ÉRETTSÉGI VIZSGA • 2019. május 7.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
$x_1 = 1$, $x_2 = -2$	2 points	
Total:	**2 points**	

2.		
3	2 points	
Total:	**2 points**	

3.		
$x = 4$	2 points	
Total:	**2 points**	

4.		
$V = 1000\ \text{cm}^3$	1 point	$V = 1\ \text{dm}^3$
$r^2\pi \cdot 20 = 1000$ ($r > 0$)	1 point	$r^2\pi \cdot 2 = 1$
$r^2 \approx 15.9$	1 point	$r^2 \approx 0.159$
$r \approx 4$ cm	1 point	$r \approx 0.4$ dm
Total:	**4 points**	

5.		
A: true **B**: false **C**: true	2 points	*Award 1 point for two correct answers, 0 points for one correct answer.*
Total:	**2 points**	

6.		
$2^3 \cdot 7^2 \cdot 19$ (= 7448)	2 points	
Total:	**2 points**	

7.		
The minimum is at 1,	1 point	
the minimum value is 5.	1 point	
Total:	**2 points**	

8.		
–1	2 points	
Total:	**2 points**	

9.		
0, π, 2π	2 points	
Total:	**2 points**	

10.		
Let q denote the common ratio of the sequence. Then $q^3 = 27$.	1 point	
$q = 3$	1 point	
The sum of the first 5 terms is $2 \cdot \frac{3^5 - 1}{3 - 1} =$	1 point	$2 + 6 + 18 + 54 + 162$
= 242.	1 point	
Total:	**4 points**	

11.		
$K(0; 3)$	2 points	
$r = 5$	1 point	
Total:	**3 points**	

12. Solution 1		
(Without considering the order of selection) there are $\binom{32}{2}$ (= 496) different ways to select two students out of 32. (This is the total number of cases.)	1 point	*(Considering the order of selection) the total number of cases is* 32 · 31 (= 992).
There are $\binom{14}{2}$ (= 91) different ways two select two girls out of 14. (Favourable cases.)	1 point	*Of which* 14 · 13 (=182) *are favourable.*
The probability is $\frac{\binom{14}{2}}{\binom{32}{2}} = \frac{91}{496} \approx 0.183$.	1 point	$\frac{182}{992}$
Total:	**3 points**	

12. Solution 2		
The probability that a girl is selected first is $\frac{14}{32}$.	1 point	
The probability that a girl is also selected for the second time is $\frac{13}{31}$.	1 point	
The final probability is the product of the above, approximately 0.183.	1 point	
Total:	**3 points**	

II. A

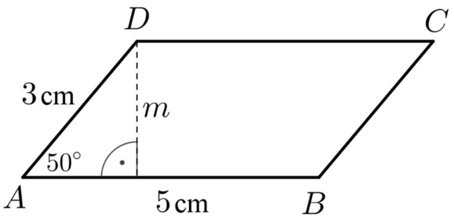
13. a) **Solution 1**		
(Let x be the cost of an adult ticket, and y be the cost of a child ticket in forints.) According to the text: $\begin{cases} x+4y=4300 \\ 2x+5y=6350. \end{cases}$	1 point	
Express x from the first equation: $x = 4300 - 4y$.	1 point	*Multiply both sides of the first equation by 2:* $\begin{cases} 2x+8y=8600 \\ 2x+5y=6350. \end{cases}$
Substitute it into the second equation: $2 \cdot (4300 - 4y) + 5y = 6350$.	1 point	*Subtract the second equation from the first:* $3y = 2250$.
Rearranged: $y = 750$ Ft, this is the price of a child ticket,	1 point	
$x = 1300$ Ft, this is the price of an adult ticket.	1 point	
Check: The cost of an adult ticket and four child tickets is (1300 + 4 · 750 =) 4300 Ft, the cost of two adult tickets and five child tickets is (2 · 1300 + 5 · 750 =) 6350 Ft.	1 point	
Total:	**6 points**	

Note: Deduce a total 1 point if the candidate does not give a textual answer (and does not clarify what each variable stands for, either).

13. a) **Solution 2**		
The combined cost of one adult and one child ticket is 6350 – 4300 = 2050 Ft.	2 points	*The combined cost of* 2 *adult and* 8 *child tickets* 2 · 4300 = 8600 Ft.
The combined cost of one adult and four child tickets is 4300 Ft, and therefore the cost of three child tickets is 4300 – 2050 = 2250 Ft.	2 points	*The combined cost of* 2 *adult and* 5 *child tickets is* 6350 Ft*, and therefore the cost of* 3 *child tickets is* 8600 – 6350 = 2250 Ft.
The price of a child ticket is 750 Ft.	1 point	
The price of an adult ticket is 1300 Ft.	1 point	
Total:	**6 points**	

Note: Deduce a total 1 point if the candidate never specifies the monetary unit in their answer.

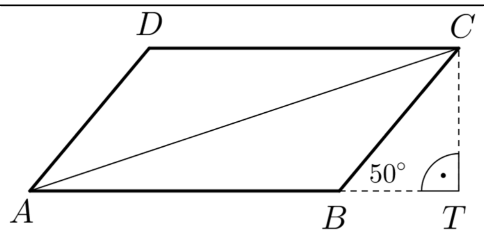
13. b)		
The gross ticket price is 1.27 times the net price.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The net price is 6350 : 1.27 = 5000 (Ft)	1 point	
The VAT content of 6350 Ft is $6350-5000=1350$ Ft.	1 point	
The VAT is $\frac{1350}{6350}\cdot 100\approx$	1 point	$\left(1-\frac{1}{1.27}\right)\cdot 100$
≈ 21.26% of the gross ticket price.	1 point	
Total:	**5 points**	

14. a) **Solution 1**		
 The height that belongs to side *AB* is $m = 3\cdot\sin 50°\approx$	1 point	
≈ 2.3 cm.	1 point	
The area of the parallelogram is $A\approx 5\cdot 2.3=$	1 point	
= 11.5 cm^2.	1 point	
Total:	**4 points**	

14. a) **Solution 2**		
The area of the parallelogram is $A = 3\cdot 5\cdot\sin 50°\approx$	1 point	
≈ 11.5 cm^2.	1 point	
The height that belongs to side *AB* is $m\approx\frac{11.5}{5}=$	1 point	
= 2.3 cm.	1 point	
Total:	**4 points**	

14. b) **Solution 1**		
The angle at vertex *B* of the parallelogram is 130°.	1 point	
Apply the Law of Cosines to side *AC* of triangle *ABC*: $AC^2=3^2+5^2-2\cdot 3\cdot 5\cdot\cos 130°$.	1 point	
$AC^2\approx 53.28$,	1 point	
$AC\approx 7.3$ cm.	1 point	
Total:	**4 points**	

14. b) Solution 2



Let point T be the base of a perpendicular drawn from vertex C of the parallelogram to the line AB. $BT = 3\cdot\cos 50° \approx 1.93$ (cm).	2 points	$BT \approx \sqrt{3^2 - 2.3^2}$
Apply the Pythagorean Theorem to the right triangle ATC: $AC^2 (= AT^2 + CT^2) \approx 6.93^2 + 2.3^2$,	1 point	
$AC \approx 7.3$ cm.	1 point	
Total:	**4 points**	

14. c)

$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DB} + \overrightarrow{BC} =$	1 point	
$= \mathbf{a} + \mathbf{b} + \mathbf{a} = 2\mathbf{a} + \mathbf{b}$	1 point	
$\overrightarrow{CD} = \overrightarrow{BA} =$	1 point	
$= -(\overrightarrow{AD} + \overrightarrow{DB}) = -\mathbf{a} - \mathbf{b}$	1 point	
Total:	**4 points**	

15. a)

The range of the data set is (11 – 3 =) 8,	1 point	
the median is 6,	1 point	
the mean is 7,	1 point	
the standard deviation is: $\sqrt{\frac{(9-7)^2 + (3-7)^2 + ... + (10-7)^2}{9}} =$	1 point	*This point is also due if the candidate obtains the correct answer using a calculator.*
$= \sqrt{\frac{64}{9}} = \frac{8}{3} \approx 2.67.$	1 point	
Total:	**5 points**	

15. b)

The frequency of event A (the sum of the numbers shown is 5, 6, 7 or 8) is 3,	1 point	
the relative frequency is $\frac{3}{9}$.	1 point	
Total:	**2 points**	

15. c)		
When two dice are rolled at the same time the number of equally likely simple outcomes is 36 (the total number of outcomes).	1 point	
5 = 1 + 4 = 2 + 3 = 3 + 2 = 4 + 1, four possible outcomes, 6 = 1 + 5 = 2 + 4 = 3 + 3 = 4 + 2 = 5 + 1, five more out-comes 7 = 1 + 6 = 2 + 5 = 3 + 4 = 4 + 3 = 5 + 2 = 6 + 1, six more outcomes 8 = 2 + 6 = 3 + 5 = 4 + 4 = 5 + 3 = 6 + 2, five more out-comes.	3 points*	
the number of favourable outcomes is the sum of the above: 20.	1 point	
The probability of event A is $\frac{20}{36} \approx 0.56$.	1 point	
Total:	**6 points**	

Notes:
*1. Award 1 of the 3 points marked by * if the candidate does not distinguish the two dice and, consequently, gives 2, 3, 3, 3 as the subtotals within this logical unit.*
2. Award full score if the candidate gives the correct answer based on a data table e.g. like the one shown below.

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

II. B

16. a)		
The statement is true,	1 point	
as the number of tickets sold on Monday, Thursday, Friday and Saturday, the days when the daily maximum was over 30 °C, was more than 1200, indeed.	1 point	
Total:	**2 points**	

16. b)		
The converse of the statement is: *If the number of tickets sold on a particular day is greater than* 1200*, then the daily maximum temperature on that day is above* 30 °C.	1 point	
The converse is false,	1 point	
as on Tuesday (or Sunday) more than 1200 tickets were sold and yet, the daily maximum temperature was below 30 °C.	1 point	
Total:	**3 points**	

16. c) **Solution 1**		
The volume of a (right-angled) trapezium-based straight prism needs to be calculated.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The bases of the trapezium are 2.1 m and 1.3 m, one other side (the height of the trapezium) is 50 m.	1 point	21 dm, 13 dm, 500 dm
The area of the trapezium is $A=(2.1+1.3)\cdot 50:2=85$ (m^2).	1 point	8500 dm^2
The height of the prism is 16.5 m,	1 point	165 dm
its volume is $V=85\cdot 16.5=1402.5$ (m^3),	1 point	1 402 500 dm^3
that is rounded to 1400 m^3.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**6 points**	

16. c) **Solution 2**		
(Calculate the sum of the volumes of a cuboid and a right triangle-based straight prism.) The volume of the cuboid is $1.3 \cdot 50 \cdot 16.5 = 1072.5$ (m^3).	1 point	1 072 500 dm^3
The area of the right triangle is $(2.1 - 1.3) \cdot 50 : 2 = 20$ (m^2).	1 point	2000 dm^2
The height of the prism is 16.5 m,	1 point	165 dm
its volume is $20 \cdot 16.5 = 330$ (m^3).	1 point	330 000 dm^3
The total volume is the sum of the above, 1402.5 (m^3),	1 point	1 402 500 dm^3
that is rounded to 1400 m^3.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**6 points**	

16. d)		
There are 8! (=40 320) different ways to sort the eight contestants into the eight lanes. (This is the total number of cases.)	1 point	
Treat Matyi and Sári as a single item, in which case there are 7! (=5040) ways to sort the „seven” swimmers.	2 points	*There are seven possible adjacent pairs of lanes for Matyi and Sári. The other six contestants could be sorted out into the remaining six lanes in* 6! (=720) *different ways in each of these cases.*
Matyi and Sári may also switch positions within a particular arrangement, so the number of favourable cases is $2 \cdot 7!$ (=10 080).	1 point	$2\cdot 7\cdot 6!$

The probability is $\frac{2\cdot 7!}{8!}=$	1 point	
$\left(=\frac{10\,080}{40\,320}\right)=0.25.$	1 point	
Total:	**6 points**	

17. a)		
The given numbers form an arithmetic progression whose common difference is 3 and the first term is 1.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$a_{56}=a_1+55d=$	1 point	
= 166	1 point	
The equation 1456 = 1 + (n – 1) · 3 is to be solved.	1 point	
$n-1=485$	1 point	
The 486th term of the progression is 1456.	1 point	
Total:	**6 points**	

17. b) Solution 1		
Rearrange the equation of the line: $-3x+y=1$.	1 point	
One normal vector of the line is (–3; 1).	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
One normal vector of the line perpendicular to this one is (1; 3).	1 point	
The equation of the perpendicular line is $x+3y=(1\cdot 14+3\cdot 56=)182$.	2 points	
Total:	**5 points**	

17. b) Solution 2		
The gradient of the given line is 3,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
the gradient of the line perpendicular to this one is $-\frac{1}{3}$.	1 point	

(Giving the equation of this line in $y = -\frac{1}{3}x + b$ form) $56 = -\frac{1}{3} \cdot 14 + b$	1 point	$y = m(x - x_0) + y_0$, *i.e.* $y = -\frac{1}{3}(x - 14) + 56$.
$b = \frac{182}{3}$	1 point	
The equation of the perpendicular line is: $y = -\frac{1}{3}x + \frac{182}{3}$.	1 point	
Total:	**5 points**	

17. c)		
The given function is (strictly monotone) decreasing wherever $x < -1$,	1 point	*These points are also due for the correct diagram.*
and is (strictly monotone) increasing if $x > -1$.	1 point	
The minimum of the function is 0 at $x = -1$.	1 point	
The function assigns 39 to –14,	1 point	$f(56) > f(-14)$
and 171 to 56.	1 point	
The range is [0; 171]	1 point	
Total:	**6 points**	

18. a)		
The number of possible passwords using six different digits is $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 151\,200$.	2 points	
The program goes through all of them in $\frac{151\,200}{1.5 \cdot 10^7} \approx$	1 point	
≈ 0.01 seconds.	1 point	
Total:	**4 points**	

18. b) Solution 1		
The number of all type **B** passwords is: 26^8.	1 point	*Trying all such passwords would take about* 3.867 hours.
The number of all type **C** passwords is: $26^{10} \cdot \binom{10}{2}$.	1 point	*Trying all such passwords would take about* 117 639 hours (*about* 13.5 years).
The ratio of these is $\frac{26^{10} \cdot \binom{10}{2}}{26^8} =$	1 point	
= 30 420. This is how many times longer it would take to try all type **C** passwords than it would take to try all type **B**-s.	1 point	
Total:	**4 points**	

18. b) **Solution 2**		
Type **C** passwords are two characters longer than type **B**-s, each of which can be any one of 26 different possibilities.	1 point	
This means a 26^2 (= 676) times longer time interval.	1 point	
Moreover, there are $\binom{10}{2}$ (= 45) different options to select the two characters out of ten that are to be capital letters.	1 point	
Therefore, it will take $26^2 \cdot \binom{10}{2} = 30\,420$ times longer to try all type **C** passwords than it would take to try all type **B**-s.	1 point	
Total:	**4 points**	

18. c)		
Let n be the number of passwords to be compared. One now has to solve the inequality $\frac{n(n-1)}{2} < 900$ (where n is a positive integer).	1 point	
$n^2 - n - 1800 < 0.$	1 point	
The roots of the equation $n^2 - n - 1800 = 0$ are $n \approx -41.9$ and $n \approx 42.9$.	1 point	
As the quadratic coefficient of the equation $n^2 - n - 1800 = 0$ is positive,	1 point	*This point is also due for the correct diagram.*
the solution of the inequality within the set of positive integers is $0 < n < 43$.	1 point	
The maximum number of passwords compared is 42.	1 point	
Total:	**6 points**	

Note: Award 2 points if the candidate gives the correct answer without reasoning (e.g. finds the solution by trial and error).

18. d)		
$\log 2^{77\,232\,917} = 77\,232\,917 \cdot \log 2 \approx$	1 point	
$\approx 23\,249\,424.7$	1 point	
The number of digits is therefore 23 249 425, indeed.	1 point	
Total:	**3 points**	