ÉRETTSÉGI VIZSGA ● 2014. október 14.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper in **ink, different in colour** from that used by the candidate. Indicate errors, incomplete solutions, etc. in the conventional way.
2. The first of the rectangles below each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to this.
3. **If the solution is perfect,** it is enough to enter the maximum score in the appropriate rectangle.
4. If the solution is incomplete or incorrect, please indicate individual **partial scores** in the body of the paper, too.
5. Do not assess anything **written in pencil** except diagrams.

Assessment of content:

1. The answer key may contain more than one marking scheme for some of the problems. If the **solution given by the candidate is different from these**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals in the answer key can be **further divided**, but the scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for that part where the error occurs. If the reasoning remains correct and the error is carried forward, the points for the rest of the solution should be awarded.
4. **In the case of a conceptual error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solution are separated by double lines in the answer key.) However, if the incorrect result obtained through the conceptual error is carried forward to the next section or the next part of the problem and it is used correctly, the maximum score is due for all parts thereafter, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key indicates a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.
6. If there are more than one different approaches to a problem, **assess only the one indicated by the candidate**.
7. **Do not give extra points** (i.e. more than the maximum score due for the problem or part of problem).
8. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
9. **Assess only two of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I.

1.		
$8x + y =$	1 point	
$= 5$	1 point	
Total:	**2 points**	

2.		
$(x-3)^2 = x^2 - 6x + 9$	1 point	
$(x-4)(x+4) = x^2 - 16$	1 point	
The simplified form: $x - 7$	1 point	
Total:	**3 points**	

3.		
The correct answer is C.	2 points	*Not to be divided.*
Total:	**2 points**	

4.		
$x_1 = 0$	1 point	
$x_2 = 4$	1 point	
$x_3 = -4$	1 point	
Total:	**3 points**	

5.		
a) $x < 3$	1 point	
b) $x = 2$	2 point	
Total:	**3 points**	

6.		
The probability: $\frac{20}{100} = 0.2$.	2 points	*20% is also acceptable.*
Total:	**2 points**	

7.		
$x = \frac{3}{2}\pi$	2 points	
Total:	**2 points**	

Note: Award 1 point only, if the answer either contains all correct real solutions, or is given as $x = 270°$.

8.		
The range of the function: [0; 2].	2 points	*The correct answer is acceptable in any other form, too.*
Total:	**2 points**	

Note: Award 1 point only if the boundaries are correct, but the interval is not a closed one.

9.		
The radius of the circle: $r = 2$,	1 point	
the equation: $(x+2)^2 + (y-3)^2 =$	1 point	
$= 4$.	1 point	
Total:	**3 points**	

10.		
The interval:]–1; 2[	2 points	*The correct answer is acceptable in any other form, too.*
Total:	**2 points**	

Note: Award 1 point only if the boundaries are correct, but the interval is not an open one.

11. **Solution 1**		
From the second equation: $y = 7 - x$.	1 point	
Substituting into the first equation: $5x + 7 - x = 3$.	1 point	
$x = -1$	1 point	
$y = 8$	1 point	
Total:	**4 points**	

11. **Solution 2**		
(Use elimination:) subtract the second equation from the first:	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
$4x = -4$.	1 point	
$x = -1$	1 point	
$y = 8$	1 point	
Total:	**4 points**	

12.		
A: false B: true C: false	2 points	*Award 1 point for two correct answers, 0 points for one correct answer.*
Total:	**2 points**	

II. A

13. a)		
The percentage of participants marking series **A**: $\frac{90}{600} \cdot 100 =$	1 point	
= 15%.	1 point	
Total:	**2 points**	

13. b) Solution 1		
To obtain the number of participants watching any one particular series only, subtract the number of those marking all three series (55) from the number of marks for that particular series.	1 point	*These 3 points may also be awarded for a Venn-diagram that reflects an understanding of the question and shows the four numbers in their appropriate slots.*
Of all participants 35 watched series **A** only, 235 watched series **B** only, and 175 watched series **C** only.	2 points	
The number of participants watching any of the series at all is 35 + 235 + 175 + 55 = 500,	1 point	
and so the number of those watching neither of the series is 600 – 500 = 100 people.	1 point	
Total:	**5 points**	

13. b) Solution 2		
On adding the number of marks on each series, the number of participants marking all three of them (55) has been counted three times.	1 point	
Subtract the double of the number of participants marking all three series from the total number of marks, thereby obtaining the number of participants watching any of the series.	2 points	*These 2 points are also due if the correct reasoning is reflected by the solution.*
The number of participants watching any of the series at all is $90+290+230-2\cdot 55=500$,	1 point	
and so the number of those watching neither of the series is 600 – 500 = 100 people.	1 point	
Total:	**5 points**	

13. c)		
The respective central angles: (A: 55°) B: 135°, C: 170°.	2 points	*These 2 points are also due if the correct reasoning is reflected by the solution.*
A 1° sector of the pie chart is equivalent to $\frac{576}{360}=1.6$ „participants”.	1 point	$\frac{360}{576}=0.625$ *degrees is the equivalent of one participant.*

The respective numbers of the marks: Series **A**: $55 \cdot 1.6 = 88$, Series **B**: $135 \cdot 1.6 = 216$, Series **C:** $170 \cdot 1.6 = 272$.	2 points	*Award 1 point in case of one wrong answer, 0 points if there are more than one wrong answers.*
Total:	**5 points**	

14. a)		
To obtain the duration of a particular stretch of the journey, divide its length by the average speed on that stretch.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
The duration on the different stretches: residential areas: 1.125 (hours), highways: 0.5 (hours), motorways: 0.875 (hours).	2 points	*Award 1 point in case of one wrong answer, 0 points if there are more than one wrong answers.*
The journey took a total 1.125 + 0.5 + 0.875 = 2.5 hours.	1 point	
Total:	**4 points**	

Note: Accept any partial and final result as long as the reasoning is correct and they are properly rounded.

14. b) Solution 1		
Gas consumption on the different stretches: residential areas: $\frac{45}{100} \cdot 8.3 = 3.735$ (litres), highways: $\frac{35}{100} \cdot 5.1 = 1.785$ (litres), motorways: $\frac{105}{100} \cdot 5.9 = 6.195$ (litres).	2 points	*Award 1 point in case of one wrong answer, 0 points if there are more than one wrong answer.*
The total gas consumption on 185 km is 11.715 litres.	1 point	
The average gas consumption on 100 km: $\frac{11.715}{185} \cdot 100$ (litres).	1 point	
The average gas consumption of the car on 100 km is approximately 6.3 litres.	1 point	
Total:	**5 points**	

14. b) Solution 2		
(The average gas consumption is the average of the respective consumptions weighed by the distance travelled on each stretch, i.e.) $\frac{45 \cdot 8.3 + 35 \cdot 5.1 + 105 \cdot 5.9}{185} \approx$	3 points	
≈ 6.332 (litres).	1 point	
The average gas consumption of the car on 100 km is approximately 6.3 litres.	1 point	
Total:	**5 points**	

14. c)		
The two cans are similar solids with a similarity ratio of 1 : 2.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
The ratio of their volumes is 1 : 8.	2 points	
The volume of the smaller can is $\frac{20}{8} = 2.5$ litres.	1 point	
Total:	**4 points**	

15. a)		
$V = 30 \cdot 40 \cdot 50 = 60\ 000\ (\text{cm}^3)$	1 point	
$V = 60\ \text{dm}^3$	1 point	
The volume of the fish tank is 60 litres.	1 point	
Total:	**3 points**	

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15. b)		
The lengths of the face diagonals: $\sqrt{50^2+40^2} = \sqrt{4100}$ (≈ 64.03) (cm), $\sqrt{50^2+30^2} = \sqrt{3400}$ (≈ 58.31) (cm), $\sqrt{30^2+40^2} = 50$ (cm).	2 points	*Award 1 point in case of one wrong answer, 0 points if there are more than one wrong answers.*
The smallest angle is opposite the shortest side of the triangle.	1 point	*This point is to be awarded if the candidate calculates all three angles correctly. ($\beta \approx 60°$, $\gamma \approx 72°$)*
(Denote the angle opposite the shortest side by α and apply the Law of Cosines:) $2500 = 4100 + 3400 - 2 \cdot \sqrt{4100} \cdot \sqrt{3400} \cdot \cos\alpha$.	2 points	
Hence $\cos\alpha \approx 0.6696$.	2 points	
The smallest angle of the triangle: $\alpha \approx 48°$.	1 point	
Total:	**8 points**	

II.B

16. a)		
(Apply the formula about the sum of the first n terms of the arithmetic sequence:) $S_{25}=\frac{2\cdot 56+24\cdot(-4)}{2}\cdot 25=$	1 point	
$=200$.	1 point	
Total:	**2 points**	

16. b)		
(Apply the formula about the sum of the first n terms of the arithmetic sequence:) $408=\frac{2\cdot 56+(n-1)\cdot(-4)}{2}\cdot n$.	1 point	
Simplified: $816=112n-4n^2+4n$.	2 points	
The quadratic equation: $4n^2-116n+816=0$.	1 point	
The roots, also the possible values of n, are 12 and 17.	2 points	
When n = 12, $a_{12}=56+11\cdot(-4)=12$.	1 point	
When n = 17, $a_{17}=56+16\cdot(-4)=-8$.	1 point	
Total:	**8 points**	

Note: Award 2 points if the candidate correctly lists the respective terms of the sequence, thereby reflecting an understanding of the problem. Award 3 additional points for obtaining n = 12 by correct addition of the first 12 terms. Award an additional 1 point each for obtaining a_{12}, n = 17, and a_{17} correctly.

16. c)		
(Apply the formula about the general term of the geometric sequence:) $100\,000=10^{25}\cdot 0.01^{n-1}$.	1 point	
Hence $10^5=10^{25}\cdot\left(10^{-2}\right)^{n-1}$.	2 points	
(Applying the properties of powers:) $10^{-20}=10^{-2n+2}$.	2 points	
(As the exponential function is strictly monotonic:) $-20=-2n+2$.	1 point	
n = 11 (i.e. 100 000 is the 11th term of the sequence).	1 point	
Total:	**7 points**	

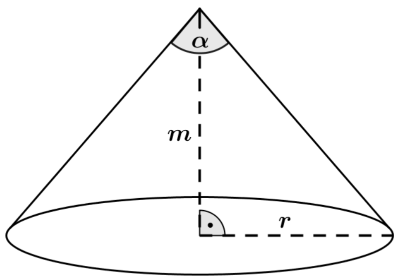
Note: Award 2 points if the candidate solves the problem by correctly listing the respective terms of the sequence. Award an additional 5 points for obtaining the correct answer.

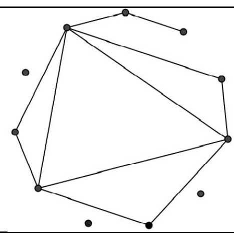
17. a)		
The number of possible ways to select the 5 balls is $\binom{15}{5}=$	2 points	
$= 3003$.	1 point	
Total:	**3 points**	

17. b) Solution 1		
The number of possible arrangements: $15\cdot 14\cdot \ldots \cdot 8\cdot 7=$	2 points	
$= 1\ 816\ 214\ 400$.	1 point	
Total:	**3 points**	

17. b) Solution 2		
The number of ways to arrange the 5 balls in the first row: $\binom{15}{5}\cdot 5!$	1 point	
The number of ways to arrange the 4 balls in the second row: $\binom{10}{4}\cdot 4!$	1 point	
(Multiply the above two numbers to obtain the total number of different arrangements.) The total number of arrangements: 1 816 214 400.	1 point	
Total:	**3 points**	

Note: Award 1 point if only $\binom{15}{5}\cdot\binom{10}{4}$ *is shown.*

17. c)		
 Diagram, correctly showing the $\alpha = 100^\circ$ angle, the $m = 85$ cm height, and the radius of the base circle.	2 points	*These 2 points are to be awarded if the candidate does not draw a diagram but all the calculations are correct.*
(Apply trigonometry in the right triangle:) tan 50° =	1 point	*This point is awarded for finding either of the acute angles correctly.*
$= \dfrac{r}{m}$.	1 point	*This point is awarded for the appropriate use of the trigonometric ratio.*
The radius of the base circle: $r \approx 101.3$ (cm).	1 point	
To answer the question the distance between the furthest points of the play area, i.e. the length e of the diagonals of the rectangle, needs to be found.	2 points	*These 2 points are also due if the correct reasoning is reflected by the solution*
$e^2 = 194^2 + 97^2$	1 point	
$e \approx 216.9$ (cm)	1 point	
As $e > 2r$,	1 point	
the lamp does not light up the whole table.	1 point	
Total:	**11 points**	

18. a)		
There are different possibilities, one of them is shown here: 	3 points	*Award 2 points in case of one wrong answer, 1 point in case of two wrong answers, 0 points if there are more than two wrong answers.*
Total:	**3 points**	

18. b)		
There were as many handshakes as many edges the graph has,	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
a total of 11.	1 point	
Total:	**2 points**	

18. c)		
The single mode of the numbers listed by the candidate is 2,	1 point	*Award these points for an incomplete or partially incorrect answer, as long as it is clear that the candidate uses these concepts correctly.*
the median is 3,	1 point	
the arithmetic mean is 4,	1 point	
the range is 5.	1 point	
The candidate lists 11 non-negative numbers in accordance with all conditions.	1 point	*One possible solution: 2, 2, 2, 2, 2, 3, 6, 6, 6, 6, 7.*
Total:	**5 points**	

18. d) Solution 1		
The probability of this player missing the goal: $(1-0.9=)\ 0.1$.	1 point	
There are three different cases.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
The player hits the goal once and misses it twice. The probability of this case: $\binom{3}{1}\cdot 0.9\cdot 0.1^2$ $(=0.027)$.	1 point	
The player hits the goal twice and misses it once. The probability of this case: $\binom{3}{2}\cdot 0.9^2\cdot 0.1$ $(=0.243)$.	1 point	
The probability of the player hitting the goal all three times: 0.9^3 $(=0.729)$.	1 point	
The probability in question is the sum of the above probabilities,	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
that is 0.999.	1 point	*99.9% is also acceptable.*
Total:	**7 points**	

18. d) Solution 2		
The probability of the player missing the goal: $(1-0.9=)\ 0.1$.	1 point	
To obtain the probability in question, subtract the probability of the player missing three times from the probability of the certain event.	2 points	*These 2 points are also due if the correct reasoning is reflected by the solution.*
The probability of the player missing the goal three times is 0.1^3.	1 point	
The probability of hitting the goal at least once: $1-0.1^3=$	2 points	
$=0.999$.	1 point	*99.9% is also acceptable.*
Total:	**7 points**	