ÉRETTSÉGI VIZSGA • 2016. október 18.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

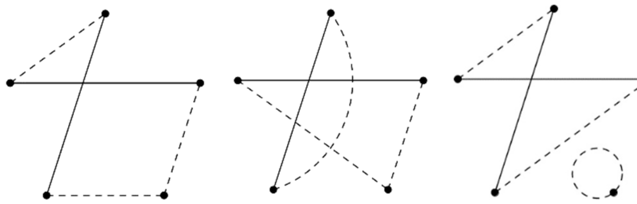
1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams.

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the markscheme.
2. Subtotals may be **further divided, unless stated otherwise in the markscheme**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the markscheme shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
11. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
12. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the markscheme.
13. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1.		
Some possible solutions (the solution does not need to be a simple graph): 	2 points	*Not to be divided.*
Total:	**2 points**	

2.		
3	2 points	
Total:	**2 points**	

3.		
38 = 7 + 31	2 points	
Total:	**2 points**	

Note: Award 0 points if the candidate writes 38 as the sum of a prime and a non-prime. Award 1 point if the answer is 19+19.

4.		
There are ($5\cdot4\cdot3\cdot2=$) 120 such numbers.	2 points	
Total:	**2 points**	

Note: Award 1 point if the candidate ignores the fact that the number needs to have different digits and, consequently, their answer is ($5^4=$) 625. *Award 1 point if the candidate ignores the fact that the number needs to have odd digits only and, consequently, their answer is* ($9\cdot9\cdot8\cdot7=$) 4536.

5.		
A) false B) false C) true	2 points	*Award 1 point for two correct answers, 0 points for one correct answer.*
Total:	**2 points**	

6. Solution 1		
(The radius of the smaller cylinder is r, the height is m, the respective values for the larger cylinder are $2r$ and $2m$.) The volume of the smaller cylinder is $r^2\pi \cdot m$,	1 point	
the volume of the larger cylinder is $(2r)^2\pi \cdot 2m =$	1 point	
$= 8r^2\pi \cdot m$.	1 point	
The volume of the larger cylinder is therefore 8 times the volume of the smaller one.	1 point	
Total:	**4 points**	

6. Solution 2		
The two cylinders are similar, the ratio of similarity is 1:2.	2 points	
The ratio of their volumes (the cube of the ratio of similarity) is 1:8.	1 point	
The volume of the larger cylinder is therefore 8 times the volume of the smaller one.	1 point	
Total:	**4 points**	

Note: Award 3 points if the candidate uses some concrete values and obtains the correct answer thereby. Award full credit if the candidate also states that their use of concrete values does not affect the general method of solution.

7.		
[–1; 3]	2 points	*The correct answer is acceptable in any other form, too.*
Total:	**2 points**	

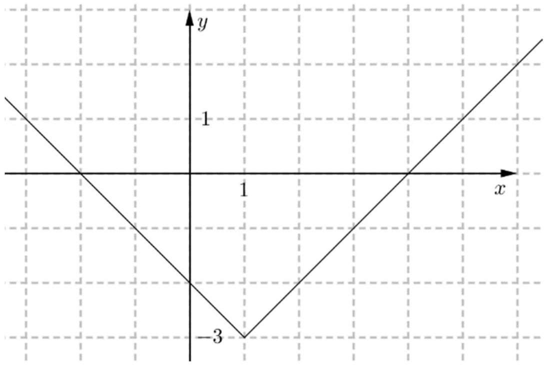
8.		
$\frac{\pi}{6}$	1 point	
$\frac{5\pi}{6}$	1 point	
Total:	**2 points**	

Note: Award 1 point if the answer is 30°and 150°. Award 1 point at most if the answers are given as real numbers but the stated interval is ignored.

9. Solution 1		
40% of 8 km is 3.2 km.	1 point	
The length of the distance still to be covered is 8 – 3.2 – 1.2 = 3.6 (km).	1 point	
$\frac{3.6}{8} = 0.45$	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
45% of the 8-km trip is still to be covered.	1 point	
Total:	**4 points**	

9. Solution 2		
1200 metres is 15% of 8000 metres.	2 points	
(15 + 40 =) 55% of the total distance has been covered so far.	1 point	
45% of the 8-km trip is still to be covered.	1 point	
Total:	**4 points**	

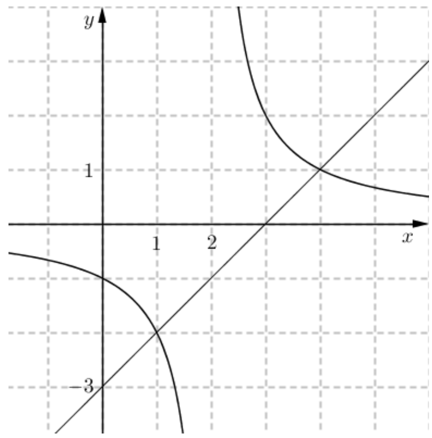
10.		
$(\log_6(2\cdot 3) =)\ 1$	2 points	
Total:	**2 points**	

11.		
The graph of function *f*: 	1 point	$0 = \|x-1\| - 3$
	1 point	$x-1=3$ or $x-1=-3$
The zeros: 4	1 point	
and –2.	1 point	
Total:	**4 points**	

12.		
D	2 points	*Not to be divided.*
Total:	**2 points**	

II. A

13. a) Solution 1		
$2=(x-2)(x-3)$	1 point	
$2=x^2-2x-3x+6$	1 point	
$x^2-5x+4=0$	1 point	
The roots of the quadratic equation: $x_1=1$, $x_2=4$.	2 points	
Check by substitution or reference to equivalent steps, while also showing the domain ($x \neq 2$).	1 point	
Total:	**6 points**	

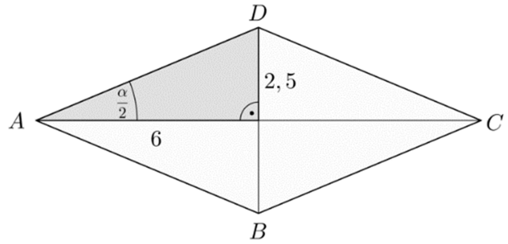
13. a) Solution 2		
Correct diagram of the graph of the function $x \mapsto \dfrac{2}{x-2}$ $(x \neq 2)$.	2 points	
Correct diagram of the graph of the function $x \mapsto x-3$ in the same coordinate system.	1 point	
The first coordinates of the points of intersection: $x_1=1$, $x_2=4$.	2 points	
Check by substitution.	1 point	
Total:	**6 points**	

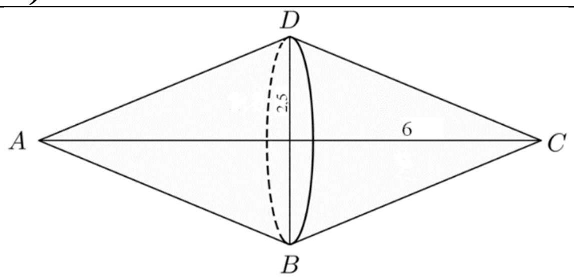
13. b)		
$9\cdot 9^x-7\cdot 9^x=54$	1 point	
$2\cdot 9^x=54$	1 point	
$9^x=27$	1 point	
$3^{2x}=3^3$	1 point	$x=\log_9 27$
(As the base 3 exponential function is a one-to-one mapping) $x = 1.5$.	1 point	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**6 points**	

14. a)		
Distances, measured in km, run at each consecutive week form (consecutive) terms of an arithmetic sequence. The first term is 15, the 11^{th} term is 60.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
About the common difference d of the arithmetic sequence: $15 + 10d = 60$.	1 point	
Which gives $d = 4.5$.	1 point	
Andrea runs 4.5 km more each week than she did during the week before.	1 point	
Total:	**4 points**	

14. b)		
$S_{11} = \frac{15+60}{2} \cdot 11 =$	2 points	*These 2 points are also due if the candidate correctly lists the distances run each week.*
= 412.5 km is the total distance run by Andrea during the 11 weeks.	1 point	
Total:	**3 points**	

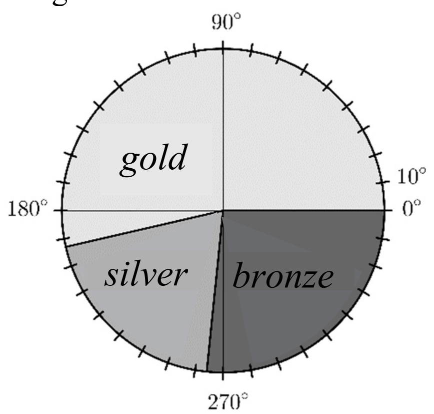
14. c)		
If Gabi increases the distance by the same percentage p each week, this means the same multiplicative factor ($q = 1 + \frac{p}{100}$) is applied each week.	1 point	*These points are also due if the candidate's solution is less detailed.*
Distances, measured in km, run at each consecutive week form (consecutive) terms of a geometric sequence. The first term is 15, the 11^{th} term is 60.	1 point	
About the common ratio q of the geometric sequence: $15 \cdot q^{10} = 60$.	1 point	
Which gives $q \approx 1.15$ (as $q > 0$).	1 point	
Gabi runs about 15% more each week than she did during the week before.	1 point	
Total:	**5 points**	

15. a)		
 (Diagonals of a rhombus are angle bisectors and also perpendicular bisectors of one another.)	1 point	
Denote the interior angle at vertex *A* by α: $\tan\frac{\alpha}{2}=\frac{2.5}{6}(\approx 0.4167)$,	1 point	
$\frac{\alpha}{2}\approx 22.6^\circ$.	1 point	
The angles at vertices *A* and *C* are about 45.2°,	1 point	
the angles at vertices *B* and *D* are about 134.8°.	1 point	
Total:	**5 points**	

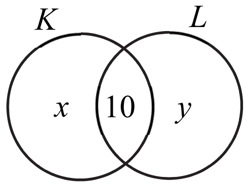
15. b)		
 The solid of revolution obtained is made by two congruent cones sharing a common base circle.	1 point	*These points are also due if the correct reasoning is reflected only by the solution.*
The radius of the base circle is 2.5 cm, the height of each cone is 6 cm.	1 point	
(The slant height of each cone is equal to the side *a* of the rhombus. Apply the Pythagorean Theorem:) $2.5^2+6^2=a^2$.	1 point	
That is: $a = 6.5$ (cm).	1 point	
The surface area of one cone: $2.5^2\cdot\pi+2.5\cdot\pi\cdot 6.5(=22.5\pi\approx 70.7\text{ cm}^2)$.	1 point	*This point is also due if the candidate gives the correct answer without calculating the surface area of the cone.*
(The surface area of the solid of revolution is obtained by subtracting the areas of the base circles from the total surface areas of the two cones:) $A=2\cdot 22.5\pi-2\cdot 2.5^2\pi=$	1 point	$A=2\cdot 2.5\cdot\pi\cdot 6.5$
$=32.5\pi\approx 102.1\text{ cm}^2$.	1 point	
Total:	**7 points**	

Note: Award a maximum of 5 points if the candidate calculates the surface area of the solid obtained by rotating the rhombus about diagonal BD $(2\cdot 6\cdot\pi\cdot 6.5=78\pi\approx 245\text{ cm}^2)$.

II. B

16. a)		
Hungary won a total 15 medals, so the equivalent of one medal in the diagram is a sector of 24°.	1 point	
Gold medals are represented by a sector of 192°, silver 72°, bronze 96°.	1 point	
A possible diagram: 	2 points	*Award 1 point for drawing the sectors with the appropriate central angles, and 1 point for the clear labelling of the diagram.*
Total:	**4 points**	

16. b) Solution 1		
Let x be the number of students watching the Olympic finals of the women's kayak only. In this case the number of students watching the soccer championship finals is $32 - x$,	1 point	
the number of students watching the kayak finals is $10 + x$.	1 point	
As stated in the problem: $2 \cdot (32 - x) = 10 + x$,	1 point	
so $x = 18$.	1 point	
There were 18 students who watched the Olympic finals of women's kayak only (and 4 students who watched the soccer championship finals only).	1 point	
Total:	**5 points**	

16. b) **Solution 2**		
Let x be the number of students watching the Olympic finals of the women's kayak only and let y be the number of students watching the soccer championship finals only. In this case $x + 10 + y = 32$,	1 point	
and also, as stated in the problem, $x + 10 = 2(y + 10)$.	1 point	
The solution of the equation system: $x = 18$ and $y = 4$.	2 points	
There were 18 students who watched the Olympic finals of women's kayak only (and 4 students who watched the soccer championship finals only).	1 point	
Total:	**5 points**	

16. b) **Solution 3**		
Add the number of students watching the Olympic finals of women's kayak and the number of students watching the soccer championship finals. In this sum the number of those watching both events is represented twice,	1 point	
so the final sum will be 10 more than the number of students in class: 42.	1 point	
Divide this in a ratio of 2 : 1, obtaining the numbers for those watching the kayak finals and those watching the soccer finals respectively.	1 point	
These numbers are 28 and 14.	1 point	
Therefore, there were 28 – 10 = 18 students watching the kayak finals only.	1 point	
Total:	**5 points**	

16. c)		
There is only one way for Péter to guess the order of all 5 other nations (excluding Hungary) right.	1 point	
He may not guess only four of them correctly, because that would mean he got the fifth nation right, too.	1 point	
He may guess three of them correctly, in this case the order of the remaining two nations is the other way 'round.	1 point	
This can be done in $\binom{5}{3} \cdot 1 =$	1 point	
= 10 possible ways.	1 point	
The number of favourable cases is 1 + 10 = 11.	1 point	
There can be 5! (= 120) different orders for 5 nations.	1 point	
The probability in question is $\frac{11}{120} \approx 0.092$.	1 point	
Total:	**8 points**	

17. a)		
One normal vector of line e is $\mathbf{n}(1; 2)$.	1 point	$y=-\frac{1}{2}x+6.5$
The gradient is $-\frac{1}{2}$.	1 point	
Substituting $x = 0$ into the equation of the line	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$y = 6.5$, and therefore the point of intersection on the y-axis is $(0; 6.5)$.	1 point	
Total:	**4 points**	

17. b)		
Rearrange the equation of circle k: $x^2+(y+1)^2=45$	1 point	
The centre is the point $(0; –1)$,	2 points	
the radius is $\sqrt{45}\ (\approx 6.71)$ (units).	1 point	
Total:	**4 points**	

17. c) Solution 1		
Prove that the equation system consisting of the equations of line e and circle k has a single solution only.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
Rearrange the equation of the line: $x=13-2y$	1 point	$y=-\frac{1}{2}x+6.5$
Substitute into the equation of the circle: $(13-2y)^2+(y+1)^2-45=0$.	1 point	$x^2+\left(-\frac{1}{2}x+7.5\right)^2-45=0$
$169-52y+4y^2+y^2+2y+1-45=0$	2 points	$x^2+\frac{1}{4}x^2-7.5x+56.25-45=0$
$5y^2-50y+125=0$	1 point	$1.25x^2-7.5x+11.25=0$
Which gives $y = 5$	1 point	$x = 3$
and $x = 3$.	1 point	$y = 5$
The equation system has only one solution and so line e and circle k really have a single common point only.	1 point	
Total:	**9 points**	

17. c) **Solution 2**		
Prove that the line *e* is tangent to the circle *k*, i.e. the distance between line *e* and the centre of circle *k* is equal to the radius of the circle.	2 points	*These 2 points are also due if the correct reasoning is reflected only by the solution.*
The equation of the line drawn through the centre *O* of the circle perpendicular to line *e*: $2x - y = 1$.	2 points*	
To obtain the coordinates of the point of intersection of the two lines, one has to solve the equation system $\left.\begin{array}{l} x+2y=13 \\ 2x-y=1 \end{array}\right\}$	1 point*	
The solution is $x = 3$ and $y = 5$, and so the point of intersection of the two lines is $M(3; 5)$.	2 points*	
The length of line segment *OM* (also the distance between line *e* and the centre of circle *k*) is $\sqrt{3^2+6^2}=\sqrt{45}$.	1 point	
As this is equal to the radius of the circle, line *e* really is tangent to circle *k*.	1 point	
Total:	**9 points**	

*Note: The 5 points marked with * are also due if the candidate calculates the distance between the centre of the circle and line e by correctly using the appropriate formula.*

18. a)		
The mean of the data: $\frac{35+40+51+55+62+67+72+84+92}{9}=$	1 point	*This point is also due if the candidate obtains the correct answer by using a calculator.*
= 62 points.	1 point	
The standard deviation of the data: $\sqrt{\frac{27^2+22^2+11^2+7^2+0+5^2+10^2+22^2+30^2}{9}}\approx$	1 point	*This point is also due if the candidate obtains the correct answer by using a calculator.*
≈ 17.9 points.	1 point	
Total:	**4 points**	

18. b) **Solution 1**		
There are $\binom{9}{3}=84$ ways to select three papers out of nine (this is the total number of cases).	2 points	
There are five papers with a score of at least 60.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*

Mr. Szabó either selects three out of these five which gives $\binom{5}{3}$(= 10) possible ways,	1 point	
or else he selects two out of these three and one from among the other four which can be done in $\binom{5}{2}\cdot\binom{4}{1}$ (= 40) different ways.	2 points	
The number of favourable cases is (10 + 40 =) 50.	1 point	
The probability is $\frac{50}{84}\approx 0.595$.	1 point	
Total:	**8 points**	

18. b) Solution 2		
(Examine the complement event, when there is only 0 or 1 paper with a score of at least 60 points.) There are five papers with scores of at least 60 points and four papers with scores less than 60.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
There are $\binom{4}{3}$(= 4) different ways to select three out of the four papers.	1 point	
There are $\binom{4}{2}\cdot\binom{5}{1}$(= 30) different ways to select two out of the four and one out of the other five papers.	2 points	
The number of favourable cases is (4 + 30 =) 34.	1 point	
There are $\binom{9}{3}= 84$ different ways to select three out of nine papers (total number of cases).	2 points	
The probability is $1-\frac{34}{84}\approx 0.595$.	1 point	
Total:	**8 points**	

18. c)		
The score of one paper is 64 (as the number of data is odd).	1 point	
The sum of the scores of the nine papers graded first is 558,	1 point	
plus 64 is 622 points.	1 point	
The sum of the scores of all 11 papers is $11\cdot 65 = 715$.	1 point	
The score of the 11th paper is therefore (715 – 622 =) 93 points (which is correct, as $93\geq 64$ and so the median is really 64).	1 point	
Total:	**5 points**	