdot 2 = 4$  of each of these and therefore the number of unfavourable cases is  $4 \cdot 4 = 16$ .*

<b>16. a) Solution 2</b>		
There are $\binom{4}{2} = 6$ cases if only the parity of the numbers is taken into account (total number of cases).	2 points	
Two of these are favourable: even-odd-even-odd and odd-even-odd-even.	1 point	
The probability: $\frac{2}{6} \left( = \frac{1}{3} \right)$ .	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>16. a) Solution 3</b>		
Any of the four numbers may be selected to be first.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
The probability that the second number's parity will be different from that of the first is $\frac{2}{3}$ .	1 point	
(Assuming that the parity of the first two numbers are different) the probability that the parity of the third number will be different from that of the second is $\frac{1}{2}$ .	1 point	
(As the parity of the last number will be correct in any case) the probability is $\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$ .	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>16. b) Solution 1</b>		
The height of the pile after $n$ cuts and replacements is $0.1 \cdot 2^n$ (mm),	2 points	
that gives about 105 000 mm for $n = 20$ .	1 point	
This is equal to 105 metres, more than 100, so Luca is right.	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>16. b) Solution 2</b>		
After 10 cuts, the pile will be $2^{10} = 1024$ (that is more than a 1000) times taller.	1 point	
After the first 10 cuts it will be more than 100 mm = 1 dm tall.	1 point	
After the next 10 cuts the height grows by a factor of 1024 again, it will be more than 1000 dm.	1 point	
This is more than 100 m, so Luca is right.	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>16. c) Solution 1</b>		
The ratios of the corresponding sides of similar rectangles are equal.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
One side of rectangle $EFGH$ is $21 - 5 = 16$ cm, the other is $29.7 - 5 = 24.7$ cm.	1 point	
The ratio of segments $EF$ and $AB$ is $\frac{16}{21}$ ( $\approx 0.76$ ).	1 point	
The ratio of segments $FG$ and $BC$ is $\frac{24.7}{29.7}$ ( $\approx 0.83$ ).	1 point	
As the two ratios are different, the rectangles are not similar, so Zsófi is wrong.	1 point	
<b>Total:</b>	<b>5 points</b>	

<b>16. c) Solution 2</b>		
(Because of their parallel sides) the rectangles may only have centro-similarity.	1 point	
(Because of the uniform margin width) the centre of the similarity may only be the common centre of the two rectangles.	1 point	
Line $AE$ , for example, does not pass through this point,	1 point	
as it is not a diagonal line of the rectangle (makes a $45^\circ$ angle with the sides).	1 point	
Since the two rectangles are not similar, Zsófi is wrong.	1 point	
<b>Total:</b>	<b>5 points</b>	

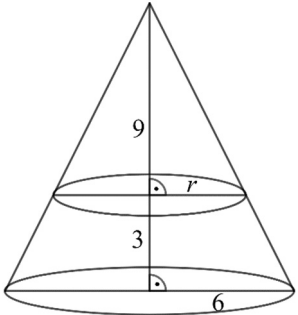
<b>16. d)</b>		
The statement is true.	1 point	
The converse of the statement: <i>If, for two quadrilaterals, the corresponding pairs of angles are congruent then these two quadrilaterals are similar.</i>	1 point	
The converse is false.	1 point	
A counterexample is a square and a (non-square) rectangle.	1 point	<i>Award this point if the candidate refers to the rectangles in part c.</i>
<b>Total:</b>	<b>4 points</b>	

<b>17. a)</b>		
The total surface area of the pyramid is equal to the sum of the areas of the base square and the two pairs of congruent right triangles.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
$A_{ABCD} = 36 \text{ (cm}^2\text{)}$	1 point	
$A_{ABE} = A_{ADE} = \frac{6 \cdot 6}{2} = 18 \text{ (cm}^2\text{)}$	1 point	
(The heights that belong to the 6 cm long sides of triangles $BCE$ and $CDE$ are $EB$ and $ED$ , respectively.) $EB = ED = 6\sqrt{2} \text{ (cm)}$	1 point	
$T_{BCE} = T_{CDE} = \frac{6 \cdot 6\sqrt{2}}{2} = 18\sqrt{2} \text{ (}\approx 25,5 \text{ cm}^2\text{)}$	1 point	
The total surface area: $A = 36 + 2 \cdot 18 + 2 \cdot 18 \cdot \sqrt{2} \approx 122.9 \text{ cm}^2.$	1 point	
<b>Total:</b>	<b>6 points</b>	

<b>17. b) Solution 1</b>		
$\vec{EC} = \vec{EA} + \vec{AD} + \vec{DC}$	1 point	
$\vec{EA} = -\vec{AE}$ and $\vec{DC} = \vec{AB}$	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
$\vec{EC} = -\vec{AE} + \vec{AD} + \vec{AB}$	1 point	
<b>Total:</b>	<b>3 points</b>	

<b>17. b) Solution 2</b>		
$\vec{EC} = \vec{AC} - \vec{AE}$	1 point	
$\vec{AC} = \vec{AB} + \vec{AD}$	1 point	
$\vec{EC} = \vec{AB} + \vec{AD} - \vec{AE}$	1 point	
<b>Total:</b>	<b>3 points</b>	

<b>17. c)</b>		
A diagram, reflecting proper understanding of the problem. Let $\alpha$ be the angle in question.	1 point	
$\tan \alpha = \frac{12}{6}$		
$\alpha \approx 63.4^\circ$	1 point	
<b>Total:</b>	<b>3 points</b>	

<b>17. d) Solution 1</b>		
<p>A diagram, reflecting proper understanding of the problem. Let <math>r</math> be the top radius of the truncated cone.</p> 	1 point	
Because of the similar triangles $\frac{r}{9} = \frac{6}{12}$ ,	1 point	
and so the top radius of the truncated pyramid is 4.5 (cm).	1 point	
The volume of the truncated pyramid: $V = \frac{3 \cdot \pi \cdot (6^2 + 6 \cdot 4.5 + 4.5^2)}{3} =$	1 point	
$= 83.25\pi \text{ cm}^3 \approx 261.5 \text{ cm}^3$ .	1 point	
<b>Total:</b>	<b>5 points</b>	

<b>17. d) Solution 2</b>		
The volume of the original cone: $V_o = \frac{6^2 \cdot \pi \cdot 12}{3} = 144\pi \approx 452.4 \text{ (cm}^3\text{)}.$	1 point	
The smaller cone at the top of the original is similar to the original cone, the ratio of similarity is $\lambda = \frac{9}{12} = \frac{3}{4}.$	1 point	
The ratio of the volume of the smaller cone to that of the original is $\lambda^3 = \frac{27}{64}$ ,	1 point	
and so the volume of the smaller cone is $\lambda^3 \cdot V_o \approx 190.9 \text{ (cm}^3\text{)}.$	1 point	<i>The volume of the truncated cone is <math>1 - \frac{27}{64} = \frac{37}{64}</math> times the volume of the original cone.</i>
The volume of the truncated cone may be obtained by subtracting the volume of the smaller cone from that of the original cone: $V \approx (452.4 - 190.9) = 261.5 \text{ cm}^3$ .	1 point	
<b>Total:</b>	<b>5 points</b>	

<b>18. a) Solution 1</b>		
Let $n$ be the number of single rooms. In this case, the number of double rooms is $3n$ and the number of triple rooms is $65 - 4n$ .	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
Based on the text: $n + 3n \cdot 2 + (65 - 4n) \cdot 3 = 125$ .	2 points	
$195 - 5n = 125$	1 point	
$n = 14$	1 point	
There are $65 - 4n = 9$ triple rooms in the hotel.	1 point	
Check: 14 single rooms, 42 doubles and 9 triples (a total of 65 rooms) will accommodate $14 + 84 + 27 = 125$ guests.	1 point	
<b>Total:</b>	<b>7 points</b>	

<b>18. a) Solution 2</b>		
(Let $e$ be the number of single rooms, let $k$ be the number of double rooms and let $h$ be the number of triple rooms.) Based on the text: $\left. \begin{array}{l} e + k + h = 65 \\ k = 3e \\ e + 2k + 3h = 125 \end{array} \right\}.$	2 points	
Express $h$ from the first equation of the system $\left. \begin{array}{l} 4e + h = 65 \\ 7e + 3h = 125 \end{array} \right\}$ and substitute it into the second equation: $7e + 195 - 12e = 125$ , therefore $-5e = -70$ .	2 points	
The solution of the system is $e = 14, k = 42, h = 9$ .	1 point	
There are 9 triple rooms in the hotel.	1 point	
Check: 14 single rooms, 42 doubles and 9 triples (a total of 65 rooms) will accommodate $14 + 84 + 27 = 125$ guests.	1 point	
<b>Total:</b>	<b>7 points</b>	

<b>18. b) Solution 1</b>		
There are $6!$ ( $=720$ ) ways for the six guests to pick up the keys (total number of cases).	1 point	
Any cases is favourable, as long as Aladár and Balázs picks up the keys for room 102 in any order.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
There are 2 different orders for them to do this.	1 point	
The others can pick up the remaining keys in $4!$ ( $=24$ ) different ways.	1 point	
The number of favourable cases is then $2 \cdot 4!$ ( $=48$ ).	1 point	
The probability: $\frac{48}{720} = \frac{1}{15}$ .	1 point	
<b>Total:</b>	<b>6 points</b>	

<b>18. b) Solution 2</b>		
The person to occupy the single room may be selected in 6 different ways.	1 point*	<i>The three people to occupy the triple room may be selected in <math>\binom{6}{3} = 20</math> different ways.</i>
There are $\binom{5}{2} = 10$ different ways to select the two occupants of the double room out of the remaining 5 people.	1 point*	<i>There are 3 different ways to select the two occupants of the double room out of the remaining 3 people.</i>
The remaining three people will take the triple room. The total number of cases is then $6 \cdot 10 = 60$ .	1 point*	$20 \cdot 3 = 60$
If Aladár and Balázs are to occupy the double room, there are 4 different ways to select the person to get the single room out of the remaining 4 people. The number of favourable cases is therefore 4.	2 points	
The probability: $\frac{4}{60} = \frac{1}{15}$ .	1 point	
<b>Total:</b>	<b>6 points</b>	

*Note: The 3 points marked by \* may also be given for the following reasoning:*

Arrange the six people in a row and assign a key to each of them.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
Calculate the number of different possible orders for a single 101, two 102, and three 103 keys (permutations with repeat).	1 point	
The total number of cases is $\frac{6!}{2! \cdot 3!} = 60$ .	1 point	



<b>18. b) Solution 3</b>		
Examine the probability that the two keys for room 102 will be given to Aladár and Balázs (the assignment of the rest of the keys is arbitrary).	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
There are $\binom{6}{2} = 15$ ways to select two keys out of six (total number of cases).	2 points	
The only favourable case is when Aladár and Balázs are given the two keys for room 102.	2 points	
The probability: $\frac{1}{15}$ .	1 point	
<b>Total:</b>	<b>6 points</b>	

<b>18. c) Solution 1</b>		
The probability that an arbitrary plate will not be broken by the waiters is $\frac{1999}{2000} = 0.9995$ .	1 point	
$P(\text{at least one plate will be broken}) =$ $= 1 - P(\text{no plate will be broken}) =$	1 point	
$= 1 - 0.9995^{150} \approx$	1 point	
$\approx 1 - 0.928 = 0.072$	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>18. c) Solution 2</b>		
The probability of the waiters breaking exactly one plate is $P(1) = \binom{150}{1} \cdot 0.0005 \cdot 0.9995^{149} \approx 0.0696$ .	1 point	
The probability of the waiters breaking exactly two plates is $P(2) = \binom{150}{2} \cdot 0.0005^2 \cdot 0.9995^{148} \approx 0.0026$ .	1 point	
$P(3) \approx 0.00006$ Compared to the answer given to the original problem the probability of breaking 3 or more plates is negligibly small.	1 point	
The final probability is the sum of the first two probabilities: 0.072.	1 point	
<b>Total:</b>	<b>4 points</b>	