ÉRETTSÉGI VIZSGA • 2017. október 17.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
$\left(\frac{5^2\pi\cdot 9}{3}=\right)75\pi\text{ cm}^3\approx 235.6\text{ cm}^3$	2 points	
Total:	**2 points**	

2.		
$A = \{1; 2; 3; 4; 6; 12\}$	1 point	
$B = \{2; 3; 5; 7; 11; 13\}$	1 point	
$A \setminus B = \{1; 4; 6; 12\}$	1 point	
Total:	**3 points**	

3.		
$x = 21$	2 points	
Total:	**2 points**	

4.		
18	2 points	
Total:	**2 points**	

5.		
The sum of the digits is $22 + 2c$.	1 point	
A number is divisible by 3 if (and only if) the sum of its digits is divisible by 3.	1 point	*The value of $22 + 2c$ must be divisible by 3.*
Possible values of c are 1; 4; 7.	1 point	
Total:	**3 points**	

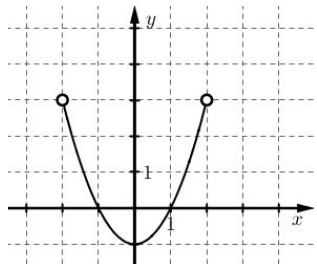
Note: Award full score if the candidate gives the correct answer by trying all 10 possible digits.

6.		
$\left(\frac{8\cdot 7}{2}=\right)28$	2 points	
Total:	**2 points**	

7.		
A: true B: false C: true	2 points	*Award 1 point for two correct answers, 0 points for one correct answer.*
Total:	**2 points**	

8. **Solution 1**		
(Consider the participants as vertices of a simple graph and the clinks of glasses as the edges of the graph.) In any graph, the sum of the degrees of the vertices must be an even number.	1 point	
$1 + 2 + 2 + 3 + 3 + 6 + 6 = 23$	1 point	
There is no such graph and therefore this situation is not possible.	1 point	
Total:	**3 points**	

8. **Solution 2**		
(Consider the participants as vertices of a simple graph and the clinks of glasses as the edges of the graph.) As there are two vertices with a degree of 6 (which are connected to all other vertices), there must not be any vertex with a degree of 1 at all.	2 points	
There is no such graph and therefore this situation is not possible.	1 point	
Total:	**3 points**	

9.		
[–1; 3[	3 points	 *The correct answer is acceptable in any other form, too.*
Total:	**3 points**	

Note: Award 1–1 point for correctly stating each endpoint of the interval (–1 and 3) and a further 1 point for correctly stating the type of each endpoint.

10.		
(The mean of the data is 2, the standard deviation is $\sqrt{\frac{2^2+1^2+0^2+1^2+2^2}{5}}=$) $\sqrt{2}\approx 1.41$	2 points	
Total:	**2 points**	

11.		
$\frac{\pi}{3}$ and $\frac{5\pi}{3}$	2 points	
Total:	**2 points**	

Note: Award 1 point if the candidate gives the (correct) solution in degrees or if the (correct) solution of the equation $\cos x = \frac{1}{2}$ *is not restricted to the given interval.*

12. **Solution 1**		
Four people may sit next to each other on a bench in $4! = 24$ different ways (total number of cases).	1 point	
The person sitting on one end may be selected out of four people, while the person sitting next to the first may be selected out of two (the opposite gender). (The order of the remaining two people is thereby determined.) The number of favourable cases is $4 \cdot 2 = 8$.	2 points*	*ABCD, CBAD, ADCB, CDAB, BADC, DABC, BCDA, DCBA*
The probability: $\frac{8}{24} = \frac{1}{3}$.	1 point	
Total:	**4 points**	

*Note: The 2 points marked by * may also be given for the following reasoning:*

Girls and boys must sit on the bench in alternating order: either BGBG or GBGB.	1 point	
In any arrangement both the girls and the boys may sit in two different orders, in which case the number of favourable cases is: $2 \cdot 2 \cdot 2 = 8$.	1 point	

12. **Solution 2**		
Considering gender only there are $\binom{4}{2} = 6$ possible ways for the four people to sit (total number of cases).	2 points	*BBGG, BGBG, BGGB, GGBB, GBGB, GBBG (The probability of each of these cases is the same.)*
Two of these arrangements are favourable: boy-girl-boy-girl or girl-boy-girl-boy.	1 point	
The probability is: $\frac{2}{6} = \frac{1}{3}$.	1 point	
Total:	**4 points**	

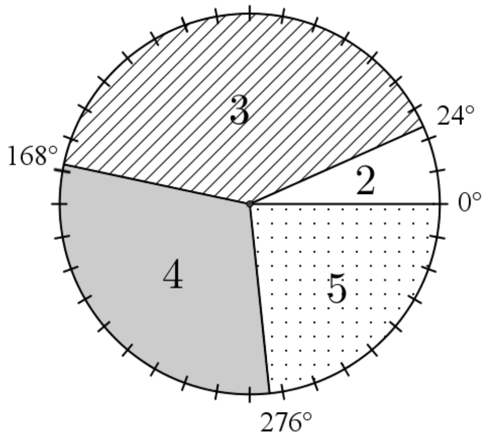
II. A

13. a) **Solution 1**		
Expand the brackets: $4x^2-12x+9=x^2$.	1 point	
Rearrange the equation: $3x^2-12x+9=0$.	1 point	$x^2-4x+3=0$
$x_1=1,\ x_2=3$	2 points	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**5 points**	

13. a) **Solution 2**		
(There will be two cases.) In case 1 the bases are equal: $2x-3=x$.	1 point	
In this case $x=3$.	1 point	
In case 2 the bases are opposites: $2x-3=-x$.	1 point	
In this case $x=1$.	1 point	
Check by substitution or reference to equivalent steps.	1 point	*This point may only be awarded if the candidate gets two solutions and checks both of them.*
Total:	**5 points**	

13. b)		
There are five different possibilities for the last digit of such numbers (1, 3, 5, 7, 9).	1 point	
The first digit may not be 0,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
(and has to be different from the last digit) so (after fixing the last digit) there are 8 different possibilities here.	1 point	
There are 8 possibilities for the second digit (which must be different from the first and last once those are fixed).	1 point	
The total number of options is the product of the above numbers: $8\cdot8\cdot5=320$.	1 point	
Total:	**5 points**	

14. a)		
The mean of the grades is $\frac{2\cdot2+12\cdot3+9\cdot4+7\cdot5}{30}=$	1 point	
= 3.7.	1 point	
The median of the grades is 4.	1 point	
The mode of the grades is 3.	1 point	
Total:	**4 points**	

14. b)		
Each person is represented in the diagram by a sector with a 12° central angle. Central angles belonging to each grade are: grade 2: 24°; grade 3: 144°; grade 4: 108°; grade 5: 84°.	2 points	
One possible chart: 	2 points	*Award 1 point for the correct central angles and 1 point for clearly identifying the 4 regions.*
Total:	**4 points**	

14. c) Solution 1		
The number of favourable cases is $\binom{12}{2}$ (= 66).	1 point	*Considering the order, too, it is* $12 \cdot 11$
The total number of cases is $\binom{30}{2}$ (= 435).	1 point	$30 \cdot 29$
The probability is the ratio of the above: $\frac{66}{435}$,	1 point	$\frac{132}{870}$
Correctly rounded: 0.152	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**4 points**	

14. c) Solution 2		
The probability that the grade on the paper that was selected first is a 3: $\frac{12}{30}$.	1 point	
(Assuming the grade on the first paper is a 3) the probability that the grade on the paper selected next is a 3, too: $\frac{11}{29}$.	1 point	
The probability asked is the product of the above: $\frac{132}{870}$,	1 point	
correctly rounded: 0.152.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**4 points**	

15. a)		
Let $BAC\angle = \alpha$, which means $ACB\angle = 90° - \alpha$.	1 point	*$BAC\angle$ and $ACD\angle$ are alternate angles and therefore congruent.*
As $BCD\angle = 90°$, $ACD\angle = \alpha$ (and $ADC\angle = 90° - \alpha$).	1 point	
Corresponding pairs of angles of the two triangles are congruent, and so the triangles are similar.	1 point	
Total:	**3 points**	

15. b) Solution 1		
$BAC\angle = \alpha,\ \cos\alpha = \frac{9}{15}$,	1 point	
$\alpha \approx 53.1°$,	1 point	
this makes the angle at vertex *A* of the trapezium to be $\alpha + 90° \approx 143.1°$,	1 point	
the angle at vertex *D* is approximately $180° - 143.1° = 36.9°$.	1 point	
Total:	**4 points**	

15. b) Solution 2		
(Because of the similarity of triangles *ABC* and *CAD*) $ADC\angle = ACB\angle = \gamma$.	1 point	
$\sin\gamma = \frac{9}{15}$,	1 point	
so the angle at vertex *D* of the trapezium is $\gamma \approx 36.9°$,	1 point	
the angle at vertex *A* is approximately $180° - 36.9° = 143.1°$.	1 point	
Total:	**4 points**	

15. c) Solution 1		
To calculate the area of the trapezium let's first determine the length of base *CD* and side *BC* (also, the height of the trapezium).	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
Because of the similarity of triangles *ABC* and *CAD*: $\frac{CD}{15} = \frac{15}{9}$,	1 point	
$CD = 25$ (cm).	1 point	
From the right triangle ABC (Pythagorean Theorem): $BC = \sqrt{15^2 - 9^2} =$	1 point	
$= 12$ (cm).	1 point	
The area of the trapezium ABCD: $\frac{25+9}{2}\cdot 12 =$	1 point	
$= 204\ \text{cm}^2$.	1 point	
Total:	**7 points**	

15. c) Solution 2		
From the right triangle *ABC* (Pythagorean Theorem): $BC=\sqrt{15^2-9^2}=$	1 point	
= 12 (cm).	1 point	
The area of triangle *ABC* is $\frac{9\cdot 12}{2}=54$ (cm^2).	1 point	
As triangles *ABC* and *CAD* are similar: $\frac{AD}{15}=\frac{12}{9}$,	1 point*	
from which *AD* = 20 (cm).	1 point*	
The area of triangle *ACD* is $\frac{20\cdot 15}{2}=150$ (cm^2).	1 point*	
The area of the trapezium is the sum of the areas of these two triangles, that is 204 cm^2.	1 point	
Total:	**7 points**	

*Note: The 3 points marked by * may also be given for the following reasoning:*

The ratio of similarity between triangles *CAD* and *ABC* (the ratio of their corresponding sides) is 15 : 9 = 5 : 3.	1 point	
The ratio of the areas of the triangles is the square of this: 25 : 9.	1 point	
The area of triangle *ACD* is $\frac{25}{9}\cdot 54=150$ (cm^2).	1 point	

Note: Award full score if the candidate correctly uses the (rounded) angles obtained in part b).

II. B

16. a)		
$\frac{12000000}{7000000}\approx 1.714$	1 point	
The number of subscriptions increased by about 71%.	1 point	
Total:	**2 points**	

16. b)		
The number of years passed is $x = 8$.	1 point	
$51\cdot 1.667^8\approx$	1 point	
≈ 3 million 41 thousand, this was the approximate number of mobile subscriptions at the end of 2000.	1 point	
Total:	**3 points**	

16. c) Solution 1		
The number of calls increased by a factor of 1.065 from one month to the next.	1 point	
(Let n be the number of months passed since January, 1991.) $350\,000 \cdot 1.065^n = 100\,000\,000$	1 point	
$n = \log_{1.065} \frac{100\,000\,000}{350\,000}$,	1 point	$n = \frac{\log \frac{100\,000\,000}{350\,000}}{\log 1.065}$
$n \approx 90$.	1 point	
The number of years passed is: $\frac{90}{12} = 7.5$.	1 point	
The month, in which the number of mobile calls first reached 100 million was in 1998.	1 point	
Total:	**6 points**	

16. c) Solution 2		
The number of calls increased by a factor of 1.065 from one month to the next,	1 point	
in one year it equals a factor of $1.065^{12} \approx 2.13$.	1 point	
(Let m be the number of years passed since January, 1991.) $350\,000 \cdot 2.13^m = 100\,000\,000$	1 point	
$m = \log_{2.13} \frac{100\,000\,000}{350\,000}$,	1 point	$m = \frac{\log \frac{100\,000\,000}{350\,000}}{\log 2.13}$
$m \approx 7.5$.	1 point	
The month, in which the number of mobile calls first reached 100 million was in 1998.	1 point	
Total:	**6 points**	

Notes: 1. Award full score if the candidate gives their answer by correctly listing the number of calls per month at the end of each year.
2. Award the appropriate score if the candidate uses an inequality instead of an equation.

16. d)		
The numbers of land-line calls form consecutive terms of a geometric sequence whose first term is 4200 (million),	1 point	*These points are also due if the correct reasoning is reflected only by the solution.*
and common ratio is $q = 0.92$.	1 point	
$4200 \cdot 0.92^9 \approx$	1 point	
$\approx$ 1983 million, this was the number of land-line calls in 2009.	1 point	
Between the beginning of 2000 and the end of 2009, a total of $4200 \cdot \frac{0.92^{10} - 1}{0.92 - 1} \approx$	1 point	
$\approx$ 29 695 million calls were initiated from land-lines.	1 point	
Total:	**6 points**	

Note: Deduct a total 1 point if the candidate gives both answers without the “millions”.

17. a) **Solution 1**		
The length of the sides of triangle *ATC* (apply the formula for the distance between two points): $AT = \sqrt{17}$, $CT = \sqrt{68}$, $AC = \sqrt{85}$.	2 points	*Award 1 point if there is 1 incorrect value, 0 points if there are more.*
As $(\sqrt{17})^2 + (\sqrt{68})^2 = (\sqrt{85})^2$,	1 point	
(apply the converse of the Pythagorean Theorem) the angle asked is a right angle.	1 point	
Total:	**4 points**	

17. a) **Solution 2**		
$\overrightarrow{AT}(4; 1)$	1 point	
$\overrightarrow{CT}(2; -8)$	1 point	
The scalar product of the two vectors is $4 \cdot 2 + 1 \cdot (-8) = 0$,	1 point	$\overrightarrow{CT}$ *is the double of the –90° rotated image of* $\overrightarrow{AT}$.
and so the angle asked is a right angle.	1 point	
Total:	**4 points**	

17. a) **Solution 3**		
$\overrightarrow{AT}(4; 1)$	1 point	
This is one normal vector of line *e*,	2 points	
and so the angle asked is a right angle.	1 point	
Total:	**4 points**	

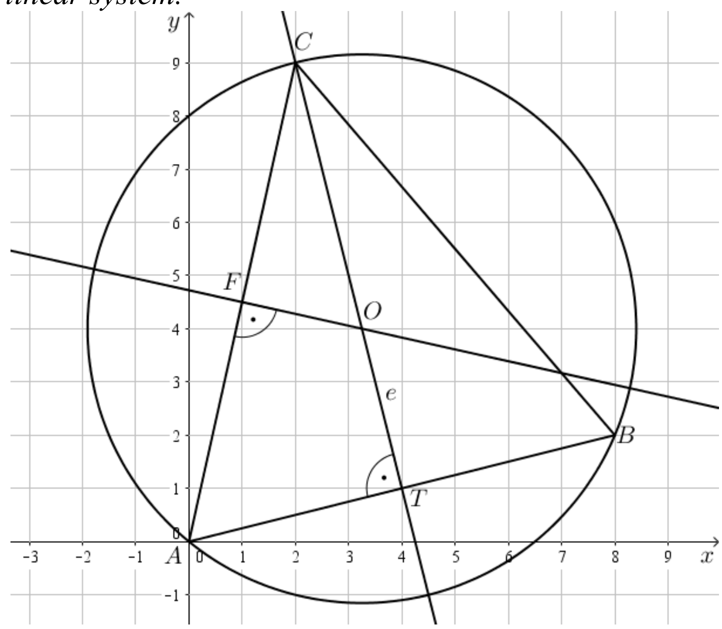
17. a) **Solution 4**		
The midpoint of line segment *AC* is *F*(1; 4.5).	1 point	
The distance between points *F* and *A* (also from *C*) is $\sqrt{21.25}$,	1 point	
which is equal to the distance between points *F* and *T*,	1 point	
so, according to Thales' Theorem, the angle asked is a right angle.	1 point	
Total:	**4 points**	

17. b)		
As angle *ATC* is a right angle, point *T* is the point of intersection of line *e* and the line drawn perpendicularly from point *A* to line *e*,	1 point	*These points are also due if the correct reasoning is reflected only by the solution.*
so point *T* bisects the line segment *AB*.	1 point	
For the coordinates of point $B(b_1; b_2)$: $\frac{0 + b_1}{2} = 4$ and $\frac{0 + b_2}{2} = 1$, $b_1 = 8$ and $b_2 = 2$ (*B*(8; 2)).	2 points	
Total:	**4 points**	

Notes:
1. Award 2 points if the candidate correctly reads the coordinates of point B off a diagram. Award a further 2 points for proving that point T bisects the line segment AB.
2. Award 1 point for the equation of line AT ($x = 4y$)*, another 1 point for the equation of a circle with centre T and radius AT* $((x-4)^2 + (y-1)^2 = 17)$.
and 2 more points for finding the point of intersection (other than A) of the above line and circle.

17. c)		
The centre of the circumcircle of triangle ABC is the point of intersection of line e and the perpendicular bisector f of the line segment AC.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
Let point F be the midpoint of segment AC: $F(1; 4.5)$.	1 point	*The midpoint of segment BC is* $G(5; 5.5)$.
One normal vector of line f is: $\overrightarrow{AC}(2; 9)$.	1 point	$\overrightarrow{BC}(-6; 7)$
The equation of line f is: $2x + 9y = 42.5$.	2 points	*The perpendicular bisector of BC:* $-6x + 7y = 8.5$.
The (x, y) coordinates of the circle are obtained by solving the equation system: $\begin{cases} 2x+9y=42.5 \\ 4x+y=17 \end{cases}$.	1 point	
Subtract the second equation from the double of the first: $17y = 68$,	1 point	
from which $y = 4$, $x = 3.25$. (The centre of the circumcircle is $O(3.25; 4)$.)	2 points	
Total:	**9 points**	

Note: If the candidate finds the coordinates of the circle by solving a quadratic system in three variables, award 2 points for giving the correct system. Award 4 more points for transforming this into a linear system.



18. a)		
The time intervals allowed for consecutive problems in this plan form terms of an arithmetic sequence whose first term is 1 (minute) and the sum of the first 25 terms is 75 (minutes).	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
Let d be the common difference of the sequence. In this case: $\frac{2\cdot 1+24d}{2}\cdot 25=75$.	1 point	
That is $d=\frac{1}{6}$ (minutes).	2 points	
The sum of the first 21 terms of the sequence: $\frac{2\cdot 1+20\cdot\frac{1}{6}}{2}\cdot 21=$	1 point*	
$=56$.	1 point*	
Vera's plan allows (75 – 56 =) 19 minutes for the last 4 problems.	1 point	
Total:	**7 points**	

*Note: The 2 points marked by * may also be awarded if the candidate correctly calculates the times allowed for problems 22, 23, 24, and 25 (* $4\frac{3}{6}$, $4\frac{4}{6}$, $4\frac{5}{6}$ *and* 5 *minutes respectively).*

18. b) Solution 1		
In Vera's case $T=25$, $C+I=22$.	1 point	
Let x be the number of correct answers. In this case $4x-(22-x)+25=93$.	2 points	
As $x=18$, the number of Vera's correct answers is 18.	1 point	
Check: $4\cdot 18-4+25=93$.	1 point	
Total:	**5 points**	

18. b) Solution 2		
Increase the score awarded by 1 in each case. This means 5 points for correct answers, 0 points for incorrect answers, 1 point for problems skipped.	2 points	
To calculate the total score, apply the formula $5C+S$, where S is the number of problems skipped.	1 point	
In Vera's case $S=3$, $5C=90$,	1 point	
so the number of correct answers is 18.	1 point	
Total:	**5 points**	

18. c) **Solution 1**		
Let x be the number of students who solved problem 24, same as the number of students solving 25, and the number of students solving neither.	1 point	*Let y be the number of students who solved problem 24 only, same as the number of those solving 25 only.*
There were $x - 1$ students solving problem 24 only, also $x - 1$ students solving 25 only.	1 point	*There were $y + 1$ students who solved neither problem.*
According to the text: $2(x - 1) + 1 + x = 11$,	1 point	$2y + (y + 1) + 1 = 11$
$x = 4$.	1 point	$y = 3$
There were (4 – 1 =) 3 students who solved problem 24 but not 25.	1 point	
Total:	**5 points**	

18. c) **Solution 2**		
Add the number of students who solved problem 24 to the number of those solving problem 25 to the number of those solving neither. The only one student who solved both problems was counted twice here, so the sum should be 12.	2 points	
As the above numbers are all equal, they must all be 4.	2 points	
There were (4 – 1 =) 3 students who solved problem 24 but not 25.	1 point	
Total:	**5 points**	

Note: Award full score if the candidate gives the correct answer using systematic trial-and-error and also proves that there can be no other solution.