ÉRETTSÉGI VIZSGA • 2015. május 5.

# MATEMATIKA ANGOL NYELVEN

# KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

# JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

## EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

# Instructions to examiners

**Formal requirements:**

1. Mark the paper in **ink, different in colour** from that used by the candidate. Indicate errors, incomplete solutions, etc. in the conventional way.
2. The first of the rectangles below each problem contains the maximum score attainable on that problem. The **points given by the examiner** are to be entered in the rectangle next to this.
3. **In case of a complete, unobjectionable solution** it is enough to enter the maximum score in the appropriate rectangle.
4. In case of incomplete or incorrect solutions, please indicate **partial scores**, too.
5. Do not assess anything **written in pencil** except diagrams.

**Assessment of content:**

1. For certain problems, the answer key may contain more than one marking schemes. If the **solution** given by the candidate **is different from those shown**, mark it by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals in the answer key **may be further divided**, but the scores awarded must always be whole numbers.
3. In case of a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward without altering the problem substantially, the points for the rest of the solution should be awarded.
4. In case of a **conceptual error**, no points should be awarded for that section of the solution, not even for steps that are formally correct. (Sections of the solution are separated by double lines in the answer key.) However, if the incorrect result obtained through the conceptual error is carried forward to the next section or the next part of the problem and it is used correctly, the maximum score is due for all parts thereafter, as long as the error has not altered the problem substantially.
5. Where the answer key indicates a unit or remark **in brackets**, the solution should be considered complete without that unit or remark as well.
6. In case of **more than one different approach** to a problem, assess only the one indicated by the candidate.
7. **Do not give extra points** (i.e. more than the maximum score for the problem or part of problem).
8. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
9. **Assess only two of the three problems in part B of Paper II.** Candidates are requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Any solution given to that problem must be ignored. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

# I.

| 1. | | |
|---|---|---|
| $A \cap B = \{3;4;5\}$ | 1 point | |
| $B \cup C = \{3;4;5;6;7;8;9;10\}$ | 1 point | |
| $A \setminus B = \{1; 2\}$ | 1 point | |
| **Total:** | **3 points** | |

| 2. | | |
|---|---|---|
| 14 | 2 points | *Not to be divided.* |
| **Total:** | **2 points** | |

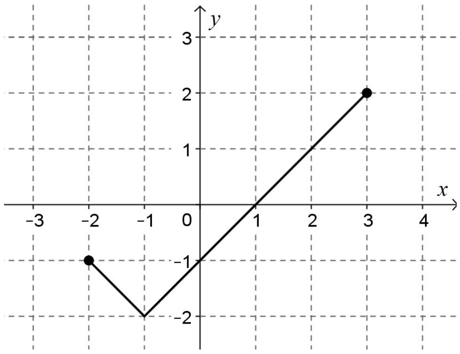
| 3. | | |
|---|---|---|
| A) true<br>B) false<br>C) true | 2 points | *Award 1 point for two correct answers, 0 points for one correct answer.* |
| **Total:** | **2 points** | |

| 4. | | |
|---|---|---|
| [–2; 2] | 2 points | *The correct answer is acceptable in any other form, too.* |
| **Total:** | **2 points** | |

| 5. | | |
|---|---|---|
| $(a+9)(a-1) = a^2 + 9a - a - 9$ | 1 point | |
| $(a-4)^2 = a^2 - 8a + 16$ | 1 point | |
| Combining the like terms: $2a^2 + 7$. | 1 point | |
| **Total:** | **3 points** | |

| 6. | | |
|---|---|---|
| **a)** –3 | 1 point | |
| **b)** –54 | 1 point | |
| **Total:** | **2 points** | |

| 7. | | |
|---|---|---|
| 17 years old | 2 points | |
| **Total:** | **2 points** | |

| 8. | | |
|---|---|---|
|  | | |
| The graph drawn is a translated image of the absolute value function. | 1 point | |
| The graph shown has a minimum at –1, the minimum value is –2. | 1 point | |
| The domain of the function is restricted to the given interval. | 1 point | |
| **Total:** | **3 points** | |



| 9. | | |
|---|---|---|
| Correct diagram. | 1 point | *This point is also due if the correct answer is given without a diagram.* |
| The height of the cone (applying the Pythagorean Theorem) $\sqrt{41^2 - 9^2} =$ | 1 point | |
| = 40 (cm). | 1 point | |
| **Total:** | **3 points** | |

| 10. | | |
|---|---|---|
| Five positive integers are given. | 1 point | |
| The median of the numbers is 4, | 1 point | |
| the mean is 3. | 1 point | |
| **Total:** | **3 points** | |

*Note: There are only two possible solutions: {1; 1; 4; 4; 5} and {1; 2; 4; 4; 4}.*

| 11. | | |
|---|---|---|
| $x^2 + (y-3)^2 =$ | 1 point | |
| $= 4$ | 1 point | |
| The radius of the circle is 2. | 1 point | |
| **Total:** | **3 points** | |

| 12. | | |
|---|---|---|
| $\frac{1}{8} (= 0.125)$ | 2 points | *The correct answer is acceptable in percentage form, too.* |
| **Total:** | **2 points** | |

# II. A

| 13. a) | | |
|---|---|---|
| $3\cdot(-7)+7p=21$ | 1 point | |
| $p=6$ | 1 point | |
| **Total:** | **2 points** | |

| 13. b) | | |
|---|---|---|
| A normal vector of line *e* is $\mathbf{n}_e(3; 7)$. | 1 point | |
| A normal vector of the perpendicular line *f* is $\mathbf{n}_f(-7; 3)$. | 1 point | |
| $-7x+3y=(-7)\cdot 1+3\cdot(-2)$ | 1 point | |
| The equation of line *f* is $-7x+3y=-13$. | 1 point | |
| **Total:** | **4 points** | |

| 13. c) **Solution 1** | | |
|---|---|---|
| The gradient of line *g* is $m_g=-\frac{3}{7}$. | 1 point | |
| Rearrange the equation of line *e*: $y=-\frac{3}{7}x+3$. | 1 point | |
| The gradient of line *e* is $m_e=-\frac{3}{7}$. | 1 point | |
| As the above gradients are equal the lines must be parallel. | 1 point | |
| **Total:** | **4 points** | |

| 13. c) **Solution 2** | | |
|---|---|---|
| Express *y* from the equation of line *g* and substitute it into the equation of line *e*. | 1 point | |
| $3x-3x+35=21$ | 1 point | |
| This equation does not have a solution, | 1 point | |
| the two lines do not have a common point and so they must be parallel. | 1 point | |
| **Total:** | **4 points** | |

*Note: Award 1 point if the candidate displays the two lines correctly in a coordinate system. Award another 1 point if, referring to the diagram but without formal proof, the candidate declares the lines to be parallel. If the candidate also proves the lines are parallel by reading off the diagram and correctly stating the gradients, the maximum score is due.*

| 14. a) | | |
|---|---|---|
| (Denote the angle in question by $\alpha$)<br>$\tan(180^\circ - \alpha) = \frac{6}{4}$ | 1 point | |
| $\alpha \approx 56.3^\circ$ | 1 point | |
| The measure of the angle is about 123.7°. | 1 point | |
| **Total:** | **3 points** | |

| 14. b) | | |
|---|---|---|
| Total number of cases: $\binom{28}{3}(=3276)$. | 1 point | |
| Number of favourable cases: $\binom{8}{1}\cdot\binom{20}{2}(=1520)$. | 2 points | |
| The probability: $\frac{\binom{8}{1}\cdot\binom{20}{2}}{\binom{28}{3}} \approx 0.464$. | 1 point | *A different answer may be accepted as long as it is rounded reasonably (at least two decimal places) and correctly. Answers given in percentage form may also be accepted.* |
| **Total:** | **4 points** | |

| 14. c) | | |
|---|---|---|
| The solid obtained consists of a cylinder and two congruent truncated cones fitted on the circular ends of the cylinder. | 1 point | *This point is also due if the correct reasoning is reflected by the solution.* |
| Both the radius of the base and the height of the cylinder are 6 cm. | 1 point | |
| The volume is $V_h = 216\pi\,(\approx 678.58)$ (cm$^3$). | 1 point | |
| Both the radius of the base and the height of the truncated cylinder are 6 cm, the radius of the top circle is 2 cm. | 1 point | |
| The volume is $V_{csk} = \frac{\pi\cdot 6}{3}\cdot(6^2 + 6\cdot 2 + 2^2) =$<br>$= 104\pi$ ($\approx 326.73$) (cm$^3$). | 1 point | |
| The volume of the solid is $V_h + 2V_{csk} = 424\pi \approx$ | 1 point | |
| $\approx 1332$ cm$^3$. | 1 point | |
| **Total:** | **7 points** | |

| **15. a)** | | |
|---|---|---|
| $f(6) = 3 \cdot 2^{6-1} =$ | 1 point | *This point is also due if the correct reasoning is reflected by the solution.* |
| $= 96$ | 1 point | |
| **Total:** | **2 points** | |

| **15. b)** | | |
|---|---|---|
| $2^{x-1} = 0.125$ | 1 point | |
| $2^{x-1} = \frac{1}{8}$ | 1 point | $\log 2^{x-1} = \log 0.125$ |
| $2^{x-1} = 2^{-3}$ | 1 point | $(x-1) \cdot \log 2 = \log 0.125$ |
| (As the exponential function is monotonic) $x - 1 = -3$. | 1 point | $x = \frac{\log 0.125}{\log 2} + 1$ |
| $x = -2$ | 1 point | |
| Check by substitution or reference to equivalent steps. | 1 point | |
| **Total:** | **6 points** | |

| **15. c)** | | |
|---|---|---|
| The first term of the sequence is $a_1 = 3$, | 1 point | |
| the quotient is $q = 2$. | 1 point | |
| The sum of the first 10 terms is $S_{10} = 3 \cdot \frac{2^{10} - 1}{2 - 1} =$ | 1 point | |
| = 3 069. | 1 point | |
| **Total:** | **4 points** | |

*Note: Award 4 points if the candidate calculates the first 10 terms of the sequence one by one and correctly adds them. Award 2 points in case of one error (miscalculation of a term or incorrect addition). Award no points for more than one error.*

# II. B

| 16. a) | | |
|---|---|---|
| The number of families without children needs to be found in 1990 and 2011. | 1 point | *This point is also due if the correct reasoning is reflected by the solution.* |
| The number of families without children in 1990 was $2896 \cdot 0.48 \approx 1\,390$ (thousands), | 1 point | |
| in 2011 it was $2713 \cdot 0.52 \approx 1\,411$ (thousands). | 1 point | |
| $\frac{1411}{1390} \approx 1.015$ | 1 point | |
| Between 1990 and 2011 the number of families without children grew by about 1.5%. | 1 point | *A different answer may also be accepted as long as it is rounded correctly to at least one decimal place.* |
| **Total:** | **5 points** | |

| 16. b) **Solution 1** | | |
|---|---|---|
| $\frac{0 \cdot 52 + 1 \cdot 25 + 2 \cdot 16 + 3 \cdot 5 + 4 \cdot 2}{100} =$ | 2 points | |
| (The average number of dependent children per family in 2011 is) = 0.8 | 1 point | *Do not accept solutions rounded to the nearest integer.* |
| **Total:** | **3 points** | |

**16. b) Solution 2**

| Number of dependent children | Number of families in 2011 (thousands) |
|---|---|
| 0 | 1411 |
| 1 | 678 |
| 2 | 434 |
| 3 | 136 |
| 4 or more | 54 |

(1 point)

| | | |
|---|---|---|
| $\frac{0 \cdot 1411 + 1 \cdot 678 + 2 \cdot 434 + 3 \cdot 136 + 4 \cdot 54}{2713} \approx$ | 1 point | |
| (The average number of dependent children per family in 2011 is about) ≈ 0.8 | 1 point | *A different answer may also be accepted as long as it is rounded correctly to at least one decimal place.* |
| **Total:** | **3 points** | |

*Note: Award 2 points if, instead of 2011, the candidate correctly calculates the 1990 average (0.84).*

| **16. c)** Solution 1 | | |
|---|---|---|
| Some quantity decreasing by 0.7% is equivalent to a multiplication by 0.993. | 1 point | *These 2 points are also due if the correct reasoning is reflected by the solution.* |
| Similarly, an increase by 6.3% is equivalent to a multiplication by 1.063. | 1 point | |
| Let $x$ be the number of (thousand) households in 1990. In this case $x \cdot 0.993 \cdot 1.063 = 4106$. | 1 point | |
| $x \approx 3890$, | 1 point | |
| so, in 1990, there were about 3 890 thousand households in the country. | 1 point | *Do not award this point if the solution is not rounded or rounded incorrectly.* |
| **Total:** | **5 points** | |

| **16. c)** Solution 2 | | |
|---|---|---|
| The number of (thousand) households in 2001 is $\frac{4106}{1.063} \approx$ | 1 point | |
| $\approx 3862.65$. | 1 point | |
| In 1990 it is $\frac{3862.65}{0.993} \approx$ | 1 point | |
| $x \approx 3890$, | 1 point | |
| so, in 1990, there were about 3 890 thousand house-holds in the country. | 1 point | *Do not award this point if the solution is not rounded or rounded incorrectly.* |
| **Total:** | **5 points** | |

| **16. d)** Solution 1 | | |
|---|---|---|
| The ratio of the areas of the discs is $\lambda^2 = \frac{1317}{946}$ $(\approx 1.39)$. | 2 points | |
| so $\lambda \approx 1.18$. | 1 point | |
| The radius in question is $(4.5 \cdot \lambda \approx)$ 5.3 cm. | 1 point | |
| **Total:** | **4 points** | |

| **16. d)** Solution 2 | | |
|---|---|---|
| The area of the circle used for the 1990 data is $A_1 = 4.5^2\pi\,(\approx 63.62)$ (cm$^2$). | 1 point | |
| The area of the other circle is $A_2 = A_1 \cdot \frac{1317}{946}\,(\approx 88.57)$ (cm$^2$). | 1 point | |
| The radius in question is $= \sqrt{\frac{A_2}{\pi}} \approx$ | 1 point | |
| $\approx 5.3$ cm. | 1 point | |
| **Total:** | **4 points** | |

*Note: A different answer may be accepted as long as it is reasonably and correctly rounded.*

| **17. a)** **Solution 1** | | |
|---|---|---|
| (Let $x$ be the length of the shorter route. In this case, the length of the other route is $(x + 140)$ km. The equation based on the text is) $\frac{x}{71} = \frac{x+140}{106}$. | 2 points | |
| $106x = 71x + 9940$ | 1 point | |
| $x = 284$ | 1 point | |
| The shorter route is 284 km long. | 1 point | |
| Check based on the text. | 1 point | |
| **Total:** | **6 points** | |

| **17. a)** **Solution 2** | | |
|---|---|---|
| (Let $y$ be the duration of the trip in hours. The equation based on the text is) $71y + 140 = 106y$ | 2 points | |
| $y = 4$ | 1 point | |
| $71 \cdot 4 = 284$ | 1 point | |
| The shorter route is 284 km long. | 1 point | |
| Check based on the text. | 1 point | |
| **Total:** | **6 points** | |

| **17. b)** | | |
|---|---|---|
| The gas consumption of the car is $\frac{396}{100} \cdot 6.5 =$ | 1 point | |
| = 25.74 litres. | 1 point | *25.7 or 26 litres is also acceptable.* |
| The cost of this is about 11 000 Ft. | 1 point | *Do not award this point if the solution is not rounded or rounded incorrectly.* |
| **Total:** | **3 points** | |

| **17. c)** **Solution 1** | | |
|---|---|---|
| (Let $v$ be the average speed for the trip. The equation based on the text is) $\frac{396}{v} = \frac{396}{v+16} + 1$. | 2 points* | |
| $396(v+16) = 396v + v(v+16)$ | 2 points* | |
| $v^2 + 16v - 6336 = 0$ | 1 point | |
| $v_1 = -88$, $v_2 = 72$ | 1 point | |
| (The solution in this context may not be negative, so) the average speed must have been 72 $\frac{\text{km}}{\text{h}}$. | 1 point | |
| Check based on the text. | 1 point | |
| **Total:** | **8 points** | |

| **17. c)** **Solution 2** | | |
|---|---|---|
| (Let $t$ be the duration of the trip. The equation based on the text is) $\frac{396}{t}+16=\frac{396}{t-1}$. | 2 points* | |
| $396(t-1)+16t(t-1)=396t$ | 2 points* | |
| $16t^2-16t-396=0$ | 1 point | $4t^2-4t-99=0$ |
| $t_1=-4.5$, $t_2=5.5$ | 1 point | |
| (The solution in this context may not be negative, so) the average speed must have been $\frac{396}{5.5}=72\ \frac{\text{km}}{\text{h}}$. | 1 point | |
| Check based on the text. | 1 point | |
| **Total:** | **8 points** | |

*The 4 points marked by * may also be given for the following reasoning:*

| | | |
|---|---|---|
| (Let $v$ be the average speed and $t$ be the duration of the trip. The equation system based on the text is) $\left.\begin{array}{l} v\cdot t=396 \\ (v+16)(t-1)=396 \end{array}\right\}$. | 2 points | |
| Expand the second equation and substitute 396 for $v\cdot t$: $16t-v-16=0$. | 1 point | |
| Express any of the variables and substitute it into the equation $v\cdot t=396$. | 1 point | |

| **18. a)** **Solution 1** | | |
|---|---|---|
| There are 5 codes containing one 2 and four 9-s. | 1 point | |
| There are 5 codes containing one 9 and four 2-s. | 1 point | |
| There are 10 codes containing two 2-s and three 9-s. | 1 point | |
| There are 10 codes containing two 9-s and three 2-s. | 1 point | |
| The total number of appropriate codes is 30. | 1 point | |
| **Total:** | **5 points** | |

| **18. a)** **Solution 2** | | |
|---|---|---|
| Count all possible 5-digit numbers containing only 2-s and 9-s and subtract the number of codes that do not contain both 2 and 9. | 1 point | *This point is also due if the correct reasoning is reflected by the solution.* |
| The number of codes containing only 2-s and 9-s is $2^5 =$ | 1 point | |
| $= 32$. | 1 point | |
| There are 2 among these codes that do not contain both 2-s and 9-s. | 1 point | |
| The total number of appropriate codes is 30. | 1 point | |
| **Total:** | **5 points** | |

| **18. b)** | | |
|---|---|---|
| The possible digits in Béla's code are: 2, 3, 5, or 7. | 1 point | |
| As it is divisible by six, it must also be divisible by both 2 and 3. | 1 point | *This point is also due if the correct reasoning is reflected by the solution.* |
| Since it is divisible by 2, the last digit must be a 2. | 1 point | |
| It will only be divisible by 3 if the other two digits are 3 and 7 | 1 point | |
| in descending order. | 1 point | *This point is also due if the correct reasoning is reflected by the solution.* |
| So the code is 732. | 1 point | |
| **Total:** | **6 points** | |

| **18. c)** **Solution 1** | | |
|---|---|---|
| The places for the 3-s may be selected in $\binom{6}{2}$ different possible ways. | 1 point | |
| The places for the 4-s may then be selected in $\binom{4}{2}$ different possible ways. | 1 point | |
| The other two digits may be placed into the remaining two slots in two different ways. | 1 point | |
| The total number of possible codes is the product of the above: $\binom{6}{2}\cdot\binom{4}{2}\cdot 2=180$ | 1 point | |
| The number of favourable cases is 1. | 1 point | |
| The probability is $\frac{1}{180}=0.00\dot{5}$. | 1 point | *A different answer may be accepted as long as it is reasonably and correctly rounded. Answers given in percentage form may also be accepted.* |
| **Total:** | **6 points** | |

| **18. c)** **Solution 2** | | |
|---|---|---|
| There are 6! different possibilities to list six different digits. | 1 point | *Award these points if the candidate refers to the formula of permutation with repeat.* |
| However, repeating digits halve this number. | 1 point | |
| Twice. | 1 point | |
| The total number of appropriate codes is 180. | 1 point | |
| The number of favourable cases is 1. | 1 point | |
| The probability is $\frac{1}{180}=0.00\dot{5}$. | 1 point | *A different answer may be accepted as long as it is reasonably and correctly rounded. Answers given in percentage form may also be accepted.* |
| **Total:** | **6 points** | |