ÉRETTSÉGI VIZSGA • 2017. május 9.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
$x_1 = -2$	1 point	
$x_2 = 0$	1 point	
Total:	**2 points**	

2.		
(23 + 19 – 29) = 13 students would attend both festivals.	2 points	
Total:	**2 points**	

3.		
10111	2 points	
Total:	**2 points**	

4.		
The number of handshakes recorded is 2 + 3 + 4 + 3 + 2.	1 point	*These 2 points are also due for the correct graph.*
However, every actual handshake was counted twice here.	1 point	
So, the total number of handshakes is 7.	1 point	
Total:	**3 points**	

5.		
$x = 16$	2 points	
Total:	**2 points**	

6.		
$x = -1$	2 points	
Total:	**2 points**	

7.		
C	2 points	
Total:	**2 points**	

Note: Award 1 point only if the candidate also marks an incorrect answer besides the correct one.

8.		
The base of the prism is a regular triangle the area of which is $\frac{4^2 \cdot \sqrt{3}}{4}$ (= $4 \cdot \sqrt{3} \approx 6.93$ cm^2).	2 points	
The volume of the prism is $4 \cdot 4 \cdot \sqrt{3} \approx$	1 point	
≈ 27.7 cm^3.	1 point	
Total:	**4 points**	

9.		
$x \geq -1.6$	2 points	
Total:	**2 points**	

10.		
A: true B: false C: true	2 points	*Award 1 point for two correct answers, 0 points for one correct answer.*
Total:	**2 points**	

11.		
$A \cap B \cap C = \{d; e; f\}$	2 points	
$(A \cup B) \setminus C = \{a; b; h\}$	2 points	
Total:	**4 points**	

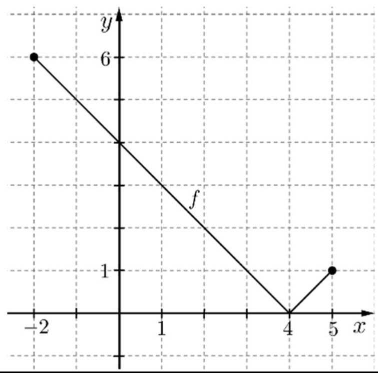
12.		
On throwing 2 dice simultaneously, the number of possible outcomes is 36 (total number of cases).	1 point	
There is only one way for the product to be 9 ($3 \cdot 3$).	1 point	
The probability is $\frac{1}{36} (= 0.02\dot{7})$.	1 point	
Total:	**3 points**	

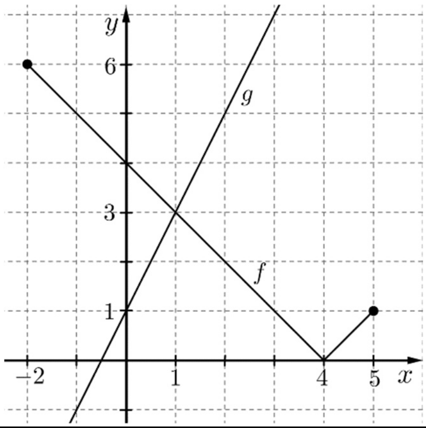
II. A

13. a) Solution 1		
From the first equation $y = 1 - 3x$,	1 point	*From the second equation* $x = 12 - 2y$.
substitute it into the second equation: $x + 2 - 6x = 12$.	1 point	$36 - 6y + y = 1$
$x = -2$,	1 point	
and $y = 7$.	1 point	
Check (e.g. substituting into both equations).	1 point	
Total:	**5 points**	

13. a) Solution 2		
Subtract the second equation from the double of the first: $5x = -10$.	2 points	*Multiply the second equation by 3 and subtract it from the first:* $-5y = -35$.
$x = -2$,	1 point	
and $y = 7$.	1 point	
Check (e.g. substituting into both equations).	1 point	
Total:	**5 points**	

13. b)		
$2 \cdot 5^x + 3 \cdot 5 \cdot 5^x = 425$	1 point	
Combine the like terms: $17 \cdot 5^x = 425$,	1 point	
$5^x = 25$.	1 point	
(As the exponential function is a one-to-one mapping) $x = 2$.	1 point	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**5 points**	

14. a)		
The graph is a transformation of the absolute value function,	1 point	
its minimum at $x = 4$ is 0,	1 point	
and it is restricted to the given domain.	1 point	
Total:	**3 points**	

14. b) **Solution 1**		
Graph function g in the same coordinate system: 	2 points	
The first coordinate of the point of intersection (as seen in the diagram) is $x = 1$.	1 point	
Check by substitution: $f(1) = g(1) = 3$.	1 point	
Total:	**4 points**	

14. b) **Solution 2**		
(We have to solve the equation $\lvert x-4\rvert = 2x+1$.) (in case of $-2 \le x < 4$) $-x+4 = 2x+1$,	1 point	
here $x = 1$ which is a correct solution (e.g. checked by substitution).	1 point	
(in case of $4 \le x \le 5$) $x-4 = 2x+1$,	1 point	
here $x = -5$, but this solution is incorrect.	1 point	
Total:	**4 points**	

14. c) **Solution 1**		
The numbers added form the first 46 terms of an arithmetic sequence	1 point	*These 2 points are also due if the correct reasoning is reflected only by the solution.*
whose first term is equal to the 5^{th} term of the original sequence and its common difference is 2.	1 point	
The 5^{th} term of the original sequence is $(3+4\cdot 2=)\ 11$.	1 point	
The sum is $\frac{2\cdot 11+45\cdot 2}{2}\cdot 46 =$	1 point	
$= 2576$.	1 point	
Total:	**5 points**	

14. c) **Solution 2**		
The sum of the first 50 terms of the sequence is $\frac{2\cdot 3+49\cdot 2}{2}\cdot 50=$	1 point	
$=2600$.	1 point	
The sum of the first 4 terms: (3 + 5 + 7 + 9 =) 24.	1 point	
The answer is the difference of these two sums: 2600 – 24 =	1 point	
= 2576.	1 point	
Total:	**5 points**	

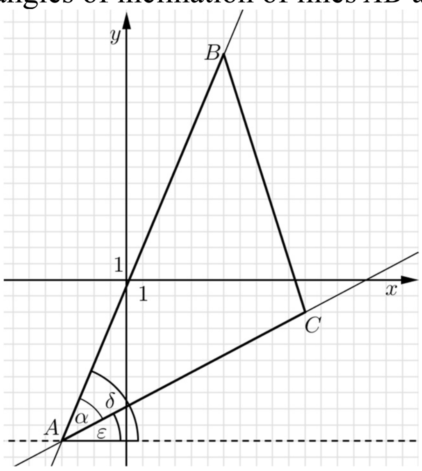
Note: Award full points if the candidate correctly lists and adds the terms of the sequence.

15. a) **Solution 1**		
The midpoint of side *AC* is (3.5; –6).	1 point	
The midpoint of side *BC* is (8.5; 6).	1 point	
The length of the midsegment is $\sqrt{(8.5-3.5)^2+(6-(-6))^2}=$	1 point	
= 13.	1 point	
Total:	**4 points**	

15. a) **Solution 2**		
The length of side *AB* is $\sqrt{(6-(-4))^2+(14-(-10))^2}=$	1 point	
= 26.	1 point	
The length of the midsegment is half of the length of the parallel side,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
so it is 13.	1 point	
Total:	**4 points**	

15. b)		
The altitude that belongs to side *AB* passes through vertex *C* and is perpendicular to side *AB*.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
One normal vector is $\overrightarrow{AB}(10;\,24)$.	2 points	$\mathbf{n}(5;\,12)$
One possible equation of the altitude is $10x+24y=$	1 point	$5x+12y=$
= 62.	1 point	= 31
Total:	**5 points**	

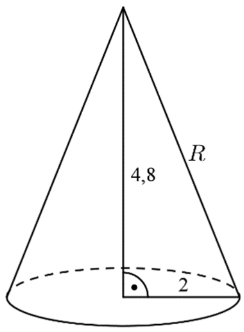
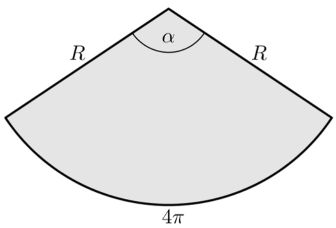
15. c) **Solution 1**		
$AB = \sqrt{(6-(-4))^2 + (14-(-10))^2} = 26$ $AC = \sqrt{(11-(-4))^2 + (-2-(-10))^2} = 17$ $BC = \sqrt{(11-6)^2 + (-2-14)^2} = \sqrt{281}\ (\approx 16.76)$	2 points	
Apply the Law of Cosines for side *BC* of the triangle *ABC*: $281 = 289 + 676 - 2 \cdot 17 \cdot 26 \cdot \cos\alpha$, where α is the angle asked.	1 point	
$\cos\alpha \approx 0.7738$,	1 point	
$\alpha \approx 39.3°$.	1 point	
Total:	**5 points**	

15. c) **Solution 2**		
The interior angle at vertex *A* is the difference between the angles of inclination of lines *AB* and *AC*. 	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
(Let δ be the angle of inclination of line *AB*) $\tan\delta = 2.4$	1 point	
(Let ε be the angle of inclination of line *AC*) $\tan\varepsilon = \frac{8}{15}$.	1 point	
$\delta \approx 67.38°$, $\varepsilon \approx 28.07°$	1 point	
So $\alpha = \delta - \varepsilon \approx 39.3°$.	1 point	
Total:	**5 points**	

15. c) **Solution 3**		
The angle is enclosed by the two side vectors $\overrightarrow{AB}(10; 24)$ and $\overrightarrow{AC}(15; 8)$.	1 point	
The scalar product of these vectors is $10 \cdot 15 + 24 \cdot 8 = 342$,	1 point	
on the other hand, it is $26 \cdot 17 \cdot \cos\alpha$.	1 point	
$\cos\alpha \approx 0.7738$,	1 point	
$\alpha \approx 39.3°$.	1 point	
Total:	**5 points**	

II. B

16. a)		
The radius of one sphere is 10 cm, the radius of the other sphere is 8 cm.	1 point	
The respective volumes of the spheres: $\frac{4}{3}\cdot 10^3\cdot\pi\approx 4189$ (cm^3), and $\frac{4}{3}\cdot 8^3\cdot\pi\approx 2145$ (cm^3),	1 point	
about 6334 (cm^3) altogether.	1 point	
This is 80% of the volume of the uncompressed fill	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
so the volume of the uncompressed fill is $\frac{6334}{80}\cdot 100\approx 7918$ (cm^3),	1 point	
which is about 7.9 litres.	1 point	
Total:	**6 points**	

16. b)		
 The radius R of the sector is the same as the slant height of the cone,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$R=\sqrt{2^2+4.8^2}=5.2$ (cm).	1 point	
The length of the arc of the sector is the same as the circumference of the base circle,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
that is $2\cdot 2\cdot\pi$ (≈ 12.57 cm).	1 point	
 Let α be the degree measure of the central angle of the sector, in which case: $4\pi=\frac{\alpha}{360°}\cdot 2R\pi$.	1 point	$\alpha=\frac{4\pi}{5.2}$ radian =
Hence $\alpha=\frac{2\cdot 360°}{5.2}\approx 138.5°$.	1 point	$\frac{4}{5.2}\cdot 180°\approx 138.5°$
Total:	**6 points**	

16. c)		
There are 6 different possibilities for the size of the eyes.	1 point	
(Denote the different buttons by the numbers 1, 2, 3, 4, 5, 6 in order of increasing size.) There is only one possibility if the top button is size 4 (4-5-6). There are 3 possibilities if the top button is size 3 (3-4-5; 3-4-6; 3-5-6).	1 point	*There are* $\binom{6}{3}$ (= 20) *different possibilities to choose the three front buttons.*
Similarly, there are 6 possibilities if the top button is size 2 and 10 different possibilities if the top button is size 1.	1 point	
The total number of different possibilities is 1 + 3 + 6 + 10 = 20.	1 point	*The buttons are then sewn on in increasing order of size.*
Mum can make $6 \cdot 20$ = 120 different plans.	1 point	
Total:	**5 points**	

17. a)		
The car travelled 70 km in the first hour and 120 km in the second hour.	1 point	
This means a total $\frac{70}{100} \cdot 6 + \frac{120}{100} \cdot 8.5 =$	1 point	
= 4.2 + 10.2 litres.	1 point	
The total distance is therefore 190 km, the total gas consumption is 14.4 litres.	1 point	
The average consumption for the whole journey is $\frac{14.4}{190} \cdot 100 \approx$	1 point	
≈ 7.6 litres (per 100 km)	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**6 points**	

17. b) **Solution 1**		
The car travels (25 · 1.6 =) 40 km on 3.8 litres of gas.	1 point	
The average gas consumption is $\frac{3.8}{40} \cdot 100 =$	1 point	
= 9.5 litres per 100 km.	1 point	
Total:	**3 points**	

17. b) **Solution 2**		
The car travels ($25 \cdot 1.6 =$) 40 km on 3.8 litres of gas.	1 point	
100 km is 2.5 times 40 km,	1 point	
and so the average gas consumption is $2.5 \cdot 3.8 = 9.5$ litres per 100 km.	1 point	
Total:	**3 points**	

17. c)		
(Let x be the number of miles travelled on the first day.) $186 = x \cdot 0.9^6$.	2 points	
Mr. Kovács drove $x = \frac{186}{0.9^6} \approx 350$ miles on the first day.	1 point	
Total:	**3 points**	

Note: Award full points if the candidate gives the right answer by calculating the distance travelled each day (correctly rounded).

17. d)		
License plates may end in 10^4 different four-digit variations.	1 point	
In $10 \cdot 9 \cdot 8 \cdot 7$ (= 5040) cases all four digits will be different.	1 point	
The probability of four different digits appearing on a randomly selected plate is $\frac{10 \cdot 9 \cdot 8 \cdot 7}{10^4} = 0.504$.	1 point	
The probability of selecting a plate with some identical digits is $1 - 0.504 = 0.496$.	1 point	$0.504 > 0.5$
Therefore, the probability of selecting a plate with four different digits is greater than the probability of selecting one with (some) identical digits.	1 point	
Total:	**5 points**	

18. a)		
(Measure all accelerations in $\frac{\text{m}}{\text{s}^2}$.) The average of the 8 values is 9.85	1 point	*Award these points if the candidate uses a calculator to obtain the standard deviation directly.*
the standard deviation is $\sqrt{\frac{0.05^2+0.1^2+0.15^2+0^2+0.05^2+0.1^2+0.1^2+0.05^2}{8}}=$ $=\sqrt{\frac{0.06}{8}}=\sqrt{0.0075}$	1 point	
≈ 0.087,	1 point	
this is less than 0.1, so the experiment is successful.	1 point	
Total:	**4 points**	

18. b)		
Calculate the average as weighed arithmetic mean.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$\frac{2\cdot 9.7+7\cdot 9.75+10\cdot 9.8+8\cdot 9.85+7\cdot 9.9+6\cdot 9.95}{40}\approx$	1 point	
$\approx 9.84\left(\frac{\text{m}}{\text{s}^2}\right)$	1 point	
Arranged in increasing order, the 20$^{\text{th}}$ and 21$^{\text{st}}$ results are both 9.85 $\frac{\text{m}}{\text{s}^2}$,	1 point	
so the median is 9.85 $\left(\frac{\text{m}}{\text{s}^2}\right)$.	1 point	
Total:	**5 points**	

18. c) **Solution 1**		
If one brass ball is loaded first, the other may be on 8 different positions.	1 point	
Similarly, if the first ball is loaded on the 2nd, 3rd, …, 8th position, the other may be on 7, 6, …, 1 different positions.	2 points	
The answer is the sum of these numbers:	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$(8 + 7 + \ldots + 1 =)$ 36.	1 point	
Total:	**5 points**	

18. c) **Solution 2**		
The number of suitable orders is the difference of the total number of orders and the number of wrong ones.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The total number of possible orders (i.e. choose 2 places for the brass balls out of 10) is $\binom{10}{2} =$	1 point	
= 45.	1 point	
Placing the brass balls next to each other, the pair may be placed in 9 different "positions".	1 point	
$45 - 9 = 36$ is the number of cases where the two brass balls are not placed next to each other.	1 point	
Total:	**5 points**	

18. c) **Solution 3**		
The 8 iron balls make 9 possible "slots" for the brass balls. (In these "slots" the brass balls are separated.)	2 points	
We have to choose 2 out of these 9 slots.	1 point	
So, there are $\binom{9}{2} =$	1 point	
= 36 different possibilities.	1 point	
Total:	**5 points**	

18. d)		
The probability of a trial being successful is $1 - 0.06 = 0.94$.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
(As all trials are independent) the probability of all 40 trials being successful is $0.94^{40} \approx$	1 point	
≈ 0.084.	1 point	
Total:	**3 points**	