ÉRETTSÉGI VIZSGA • 2018. május 8.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
–2	2 points	
Total:	**2 points**	

2.		
6	2 points	
Total:	**2 points**	

3.		
$2\cdot 3^2$ (= 18)	2 points	
Total:	**2 points**	

4.		
A: true B: false C: true	2 points	*Award 1 point for two correct answers, 0 points for one correct answer.*
Total:	**2 points**	

5.		
3	2 points	
Total:	**2 points**	

6. Solution 1		
The discount price is $\frac{95\,200}{112\,000}\cdot 100 =$	1 point	
= 85% of the initial price.	1 point	
Therefore, the discount price is 15% lower than the initial price.	1 point	
Total:	**3 points**	

6. Solution 2		
The discount is 112 000 – 95 200 = 16 800 (Ft).	1 point	
The discount price is $\frac{16\,800}{112\,000}\cdot 100 =$	1 point	
= 15% lower than the initial price.	1 point	
Total:	**3 points**	

7.		
$3^{x-4} = 27$	1 point	
$x - 4 = 3$	1 point	
$x = 7$	1 point	
Total:	**3 points**	

8.		
$\left(\dfrac{a^2b+ab^2}{a+b}=ab=\right)4$	2 points	
Total:	**2 points**	

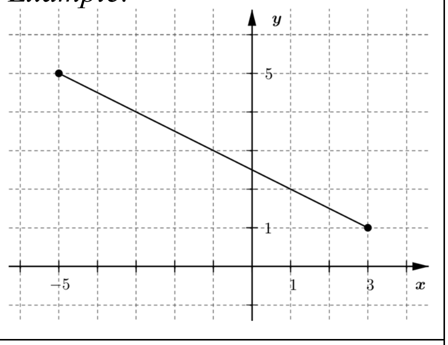
Note: Award at most 1 point if the answer is not the exact value of the expression.

9.		
331 224 Ft	2 points	
Total:	**2 points**	

Note: Accept 331 225 Ft or any correct answer that is not rounded to the nearest forint.

10. Solution 1		
$\log_2 32 = 5$	1 point	
$x = 8^5 = 32768$	1 point	
So, it is true that $x > 32\,000$.	1 point	
Total:	**3 points**	

10. Solution 2		
$\log_8 32000 \approx 4.99$	1 point	
$\log_2 32 = 5$	1 point	
(As the base 8 logarithm function is a strictly monotone increasing) it is true that $x > 32\,000$.	1 point	
Total:	**3 points**	

11.		
The domain of the graph is [–5; 3],	1 point	*Example:* 
the range is [1; 5],	1 point	
the graph is strictly monotone decreasing.	1 point	
Total:	**3 points**	

Note: Award at most 2 points if the candidate switches the domain and the range. Deduce a total of 1 point if the candidate draws the graph over an open, or partially open interval.

12.		
After throwing twice, the total number of possible outcomes is $6^2 = 36$.	1 point	
The ones divisible by 7 are: 14, 21, 35, 42, 56 and 63.	1 point	
The number of favourable cases is therefore 6.	1 point	
The probability is $\frac{6}{6^2} = \frac{1}{6}$.	1 point	
Total:	**4 points**	

II. A

13. a)		
Expand the brackets: $\frac{1-2x-2}{5} + \frac{18-x}{11} = -2$.	1 point	
Multiply both sides by the common denominator: $-11 - 22x + 90 - 5x = -110$.	1 point	
Rearranged: $-27x = -189$.	1 point	
$x = 7$.	1 point	
Check by substitution of reference to equivalent steps.	1 point	
Total:	**5 points**	

13. b)		
Square both sides of the equation: $7 - x = x^2 + 10x + 25$.	2 points	
Rearranged: $x^2 + 11x + 18 = 0$.	1 point	
$x = -2$ or $x = -9$.	2 points	
By means of substitution –2 is a correct solution,	1 point*	
–9 is not a correct solution.	1 point*	
Total:	**7 points**	

*Note: Award the 2 points marked by * if the candidate states that the solutions must fall within the interval [–5; 7] and refers to squaring both sides being an equivalent step over this interval, thereby determining that –2 is a correct solution while –9 is not.*

14. a)		
There are 15 numbers among the 90 that are divisible by 6,	1 point	
and 10 numbers that are divisible by 9.	1 point	
Among the 90 numbers 5 are divisible by both 6 and 9 (i.e. by 18, these have been counted in both cases).	2 points	
Áron may choose from among 90 – 15 – 10 + 5 = 70 numbers.	1 point	
Total:	**5 points**	

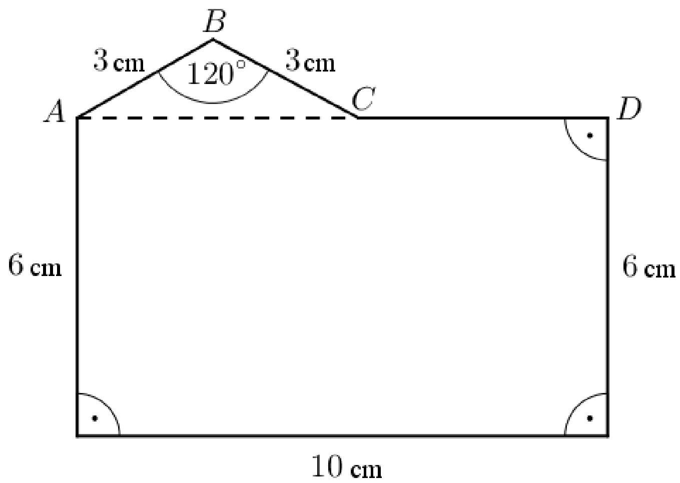
Note: The 70 numbers Aron may choose from are shown (in white) in the table:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90

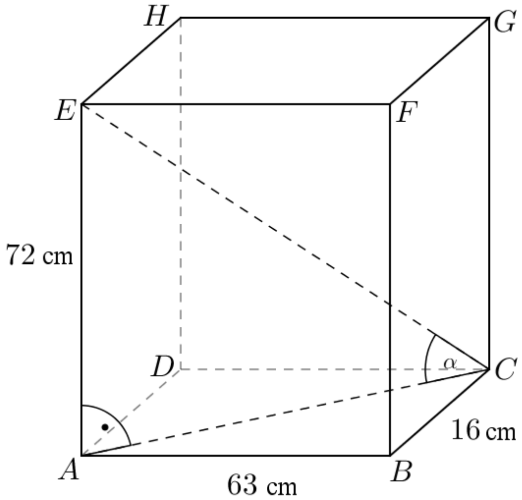
14. b) **Solution 1**		
There are 86 numbers satisfying the condition.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The number of favourable cases: $\binom{86}{5}$(= 34 826 302)	1 point	
The number of possible cases: $\binom{90}{5}$(= 43 949 268)	1 point	
The probability: $\frac{34\,826\,302}{43\,949\,268} \approx$	1 point	
≈ 0.79.	1 point	
Total:	**5 points**	

14. b) **Solution 2**		
There are 86 numbers satisfying the condition.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The probability that the first number drawn is at least 5 is $\frac{86}{90}$.	1 point	
The probabilities that the second, third, fourth, and fifth numbers are also at least 5 are $\frac{85}{89}, \frac{84}{88}, \frac{83}{87}, \frac{82}{86}$, respectively.	1 point	
The final probability is the product of the above,	1 point	
approximately 0.79.	1 point	
Total:	**5 points**	

Note: Award a maximum of 3 points if the candidate works with a probability model that involves the replacement of the numbers drawn.

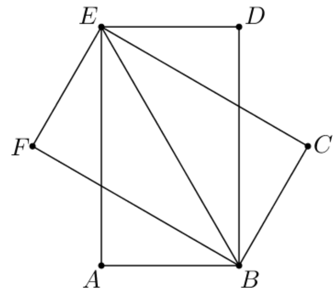
15. a)		
(Break the hexagon into a triangle and a rectangle.)  	1 point	
Apply the Law of Cosines in triangle *ABC*: $AC^2 = 3^2 + 3^2 - 2\cdot 3\cdot 3\cdot \cos 120°$.	1 point	$AC = 2\cdot 3\cdot \sin 60°$
$AC = \sqrt{27}$ (≈ 5.2 cm),	1 point	
$CD = 10 - \sqrt{27}$ (≈ 4.8 cm).	1 point	
The perimeter of the hexagon: $K = 2\cdot 6 + 10 + 2\cdot 3 + (10 - \sqrt{27}) =$	1 point	
$= 38 - \sqrt{27}$ (≈ 32.8) cm.	1 point	
The area of the triangle: $\frac{3\cdot 3\cdot \sin 120°}{2} \approx$	1 point	*The area of triangle ABC is also equal to that of a regular triangle of* 3 cm *sides:* $\frac{3^2\sqrt{3}}{4} \approx$
≈ 3.9 (cm^2).	1 point	
The area of the rectangle is 60 (cm^2),	1 point	
and so the total area of the hexagon is approximately 63.9 cm^2.	1 point	
Total:	**10 points**	

Note: Deduce a total of 1 point if the candidate does not use units in either of their answers.

15. b)		
	1 point	*Award this point for correctly identifying the angle.*
Apply the Pythagorean Theorem: $AC = \sqrt{63^2 + 16^2} = 65$ (cm).	1 point	*The solid diagonal of the cuboid is* $EC = \sqrt{63^2 + 16^2 + 72^2}$ = 97 (cm).
Let α be the missing angle: $\tan\alpha = \frac{72}{65}$ (≈ 1.108).	1 point	$\sin\alpha = \frac{72}{97}$ (≈ 0.742)
α ≈ 47.9°.	1 point	
Total:	**4 points**	

II. B

16. a) Solution 1		
Add the numbers of games played by the various players. (As every game was counted twice) the total must be an even number.	1 point	
Should player F have played 3 games, the total would have been 19.	1 point	
As 19 is an odd number, it is not possible that player F has played 3 games.	1 point	
Total:	**3 points**	

16. a) **Solution 2**		
As players B and E have each played everybody else, players A, C, and D must have played their two games with these two.	1 point	
Player F could only play with B and E, playing just two games,	1 point	
so it is not possible that player F has played 3 games.	1 point	
Total:	**3 points**	

16. b)		
The sum of the heights of the 11 players at the beginning of the game is $186 \cdot 11 = 2046$ (cm).	1 point	*The substitute player is* $((188 - 186) \cdot 11 =)$ $2 \cdot 11 =$
The sum after substitution is: $188 \cdot 11 = 2068$ (cm)	1 point	
The substitute player is 2068 – 2046 =	1 point	
= 22 cm taller than the one he replaced.	1 point	
Total:	**4 points**	

16. c)		
One second after being kicked the ball was at a height of $h(1) = -5 \cdot 1^2 + 15 \cdot 1 = 10$ metres.	2 points	
Total:	**2 points**	

16. d)		
(The moment when the ball was kicked is t = 0 (s) and so) the positive zero of the function will give the total time of the ball being in the air.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The solutions of the equation $-5t^2 + 15t = 0$ are ($t_1 = 0$ and) $t_2 = 3$.	2 points	
The ball was in the air for 3 seconds.	1 point	
Total:	**4 points**	

16. e) **Solution 1**		
Complete the square: $-5t^2+15t = -5(t^2-3t) =$	1 point	
$=-5(t-1{,}5)^2+11{,}25$.	2 points	
(Function h has a maximum at $t = 1.5$, the maximum value is 11.25,) so the maximum height of the trajectory was 11.25 m.	1 point	
Total:	**4 points**	

16. e) **Solution 2**		
The quadratic function $t \mapsto at^2+bt+c \quad (a \neq 0)$ reaches its extreme at $t=-\dfrac{b}{2a}$.	1 point	*The graph of this function is an "inverted" parabola. It reaches its maximum at the arithmetic mean of its zeros.*
The maximum value of function h at $t=-\dfrac{15}{2\cdot(-5)}=1.5$ is	1 point	$\dfrac{0+3}{2}=1.5$
$h(1.5) = 11.25$.	1 point	
Therefore, the maximum height of the trajectory was 11.25 m.	1 point	
Total:	**4 points**	

17. a)		
There are 6 different ways to choose the five questions that would be answered correctly (or to choose the one that would be answered wrong). (These 5 questions may only have one answer.)	1 point	
There are two possible wrong answers for the question that is answered incorrectly.	1 point	
The total number of different possibilities is $6\cdot 2 = 12$.	1 point	
Total:	**3 points**	

17. b) Solution 1		
Eszter may select two out of the eight problems in $\binom{8}{2}$ (= 28) different ways (total number of cases).	1 point	
If at least one problem requires the ability to find the point of intersection of two lines then either just one or both of them are such problems.	1 point*	*This point is also due if the correct reasoning is reflected only by the solution.*
If both problems are like that, they may be selected in $\binom{3}{2}$ (= 3) different ways..	1 point*	
If only one problem is like that, it gives $\binom{3}{1}\cdot\binom{5}{1}$ (= 15) possible cases.	1 point*	
The number of favourable cases is 3 + 15 = 18.	1 point*	
The probability is $\frac{18}{28}$ (≈ 0.64).	1 point	
Total:	**6 points**	

*The 4 points marked by * may also be given for the following reasoning:*

The number of favourable cases may also be obtained by subtracting the number of unfavourable cases (when neither of the two problems requires such skill) from the total.	2 points	*These 2 points are also due if the correct reasoning is reflected only by the solution.*
The number of unfavourable cases is $\binom{5}{2}=10$.	1 point	
The number of favourable cases is 28 – 10 = 18.	1 point	

17. b) **Solution 2**		
There are three different cases based on which of the two problems requires the ability to find the point intersection of two lines.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The probability that both problems require it is $\frac{3}{8}\cdot\frac{2}{7}$.	1 point	
The probability that only the first one requires it is $\frac{3}{8}\cdot\frac{5}{7}$.	1 point	
The probability that only the second requires it is $\frac{5}{8}\cdot\frac{3}{7}$.	1 point	
The final probability is the sum of the above:	1 point	
$\frac{36}{56}=\frac{9}{14}$ (≈ 0.64).	1 point	
Total:	**6 points**	

17. b) **Solution 3**		
Calculate the probability of the complement (i.e. none of the problems selected requires such skill).	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The probability that the first of the selected problems will not require the skill is $\frac{5}{8}$.	1 point	
The probability that the second problem is also like that is $\frac{4}{7}$.	1 point	
The probability of the complement is $\frac{5}{8}\cdot\frac{4}{7}$.	1 point	
The final probability is $1-\frac{5}{8}\cdot\frac{4}{7}=$	1 point	
$=\frac{9}{14}$ (≈ 0.64).	1 point	
Total:	**6 points**	

17. c)		
The image of point *A* about line *e* is *A'*(11; 36).	2 points	
One normal vector of line *A'B* is **n**(25; –20) (one direction vector is **v**(20; 25), the gradient is 1.25).	2 points	**n**(5; –4) or **v**(4; 5)
The equation of the line *A'B*: $25x-20y=-445$.	1 point	$5x-4y=-89$
The first coordinate of point *E* is $x = 3$.	1 point	
Substitute it into the equation of line *A'B*: $25\cdot 3-20y=-445$.	1 point	$5\cdot 3-4y=-89$
The second coordinate of *E* is $y = 26$.	1 point	
Total:	**8 points**	

18. a)		
One machine harvests $\frac{1}{8}$ of the field in an hour.	1 point	
Let x be the number of hours the first machine worked. In this case the second worked for $(x-3)$ hours, and so $\frac{1}{8}\cdot x+\frac{1}{8}\cdot(x-3)=1$.	1 point	*The first machine harvested $\frac{3}{8}$ of the field by 10 o'clock.*
$2x-3=8$,	1 point	*After 10 the machines harvested 2/8 of the remaining 5/8 of the field every hour.*
that is: $x = 5.5$ hours.	1 point	*That takes another 2.5 hours until they finish the whole field.*
Check by substituting into the original text. (The first machine harvested $\frac{11}{16}$ of the field in 5.5 hours, the second did $\frac{5}{16}$ of it in 2.5 hours. $\frac{11}{16}+\frac{5}{16}=1$, so the whole field will indeed be harvested.	1 point	*Award this point if the candidate gives the correct answer without solving an equation.*
The work is finished by 12:30.	1 point	
Total:	**6 points**	

18. b)		
The radius of the base circle of the bale is 0.6 m, the height is 1.2 m.	1 point	
The volume of one bale is $0.6^2\cdot\pi\cdot1.2\approx$	1 point	
$\approx 1.36\ \text{m}^3$.	1 point	
The mass of one bale is about $1.36\cdot160$ (= 217.6 kg),	1 point	
rounded to 220 kg.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**5 points**	

18. c)		
The mean of the sample is 119 cm	1 point	
which is within limits.	1 point	
The standard deviation is $\sqrt{\frac{4^2+3^2+0^2+5^2+3^2+1^2+5^2+3^2+1^2+7^2}{10}}\approx$	1 point	*Award this point if the candidate correctly uses a calculator to give the standard deviation.*
≈ 3.79,	1 point	
which is less than 4, so the standard deviation is also acceptable.	1 point	
The hay baler will pass quality control.	1 point	
Total:	**6 points**	