ÉRETTSÉGI VIZSGA • 2018. október 16.

MATEMATIKA
ANGOL NYELVEN

KÖZÉPSZINTŰ
ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

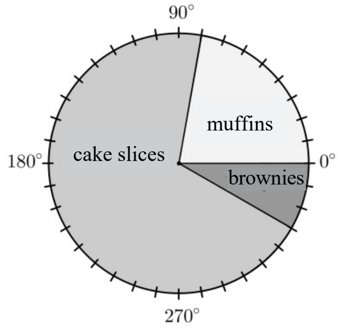
1.		
17	2 points	
Total:	**2 points**	

2.		
$\frac{1}{4}$	2 points	
Total:	**2 points**	

3.		
12	2 points	
Total:	**2 points**	

4.		
It intersects the x-axis at 3,	1 point	*at point* (3;0)
It intersects the y-axis at 6.	1 point	*at point* (0;6)
Total:	**2 points**	

5.		
A) false B) true C) true	2 points	*Award 1 point for two correct answers, 0 points for one correct answer.*
Total:	**2 points**	

6.		
There are 144 products in the cake shop.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
In the pie chart 1 product is represented by a central angle of $\frac{360}{144} = 2.5^\circ$.	1 point	
The central angles representing the three types of products are: muffins: 80°; cake slices: 250°; brownies: 30°.	1 point	
Correctly labelled pie chart.	1 point	
Total:	**4 points**	

7.		
$A \cap B = [2; 8]$	2 points	*Accept any other correct notation.*
Total:	**2 points**	

8.		
C	2 points	*Do not divide these points.*
	2 points	

9.		
[2; 4]	2 points	*Accept any other correct notation.*
Total:	**2 points**	

10.		
$\left(\binom{31}{2} =\right) 465$	2 points	
Total:	**2 points**	

11.		
The common difference of the sequence is $(d = a_5 - a_4 = 11 - 8 =)$ 3.	1 point	
The first term is $(a_1 = a_4 - 3d = 8 - 3 \cdot 3 =) -1$.	1 point	
The sum of the first ten terms is $\left(\frac{2a_1 + (n-1)d}{2} \cdot n =\right) \frac{2 \cdot (-1) + 9 \cdot 3}{2} \cdot 10 =$	1 point	$-1 + 2 + 5 + \ldots + 26 =$
= 125.	1 points	
Total:	**4 points**	

12.		
Range: 0.6 (grams).	1 point	
Mean: 15 (grams).	1 point	
Standard deviation: 0.2 (grams).	2 points	
Total:	**4 points**	

II. A

13. a) **first solution**		
(Let's find the fraction in the form of $\frac{x}{y}$) According to the problem: $\begin{cases} \frac{x}{y} = \frac{4}{11} \\ x = y - 119 \end{cases}$	1 point	
The solution of the system of equations is found to be (e.g. by substituting the second equation into the first): $y = 187$, $x = 68$.	2 points	
The fraction in question is: $\frac{68}{187}$.	1 point	
Check by substitution into the original problem: The numerator is less than the denominator by 119. The value of the fraction is $\frac{4}{11}$.	1 point	
Total:	**5 points**	

13. a) **second solution**		
In case of fraction $\frac{4}{11}$ the numerator is less than the denominator by 7.	1 point	*The original form of fraction is* $\frac{4n}{11n}$ $(n \neq 0)$.
The denominator and numerator both have to be multiplied by an integer that makes this difference 119. Thus we have to multiply them by $\left(\frac{119}{7} =\right) 17$.	2 points	$11n - 4n = 119$, *from which we get* $n = 17$.
Therefore the fraction in question is: $\frac{68}{187}$.	2 points	
Total:	**5 points**	

Note: If the candidate finds the correct answer by trying equivalent fractions of $\frac{4}{11}$ *but does not prove that there are no other solutions, a maximum of 4 points can be awarded.*

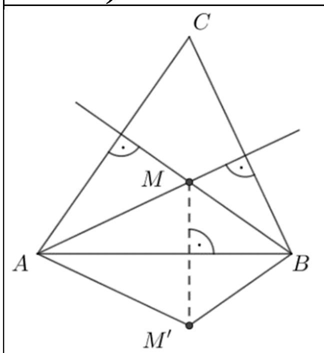
13. b)		
There are 100 different numbers that can be substituted for the denominator (total number of outcomes)	1 point	
The value of the fraction will be an integer if n is a divisor of 100.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The (positive) divisors of 100 are: 1; 2; 4; 5; 10; 20; 25; 50; 100.	1 point*	*Award 1 point in case of one or two incorrect solutions for n, and award 0 for more than two incorrect solutions for n.*
Therefore there are 9 favourable outcomes.	1 point	

The probability is $\frac{9}{100} = 0.09$.	1 point	
Total:	**5 points**	

*The point marked with * should also be awarded for the following reasoning:*
$100 = 2^2 \cdot 5^2$, *therefore the number of its positive divisors is* $(2 + 1) \cdot (2 + 1)$.

14. a)		
Let P' be the image of P after the reflection. Due to the properties of reflection about a point, K is the midpoint of segment PP', therefore $\overrightarrow{PK} = \overrightarrow{KP'}$.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$\overrightarrow{PK} = (5;\ 12)$	1 point	*The coordinates of point* $P'(p_1;\ p_2)$ *can be found using:* $\frac{-2 + p_1}{2} = 3$, $\frac{3 + p_2}{2} = 15$.
(Adding the coordinates of vector $\overrightarrow{PK}$ to the coordinates of point K, we get the coordinates of point P') $P'(8;\ 27)$.	2 points	$p_1 = 8$; $p_2 = 27$, *thus* $P'(8;\ 27)$.
Total:	**4 points**	

Note: If the candidate finds the correct answer by reading it from a diagram, 2 points should be awarded. If the candidate checks that the values read from the diagram are correct, 4 points should be awarded.

14. b)		
 Correct diagram.	1 point	*This point is also due if the candidate calculates correctly without drawing a diagram.*
The sum of the interior angles of a triangle is 180°, so $\angle ACB = 60°$.	1 point	
(Calculating angles in the right-angled triangles leads to) $\angle MAB = 90° - 65° = 25°$ and $\angle MBA = 90° - 55° = 35°$,	2 points	$\angle CAM = \angle CBM = 30°$, $\angle MAB = 55° - 30° = 25°$, $\angle MBA = 65° - 30° = 35°$.
therefore (because of the reflection) $\angle CAM' = 55° + 25° = 80°$, and $\angle CBM' = 65° + 35° = 100°$.	2 points	
$\angle AM'B = 360° - (60° + 80° + 100°) =$	1 point	$\angle AMB = 180° - (25° + 35°) = 120°$.

$= 120°$.	1 point	*Because of the reflection $\angle AM'B = 120°$.*
Total:	**8 points**	

Note: If the candidate solves the problem by reflecting any special point of the triangle other than the orthocentre, a maximum of 5 points can be awarded.

15. a)		
$x \neq -2$, $x \neq 2$	1 point	*This point is also due if the candidate checks the solution by substituting into the original equation.*
Finding the common denominator: $\frac{x(x-2)}{(x+2)(x-2)} = \frac{8}{(x+2)(x-2)}$.	1 point	*This point is also due if the candidate multiplies both sides by the common denominator.*
$x(x-2) = 8$	1 point	
Rearranging the equation: $x^2 - 2x - 8 = 0$.	1 point	
The roots of the equation are $x = 4$ and $x = -2$.	1 point	
Check by substitution or reference to equivalent steps in the domain of the equation: $x = -2$ is a false root, while, $x = 4$ is a correct solution.	1 point	
Total:	**6 points**	

15. b)		
The inequality is true, if $x > 0$ and $x + 2 < 0$,	1 point	
or $x < 0$ and $x + 2 > 0$.	1 point	
There is no real number satisfying the first set of inequalities,	1 point	
the solution of the second set of inequalities is: $-2 < x < 0$ ($x \in \mathbf{R}$).	1 point	
Total:	**4 points**	

Note: If the candidate accepts 0 and/or -2 as a solution, deduce only 1 point.

15. c)		
Completing the square leads to: $f(x) = (x-3)^2 - 4$.	2 points*	*The zeros of f can be found by solving the quadratic equation $x^2 - 6x + 5 = 0$ $x = 1$ and $x = 5$.*
The minimum of f is at $x = 3$.	1 point	*The minimum of f is at the mean of zeros: 3.*
The minimum value is: -4.	1 point	
Total:	**4 points**	

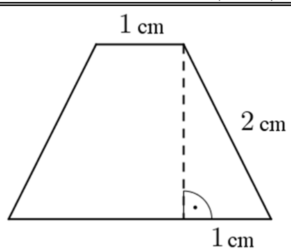
*Note: The 2 points marked with * should also be awarded if the candidate states that the minimum of a quadratic function $f(x) = ax^2 + bx + c$ $(a > 0)$ is at $x = -\frac{b}{2a}$.*

II. B

16. a)		
The distances covered by Cili are consecutive terms in a geometric sequence with first term $a_1 = 20$, and common ratio $q = 1.15$.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
If the distance gets at least 1000 m on the nth day, then $a_n = 20 \cdot 1.15^{n-1} = 1000$.	1 point	
(Dividing each side by 20 and taking their logarithm leads to) $\log 1.15^{n-1} = \log\ 50$.	1 point	$n-1 = \log_{1.15} 50$
$(n-1) \cdot \lg 1.15 = \lg 50$	1 point	
$n-1 = \dfrac{\log 50}{\log 1.15} \approx 27.99$, thus $n \approx 29$.	1 point	
It was the 29th day when Cili could say that she walked 1000 m already that given day.	1 point	
Total:	**6 points**	

Notes:
1. If the candidate lists the terms of the sequence using reasonable rounding, then gives a correct answer based on these calculations, full points should be awarded (e.g. by rounding each term to the nearest meter, the correct answer is the 30th day)
2. If the candidate solves an inequality instead of an equation, the points given for solving an equation should be awarded.

16. b)		
If the volume of 20 drops is 1 ml, then the volume of the daily 50 drops is 2.5 ml.	1 point	
This volume contains $2.5 \cdot 100 = 250$ mg active ingredient.	1 point	
Total:	**2 points**	

16. c)		
The volume of the cylinder is $1.5^2 \cdot \pi \cdot 7 \approx 49.5$ (cm^3).	1 point	
The height of the truncated cone (applying the Pythagorean theorem) is $\sqrt{2^2 - 1^2} = \sqrt{3} \approx 1.73$ (cm). 	2 points	
The volume of the truncated cone is $\dfrac{1.73 \cdot (1.5^2 + 0.5^2 + 1.5 \cdot 0.5) \cdot \pi}{3} \approx$	1 point	
≈ 5.9 (cm^3).	1 point	

The total volume of the liquid is $49.5 + 5.9 = 55.4\ \text{cm}^3$,	1 point	
so a full bottle contains 55.4 ml of liquid vitamin drops.	1 point	
This is the volume of $55.4 \cdot 20 \approx 1108$ drops,	1 point	
which is enough for $\frac{1108}{50} \approx 22$ days.	1 point	
Total:	**9 points**	

Note: The last 2 points should also be awarded, if the candidate finds the number of days using the volume of a daily dose calculated in part b) $(55.4 : 2.5 \approx 22)$.

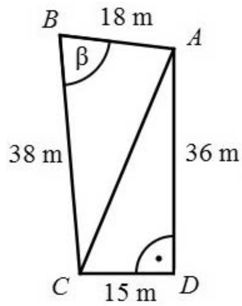
17. a)		
The diagonal of the screen is $5.4 \cdot 25.4 \approx 137.2$ mm.	1 point	
(Let the sides of the screen be $16x$ and $9x$, measured in millimeters. Applying the Pythagorean theorem:) $(16x)^2 + (9x)^2 = 137.2^2$,	1 point	
from which we get $x \approx 7.47$.	2 points	
The sides of the screen are 120 (mm) and 67 (mm) long.	1 point	
Adding the rims, we get that the sides of the front panel are 144 mm and 73 mm long.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**6 points**	

17. b)		
The probability that a candidate switches off the mobile phone is $(1 - 0.02 =)\,0.98$.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The probability that every candidate switches off his or her mobile phone is 0.98^{12} (≈ 0.785).	1 point	
Therefore the probability that at least one candidate forgets to switch off the mobile phone is $1 - 0.98^{12} \approx 0.215$.	1 point	
Total:	**3 points**	

17. c)		
If the two girls sit in the first row, they can choose two neighbouring desks in 3 different ways.	1 point	
If they sit in the second or third row, they can choose the desks in 3 ways as well in each case. This means that the two neighbouring desks can be chosen in 9 different ways.	1 point	
In all these 9 cases there are two different arrangements of Julcsi and Tercsi, which gives 18 different ways.	1 point	
The remaining 10 candidates can be seated in 10!(= 3 628 800) different ways.	1 point	
Thus the total number of different arrangements for Julcsi and Tercsi to be seated next to each other is $(18 \cdot 10! =)$ 65 318 400.	1 point	
Total:	**5 points**	

17. d)		
The mean will take its highest possible value, if all data are assumed to be equal to the highest value in their interval.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
Therefore the highest possible mean is: $\frac{1}{100}\cdot(30\cdot 8+40\cdot 12+50\cdot 8+60\cdot 18+70\cdot 20+$ $+80\cdot 12+90\cdot 16+100\cdot 6)=$	1 point	*Award this point if the candidate uses a calculator and finds the correct mean.*
= 66.	1 point	
Total:	**3 points**	

18. a)		
Calculating compound interest for 5 years: $8\,115\,000 = 7\,000\,000 \cdot x^5$	1 point	
$x \left(= \sqrt[5]{\frac{8115}{7000}} \right) \approx 1.03$	2 points	
The bank payed an annual interest rate of approximately 3%.	1 point	
Total:	**4 points**	

18. b)		
(Applying the Pythagorean theorem in triangle *ACD*:) $AC = \sqrt{36^2 + 15^2} = 39$ (m). 	1 point	
(Applying the cosine-rule in triangle ABC:) $39^2 = 18^2 + 38^2 - 2 \cdot 18 \cdot 38 \cdot \cos\beta$.	1 point	
Therefore $\cos\beta = \frac{13}{72}$ (≈ 0.1806),	1 point	
from which we get $\beta = 79.6°$.	1 point	
The area of triangle *ACD* is $\frac{36 \cdot 15}{2} = 270$ (m^2).	1 point	
The area of triangle *ABC* is $\frac{18 \cdot 38 \cdot \sin 79.6°}{2} \approx 336.4$ (m^2).	1 point	*Using Heron's formula* $\sqrt{47.5 \cdot 8.5 \cdot 9.5 \cdot 29.5} \approx$ ≈ 336.4 (m^2).
Therefore the area of the plot of land is (270 + 336.4 =) 606.4 (m^2).	1 point	
The area that can be built on is $606.4 \cdot 0.2 \approx$	1 point	
≈ 121 m^2.	1 point	
Total:	**9 points**	

18. c) **first solution**		
Mr Molnár can pick a wrong key for the first attempt and pick the right one for the second in 3 different ways. (Number of favourable outcomes.).	2 points	
He can pick two keys in $4 \cdot 3 = 12$ different ways.	1 point	
Therefore the probability in question is $\frac{3}{12} = \frac{1}{4}$.	1 point	
Total:	**4 points**	

18. c) **second solution**		
The probability that the first key will not open the lock is $\frac{3}{4}$.	1 point	
The probability that the second key will open the lock is $\frac{1}{3}$.	1 point	
Therefore the probability in question is $\frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$.	2 points	
Total:	**4 points**	

18. c) **third solution**		
Trying all four keys can be done in $4! = 24$ different ways. (Total number of possible outcomes.)	1 point	
There are $3 \cdot 1 \cdot 2 \cdot 1 = 6$ cases in which the second key is the one that opens the lock. (Number of favourable outcomes.)	2 points	
Therefore the probability in question is $\frac{6}{24} = \frac{1}{4}$.	1 point	
Total:	**4 points**	

Note: If the candidate states that choosing right key for any of the four attempts has the same probability, therefore (since these events are equally likely and mutually exclusive) the probability in question is $\frac{1}{4}$*, full points should be awarded.*