ÉRETTSÉGI VIZSGA ● 2015. október 13.

MATEMATIKA
ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI
ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

EMBERI ERŐFORRÁSOK
MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect,** enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the markscheme.
2. Subtotals may be **further divided, unless stated otherwise in the markscheme**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the markscheme shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
11. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
12. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the markscheme.
13. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

Attention! The **Instructions to examiners** section at the beginning of this marking scheme has changed substantially! Please, read it carefully before starting correction.

I.

1.		
$x_1 = -3$	1 point	
$x_2 = 7$	1 point	
Total:	**2 points**	

2. Solution 1		
The interior angle next to the 104° exterior angle is 76°.	1 point	
The measure of the interior angle at vertex *C* is $180° - (76° + 74°) = 30°$,	1 point	
and the measure of the corresponding exterior angle is 150°.	1 point	
Total:	**3 points**	

2. Solution 2		
The interior angle next to the 104° exterior angle is 76°.	1 point	
The measure of the exterior angle at vertex *C* is equal to the sum of the interior angles at the other two vertices,	1 point	
which is 150°.	1 point	
Total:	**3 points**	

2. Solution 3		
The exterior angle next to the 74° interior angle is 106°.	1 point	
The sum of the exterior angles of the triangle is 360°,	1 point	
and so the measure of the exterior angle at vertex *C* is $(360° - 104° - 106° =)150°$.	1 point	
Total:	**3 points**	

3.		
[0; 2]	2 points	*The correct answer is acceptable in any other form, too.*
Total:	**2 points**	

Note: Award 1 point only, if the solution given by the candidate is an open or semi-open interval but the boundaries of the interval are otherwise correct.

4.		
h	2 points	*Not to be divided.*
Total:	**2 points**	

5.		
$A = \{1; 2; 4; 7; 14; 28\}$ $B = \{1; 7; 49\}$	1 point	
$A \cap B = \{1; 7\}$	1 point	
$B \setminus A = \{49\}$	1 point	
Total:	**3 points**	

6.		
10	2 points	
Total:	**2 points**	

Note: Award 1 point for a correct list of all subsets with exactly two elements.

7.		
A) true B) false C) true	2 points	*Award 1 point for two correct answers, 0 point for one correct answer.*
Total:	**2 points**	

8.		
$x_1 = 4$	1 point	
$x_2 = -8$	1 point	
Total:	**2 points**	

9.		
Range: 6	1 point	
Mean: 3	1 point	
Standard deviation: 2	2 points	
Total:	**4 points**	

Note: Award 1 point if the candidate correctly substitutes all data into the formula of the standard deviation, or if the variance is given.

10.		
The total number of cases is 25.	1 point	
12 of these are divisible by four (this is the number of favourable cases).	1 point	
The probability asked is therefore $\frac{12}{25} = 0.48$.	1 point	
Total:	**3 points**	

11. **Solution 1**		
Gross price is 1.27 times the net price.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The net price of the product is $\frac{6350}{1.27} = 5000\,(\text{Ft})$,	1 point	
and so the VAT is (6350 – 5000 =) 1350 Ft.	1 point	
Total:	**3 points**	

11. **Solution 2**		
VAT is $\frac{6350}{127} \cdot 27 =$	2 points	
= 1350 Ft.	1 point	
Total:	**3 points**	

12.		
Flóra has played 2 games so far.	2 points	
Total:	**2 points**	

II. A

13. a) Solution 1		
$18 - 32 = -14$	1 point	$32 + 2d = 18$
The common difference is –7.	1 point	
$a = 25$	1 point	
Total:	**3 points**	

13. a) Solution 2		
$a = \frac{18+32}{2} =$	1 point	
$= 25$	1 point	
The common difference is –7.	1 point	
Total:	**3 points**	

13. b)		
Denote the common ratio by q, in which case $q^2 = \frac{18}{32}$.	1 point	$b^2 = 32 \cdot 18 = 576$
That is, $q_1 = \frac{3}{4}$	1 point	
and $b_1 = 24$;	1 point	
or $q_2 = -\frac{3}{4}$	1 point	
and $b_2 = -24$.	1 point	
Total:	**5 points**	

13. c)		
The median of the three numbers is c,	1 point	*These 2 points are also due if the correct reasoning is reflected only by the solution.*
their mean is $\frac{32+c+18}{3}$.	1 point	
As $\frac{32+c+18}{3} = c - 2$,	1 point	
$50 + c = 3c - 6$,	1 point	
and therefore $c = 28$ (which really is the median of the three numbers).	1 point	
Total:	**5 points**	

14. a)		
Péter won 25 times out of 30.	1 point	
His score obtained at the fencing event: $250 + 4 \cdot 7 =$	1 point	
$= 278$.	1 point	
Total:	**3 points**	

14. b)		
250 – 215 = 35	1 point	
Bence misses (35:7 =) 5 bouts to make 21,	1 point	
so the total number of bouts won is 16.	1 point	
Total:	**3 points**	

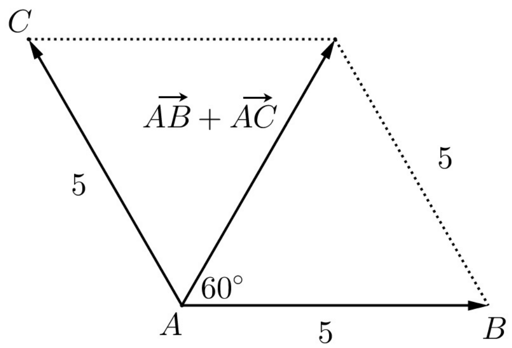
14. c)		
C	1 point	
C	1 point	
Total:	**2 points**	

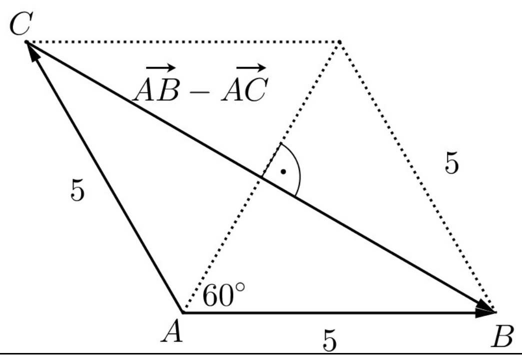
14. d)		
The number of possible orders for obstacles of category *A* is 5!, the same for category *B* is 4!, and for category *C* is 3!.	2 points	
The total number of different orders for the 12 obstacles is the product of the above,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
which is 17 280.	1 point	
Total:	**4 points**	

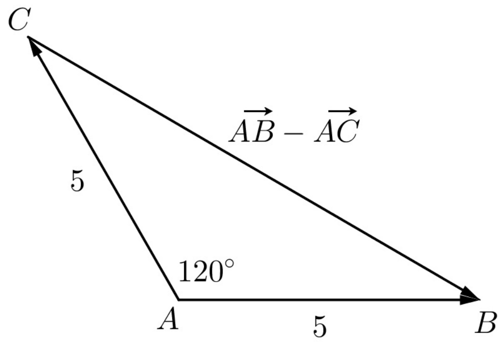
15. a)		
Denote the angle at vertex *A* by α: $\tan\alpha = \frac{8}{6}$,	1 point	
from which α ≈ 53.13°.	1 point	
(Angle β is at vertex *B* and completes angle α to 90°. Therefore) β ≈ 36.87°.	1 point	
Total:	**3 points**	

15. b)		
Let *x* be the length of leg *DF* (in cm-s). *DE* = *x* – 7, *EF* = *x* + 2.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
According to the Pythagorean Theorem: $x^2 + (x-7)^2 = (x+2)^2$.	1 point	
Expanding the brackets: $x^2 + x^2 - 14x + 49 = x^2 + 4x + 4$.	1 point	
Rearranged: $x^2 - 18x + 45 = 0$.	1 point	
One solution of the equation is *x* = 3,	1 point	
which is incorrect, as the length of the other leg would then be negative.	1 point	
The other solution is *x* = 15,	1 point	
so the three sides of the triangle are *DE* = 8cm, *DF* = 15 cm, *EF* = 17 cm.	1 point	
Total:	**8 points**	

II. B

16. a)		
Diagram with vector $\overrightarrow{AB}+\overrightarrow{AC}$. 	1 point	
Vectors $\overrightarrow{AB}+\overrightarrow{AC}$ and $\overrightarrow{AB}$ determine two sides of an isosceles triangle, one angle of which is 60°, thus making it a regular triangle.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The length of the resultant sum vector is therefore 5 units.	1 point	
Total:	**3 points**	

16. b) Solution 1		
Diagram with vector $\overrightarrow{AB}-\overrightarrow{AC}$. 	1 point	
The length of the difference vector is the double of the height of a regular triangle of side 5 units,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
that is, $2\cdot 5\cdot\frac{\sqrt{3}}{2}\approx$	1 point	
≈ 8.66 units.	1 point	
Total:	**4 points**	

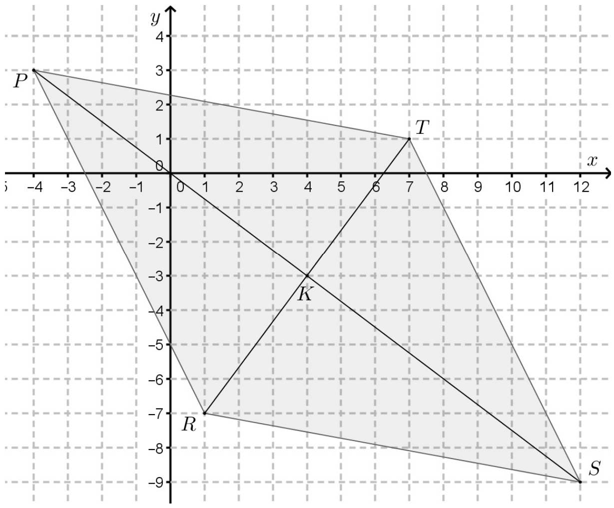
16. b) **Solution 2**		
Diagram with vector $\overrightarrow{AB}-\overrightarrow{AC}$. 	1 point	
Applying the Law of Cosines:	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$\left\lvert\overrightarrow{AB}-\overrightarrow{AC}\right\rvert=\sqrt{5^2+5^2-2\cdot5\cdot5\cdot\cos 120°}\approx$	1 point	
≈ 8.66 units.	1 point	
Total:	**4 points**	

16. c)		
(The diagonals of the rhombus bisect each other perpendicularly in point *K*, so *K* is the midpoint of diagonal *TR*.) For the coordinates of point $R(x_R;y_R)$ $\frac{7+x_R}{2}=4$, and $\frac{1+y_R}{2}=-3$ is true.	1 point	
Hence $x_R=1$ and $y_R=-7$, i.e. *R*(1; –7).	1 point	
$\overrightarrow{KT}(3;4)$.	1 point*	
Rotate this by 90° thus obtaining the vector (–4; 3).	2 points*	
The double of which is the vector $\overrightarrow{KP}$ (–8; 6);	1 point*	
the opposite of which is the vector $\overrightarrow{KS}$ (8; –6).	1 point*	
Add the appropriate coordinates of these vectors to the coordinates of the point *K*, thus obtaining the coordinates of the missing vertices.	1 point*	*This point is also due if the correct reasoning is reflected only by the solution.*
P(–4; 3)	1 point	
and *S*(12; –9).	1 point	
Total:	**10 points**	

*The 6 points marked by * may also be given for the following reasoning:*

The length of the half of the shorter diagonal of the rhombus is $d_{KT} = \sqrt{3^2+4^2} = 5$ units long.	1 point	
The equation of the circle k of centre K and radius 10 units is: $(x-4)^2+(y+3)^2=100$.	1 point	
Line e of the diagonal PS passes through point K. One of its normal vectors is $\overrightarrow{KT}(3;4)$.	1 point	
The equation of line e is then: $3x+4y=0$.	1 point	
The solutions of the system made by the equations of line e and circle k: $x_1=-4$ and $y_1=3$,	1 point	
or $x_2=12$ and $y_2=-9$.	1 point	

Note: Award 4 points at most, if the candidate correctly, but without any further explanation, reads the coordinates of the missing points off a diagram.



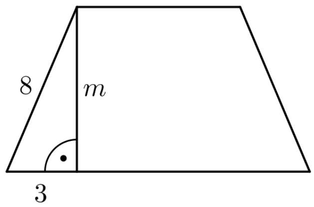
17. a) Solution 1		
$t(0) = 3600$	1 point	
$t(2) \approx 2626$	1 point	
$\frac{2626}{3600} \approx 0.73$	1 point	
The number of tigers decreases by about 27%.	1 point	
Total:	**4 points**	

17. a) Solution 2		
Each year, the number of tigers drops to 0.854 times that of the previous year.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$0.854^2 \approx$	1 point	
≈ 0.73	1 point	
The number of tigers decreases by about 27%.	1 point	
Total:	**4 points**	

17. b)		
Solve the equation $3600 \cdot 0{,}854^x = 900$ (here x refers to the number of years passed since 2014).	1 point	
$0.854^x = 0.25$	1 point	
$x = \frac{\log 0.25}{\log 0.854} \approx$	1 point	$x = \log_{0.854} 0.25$
≈ 8.78	1 point	
The number of tigers is expected to drop below 900 in (9 years, that is, in) 2023.	1 point	
Total:	**5 points**	

Notes: 1. Award the appropriate points if the candidate solves an inequality, rather than an equation.
2. Award maximum points if the candidate correctly calculates and rounds the number of tigers year by year until they obtain the correct number.

17. c)		
According to (I) and (II) there can be three or four tigers in the smaller pen.	1 point	
Assuming this number is three, (III) and (IV) guarantees that there may only be two females and one male.	1 point	
Two females and one male may be selected in $\binom{5}{2} \cdot 4$ (= 40) different ways (and such a selection will also clearly determine the six tigers that will be placed into the other pen, too).	2 points	
If the number of tigers in the smaller pen is four, (III) and (IV) (and considering what tigers will remain in the other pen) will guarantee that these will be two females and two males.	1 point	
Two females and two males may be selected in $\binom{5}{2} \cdot \binom{4}{2}$ (= 60) different ways (which also determines the five tigers placed into the other pen).	2 points	
The total number of cases is therefore 40 + 60 = 100.	1 point	
Total:	**8 points**	

18. a)		
The area of a regular hexagon of side a can be obtained by adding the areas of six regular triangles of side a.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
The area of the top of the truncated pyramid: $A_1 = 6 \cdot \frac{7^2 \cdot \sin 60°}{2} \approx$	1 point	
≈ 127.3 (cm^2).	1 point	
Let m be the height of one of the lateral faces of the truncated pyramid: 	1 point	
Use the Pythagorean Theorem: $3^2 + m^2 = 8^2$.	1 point	
$m \approx 7.42$ (cm)	1 point	
The area of one lateral face: $A_2 = \frac{7+13}{2} \cdot 7.42 = 74.2$ (cm^2).	1 point	
The total surface area: $F = A_1 + 6 \cdot A_2 = 572.5$ (cm^2).	1 point	
0.93 m^2 = 9300 cm^2	1 point	
9300 : 572.5 ≈ 16.24	1 point	
1 kg of plastic granulate is enough to make 16 boxes.	1 point	
Total:	**11 points**	

18. b)		
"At least 8 bulbs spring into flower" means 8, 9 or 10 bulbs.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
The probability of all 10 bulbs springing into flower: 0.91^{10} (≈ 0.3894).	1 point	
The probability of exactly 9 bulbs springing into flower: $\binom{10}{9} \cdot 0.91^9 \cdot 0.09 (\approx 0.3851)$.	1 point	
The probability of exactly 8 bulbs springing into flower: $\binom{10}{8} \cdot 0.91^8 \cdot 0.09^2$ (≈ 0.1714).	1 point	
The probability asked is the sum of the above three,	1 point	
about 0.946.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**6 points**	

Note: Accept 0.945 if the candidate rounds each probability to three decimal places correctly and then adds them.