ÉRETTSÉGI VIZSGA ● 2016. május 3.

MATEMATIKA
ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI
ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

EMBERI ERŐFORRÁSOK
MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the markscheme.
2. Subtotals may be **further divided, unless stated otherwise in the markscheme**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the markscheme shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
11. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
12. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the markscheme.
13. **Assess only two out of the three problems in part B of Paper II**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

Attention! The **Instructions to examiners** section at the beginning of this marking scheme has changed substantially! Please, read it carefully before starting correction.

I

1.		
$x_1 = 0$	1 point	
$x_2 = \frac{5}{2}$	1 point	
Total:	**2 points**	

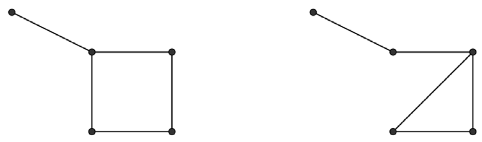
2.		
Statement 1: false.	1 point	
Statement 2: true.	1 point	
Statement 3: true.	1 point	
Total:	**3 points**	

3.		
$x = 25$	2 points	*1 point for* $\log_3 9 = 2$.
Total:	**2 points**	

4.		
The sum of the given digits (18) is divisible by 3.	1 point	
(The digits 3, 4, 6 may appear in the first three positions in any order, so there are) $3 \cdot 2 \cdot 1 =$	1 point	
= 6 such numbers.	1 point	
Total:	**3 points**	

Note: Award 2 points if the candidate lists the 6 appropriate four-digit numbers without an explanation.

5.		
$b_2 = -2$	2 points	*Do not divide.*
Total:	**2 points**	

6.		
A graph satisfying the conditions. Possible solutions: 	2 points	*Do not divide.*
Total:	**2 points**	

7.		
$r^2 = CE^2 = (-2-1)^2 + (3-(-1))^2$	1 point	
$r^2 = 25$ (or $r = 5$)	1 point	
The equation of the circle: $(x-1)^2 + (y+1)^2 = 25$.	1 point	
Total:	**3 points**	

8.		
$P(A) = \frac{1}{6}$	1 point	
$P(B) = \frac{4}{36}\left(=\frac{1}{9}\right)$	2 points	*Do not divide.*
Total:	**3 points**	

Remark. A correct answer given in decimal form or as a percentage is also acceptable.

9.		
Writing down any non-negative number.	2 points	*Do not divide.*
Total:	**2 points**	

Remark. Award the 2 points if the candidate does not name a particular number but mentions that any non-negative (positive) number is suitable.

10.		
$x_1 = -\pi$	1 point	
$x_2 = \pi$	1 point	
Total:	**2 points**	

Note: Award 1 point for an answer in degrees (−180°, 180°). Award 1 point for (correctly) identifying all the zeros of the function in the set of real numbers. ($x = \pi + k \cdot 2\pi$, $k \in \mathbf{Z}$)

11. **Solution 1**		
The ratio of the sides of the two squares (the scale factor of the similarity) is also 1:4.	1 point	*This point is also due if the correct reasoning is revealed by the solution.*
The ratio of the areas of the two squares is 1:16.	1 point	
The area of the larger square is 400 cm^2.	1 point	
Total:	**3 points**	

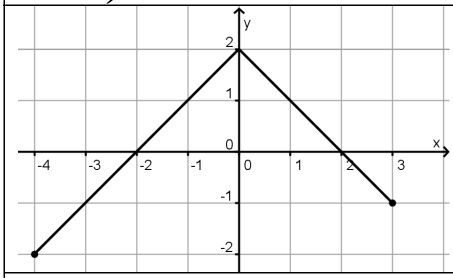
11. **Solution 2**		
The side of the smaller square is 5 cm long.	1 point	
A side of the larger square is 20 cm long.	1 point	
The area of the larger square is 400 cm^2.	1 point	
Total:	**3 points**	

12.		
$1000 - 240 = 760$ participants of the survey have some kind of insurance.	1 point	
The number of all insurances is $470 + 520 = 990$.	1 point	
$990 - 760 = 230$ of the participants have both kinds of insurance.	1 point	
Total:	**3 points**	

Note: Award full score if the correct answer is read off a correct Venn diagram.

II A

13. a)		
$f(-2.85) = 2 - \lvert -2.85 \rvert =$	1 point	*This point is also due if the correct reasoning is revealed by the solution.*
$= -0.85.$	1 point	
Total:	**2 points**	

13. b)		
		*3 points altogether for graphing the function correctly.*
An absolute-value function is graphed, where the gradients of the branches are 1 and –1.	1 point	
The domain of the function graphed is the interval $[-4;3]$.	1 point	
The function graphed has a maximum at $x = 0$, and the value of the maximum is 2.	1 point	
The range of the function is the interval $[-2;2]$.	2 points	*Award 1 point if the answer is wrong but the response reveals the knowledge of the concept of range.*
Total:	**5 points**	

13. c)		
$\frac{1}{5} = 5^{-1}$	1 point	
(Since the base 5 exponential function is strictly increasing,) $2 - \lvert x \rvert = -1$.	1 point	
$\lvert x \rvert = 3$	1 point	
$x_1 = 3$; $x_2 = -3$	1 point	*Award 1 point if the candidate only finds one root and checks that.*
Checking (of both roots).	1 point	
Total:	**5 points**	

Note: Award at most 3 points if the candidate attempts to solve part c) graphically but uses the wrong graph.

<table>
<tr><td colspan="3">14. a)</td></tr>
<tr><td>

	Blood Group			
	0	A	B	AB
Relative frequency	0.31	0.45	0.16	0.08

</td><td>3 points</td><td>0 points for 0 or 1 correct answer.
1 point for 2 correct answers.
2 points for 3 correct answers.

In case of less than 3 correct answers, 1 further point is due if the four numbers in the table add up to 1.</td></tr>
<tr><td align="right">Total:</td><td>3 points</td><td></td></tr>
</table>

14. b) Solution 1		
There are $\binom{125}{2}(=7750)$ different ways to select two out of the 125 donors of blood group zero.	1 point	
The two donors of blood group zero with different Rh factors can be selected in $100\cdot 25\,(=2500)$ different ways.	1 point	
The probability in question: $\dfrac{100\cdot 25}{\binom{125}{2}}\left(=\dfrac{2500}{7750}=\dfrac{10}{31}\right)$.	1 point	
Rounded to two decimal places, it is 0.32.	1 point	
Total:	**4 points**	

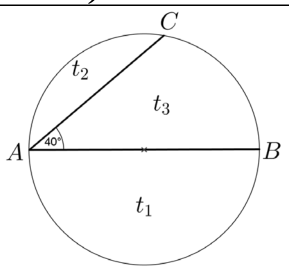
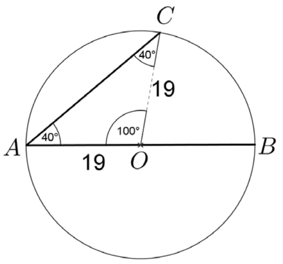
14. b) Solution 2		
The probability of selecting Rh positive first and then selecting Rh negative is $\dfrac{100\cdot 25}{125\cdot 124}$.	1 point	
The probability of selecting Rh negative first and then selecting Rh positive is $\dfrac{25\cdot 100}{125\cdot 124}$.	1 point	
The probability in question is the sum of these.	1 point	
Rounded to two decimal places, it is 0.32.	1 point	
Total:	**4 points**	

Note: Award 2 points if the candidate uses the model of sampling with replacement, and calculates correctly in the model chosen $\left(p=\dfrac{2\cdot 100\cdot 25}{125\cdot 125}=0.32\right)$.

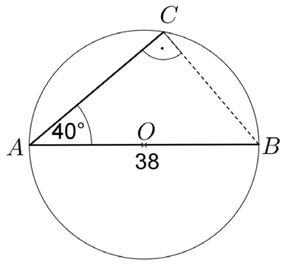
14. c)

<table>
<tr><td>
<table>
<tr><td></td><td>Is the value given in the diagram correct? (yes/no)</td><td>If the value in the diagram is not correct, the correct value is this:</td></tr>
<tr><td>Percentage of donors with Rh positive blood type</td><td>no</td><td>81.25 %</td></tr>
<tr><td>Percentage of donors with Rh negative blood type</td><td>yes</td><td>–</td></tr>
<tr><td>Central angle of sector representing Rh positive</td><td>yes</td><td></td></tr>
<tr><td>Central angle of sector representing Rh negative</td><td>no</td><td>67.5°</td></tr>
</table>
</td><td>1 point each</td><td></td></tr>
<tr><td>Total:</td><td>5 points</td><td></td></tr>
</table>

15. a)

 The area of the semicircle (marked with t_1) is $\frac{19^2\pi}{2}\approx$	1 point	*This point is due for substituting correctly in the appropriate formula.*
$\approx 567\text{ m}^2$.	1 point	
 (Triangle *AOC* is isosceles, therefore) the central angle subtended by line segment *AC* is 100°.	1 point	
The area of the circular sector *AOC* with a central angle of 100° is $\frac{19^2\pi\cdot 100°}{360°}$ ($\approx 315\text{ m}^2$).	1 point	*The area of the circular sector BOC with a central angle of 80° is* $\frac{19^2\pi\cdot 80°}{360°}$ *(≈ 252 m²).*
The area of triangle *AOC* is $\frac{19^2\cdot\sin 100°}{2}$ ($\approx 178\text{ m}^2$).	1 point	*This point is due for substituting correctly in the appropriate formula.*

The area of the circular segment is the difference of the areas of the sector and the triangle.	1 point	*This point is also due if the correct reasoning is revealed by the solution.*
Thus the area denoted by t_2 is (315 − 178 =) 137 m^2,	1 point	*$t_3 = 252 + 178 = 430\ m^2$*
and the area denoted by t_3 is 567 − 137 = 430 m^2.	1 point	*$t_2 = 567 – 430 = 137\ m^2$*
Total:	**8 points**	

15. b)		
 (It follows from Thales' theorem that) triangle ABC is right-angled.	1 point	
$\sin 40° = \frac{BC}{38}$	1 point	
$BC = 38 \cdot \sin 40°$	1 point	
$BC \approx 24.4$ m.	1 point	
Total:	**4 points**	

Note: Take off 1 point altogether in the entire problem if any answer is not rounded or is rounded incorrectly.

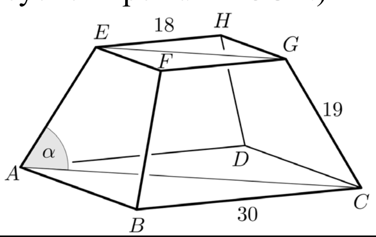
II B

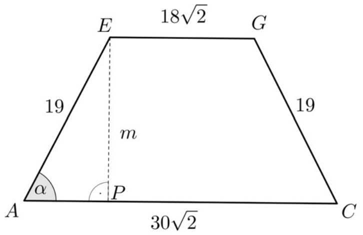
16. a)		
The numbers of seats in the successive rows form an arithmetic progression with first term $a_1 = 60$ and common difference $d = 6$.	1 point	*This point is also due if the correct reasoning is revealed by the solution.*
The 17th term of the sequence is $a_{17} = a_1 + 16d =$	1 point	
$= 156$. (There are 156 seats in row 17.)	1 point	
Total:	**3 points**	

16. b)		
$6786 = \frac{2\cdot 60+(n-1)\cdot 6}{2}\cdot n$	1 point	*This point is due for substituting correctly in the appropriate formula.*
$6n^2+114n-13572=0$	2 points	*Do not divide.*
$n_1=39$	1 point	
$n_2(=-58)<0$, this does not satisfy the conditions of the problem.	1 point	
There are 39 rows of seats in the theatre.	1 point	
Checking: $S_{39}=6786$.	1 point	
Total:	**7 points**	

16. c)		
$6786=\frac{60\cdot(1.1^n-1)}{1.1-1}$	1 point	*This point is due for substituting correctly in the appropriate formula.*
$1.1^n=12.31$	2 points	*Do not divide.*
$n=\frac{\log 12.31}{\log 1.1}$	2 points	$n=\log_{1.1}12.31$
$n\approx 26.34$	1 point	
(Since each term of the sequence is positive,) the terms need to be added up to at least the 27th term to reach 6786.	1 point	
Total:	**7 points**	

Note: Award the appropriate points if the candidate calculates with an inequality instead of an equation (equality).
Also, award the 7 points if the candidate evaluates the first 27 terms and gives a correct answer based on that.

17. a)		
(The angle enclosed by the lateral edges with the base is the angle lying on the longer base of the cyclic trapezium *ACGE*.) 	2 points	*These 2 points are due for identifying the angle in question (even if there is no diagram).*

The lengths of the bases of the cyclic trapezium *ACGE* are $30\sqrt{2}$, and $18\sqrt{2}$.	2 points	
Triangle *APE* is obtained by drawing the height of the trapezium from vertex *E*. 	1 point	
In this triangle, $AP = 6\sqrt{2}$.	1 point	
So $\cos\alpha = \frac{6\sqrt{2}}{19}$ (≈ 0.4466).	1 point	
The angle of the lateral edge and the plane of the base: $\alpha \approx 63.5°$.	1 point	
Total:	**8 points**	

17. b)		
The height *m* of the truncated pyramid can be calculated by using a trigonometric ratio or the Pythagorean theorem.	1 point	*This point is due for setting up the appropriate equation.*
$m = 17$ (cm).	1 point	
$V = \frac{17}{3} \cdot (30^2 + 30 \cdot 18 + 18^2) =$	1 point	*This point is due for substituting correctly in the appropriate formula.*
= 9996 cm^3.	1 point	
Total:	**4 points**	

Note: Other answers and intermediate results are also acceptable if the reasoning and the roundings are correct.

17. c)		
Every vertex of the original graph needs to be connected to four further vertices.	1 point	
That would make 32 new edges to draw,	1 point	
but, since in this way every edge is counted twice,	2 points	
the number of new edges is 16.	1 point	
Total:	**5 points**	

Note: The 5 points are also due if the candidate draws the further edges in the diagram and counts them to get 16. If the candidate answers by using the diagram, take off 1 point for each missing edge of superfluous edge.
Take off (another) 1 point if the candidate answers by using the diagram, but the answer is inconsistent with the diagram.
(The total score awarded for the solution may not be negative.)

18. a)		
The candidate knows that in order to increase a quantity by 2.4% it is multiplied by 1.024.	1 point	
The candidate knows that in order to decrease a quantity by 3.8% (or by 4.7%) it is multiplied by 0.962 (0.953).	1 point	
The population of Győr-Moson-Sopron County in 2001: $449 : 1.024 \approx 438$ (thousand). The population of Vas County in 2001: $258 : 0.962 \approx 268$ (thousand). The population of Zala County in 2001: $283 : 0.953 \approx 297$ (thousand).	2 points	*Award 1 point if there is one error, 0 points for more than one error.*
The population of the entire region in 2001: 1003 thousand, The population of the entire region in 2011: 990 thousand.	1 point	
$\frac{990}{1003} \cdot 100 \approx 98.7$	1 point	
The population of the region decreased by 1.3% between 2001 and 2011.	2 points	*Award at most 1 point if there is no rounding or the rounding is wrong.*
Total:	**8 points**	

Note: Other answers and intermediate results are also acceptable if the reasoning and the roundings are correct.

18. b)		
If there are x thousand men living in Budapest, then the number of women is $1.21x$ thousand. If there are y thousand men living in Pest County, then the number of women is $1.084y$ thousand.	1 point	*This point is also due if the correct reasoning is revealed by the solution.*
Thus, according to the data in the table, $x + 1.21x = 1737$.	1 point	
Hence $x \approx 786$ (thousand men).	1 point	
The number of women in Budapest is about $1737 - 786 = 951$ (thousand).	1 point	
Based on the Pest County data: $y + 1.084y = 1223$.	1 point	
Hence $y \approx 587$ (thousand men).	1 point	
The number of women in Pest County is about $1223 - 587 = 636$ (thousand).	1 point	
The ratio of the number of women to men in the region is $\frac{951+636}{786+587} \approx 1.156$,	1 point	
Therefore the number of women per thousand men in the entire region is about 1156.	1 point	
Total:	**9 points**	