ÉRETTSÉGI VIZSGA • 2014. május 6.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors, incomplete solutions, etc. in the conventional way.
2. The first one of the rectangles under each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to that.
3. **If the solution is perfect,** it is enough to enter the maximum scores in the appropriate rectangles.
4. If the solution is incomplete or incorrect, please indicate the individual **partial scores** in the body of the paper, too.
5. Do not assess anything except diagrams that is written in pencil.

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution by the candidate is different**, allocate the points by identifying the parts of the solution equivalent to those of the one given in the markscheme.
2. The subtotals in the markscheme can be **further divided**, but the scores awarded should always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error occurs. If the reasoning remains correct and the error is carried forward, the points for the rest of the solution should be awarded.
4. **In the case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information obtained owing to the principal error is carried forward to the next section or in the next part of the problem and it is used correctly, the maximum score is due for the next part, provided that the error has not changed the nature of the task to be completed.
5. Where the markscheme shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.
6. If there are more than one different approaches to a problem, **assess only the one indicated by the candidate**.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
9. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I.

1.		
There are 15 boys in the class.	2 points	*1 point for knowing that 35 needs to be divided into seven equal parts.*
Total:	**2 points**	

2.		
$x = 1$	2 points	*1 point for knowing that* $\sqrt{2} = 2^{\frac{1}{2}}$.
Total:	**2 points**	

3.		
a) At the point (0; 4) or at ($y =$) 4.	1 point	
b) $-2x + 4 = 6$	1 point	*These 2 points are also due if the result is read from a graph.*
$x = -1$	1 point	
Total:	**3 points**	

4.		
The test was taken by $(3^3 =)27$ students.	2 points	*Do not divide.*
Total:	**2 points**	

5.		
The sum of the degrees is 14.	2 points	*Do not divide.*
Total:	**2 points**	

6.		
$5 - x \geq 0$	1 point	
$(0 \leq)\ x \leq 5,\ (x \in \mathbf{Z})$	1 point	
$A = \{0; 1; 2; 3; 4; 5\}$	1 point	
Total:	**3 points**	

7.		
270º is $\frac{3}{4}$ of 360º.	1 point	
The area of the circle is $3^2 \cdot \pi (\approx 28.27\ \text{cm}^2)$.	1 point	*These 2 points are also due if the candidate uses a value of π rounded correctly.*
The area of the sector is $\frac{27}{4} \cdot \pi (\approx 21.2)\ \text{cm}^2$.	1 point	
Total:	**3 points**	

8.

grade	1	2	3	4	5
relative frequency	0	0.1	0.35	0.4	0.15

(table above)	2 points	*The 2 points are also due if the data are given in some other correct form (fraction or percentage). 1 point for one wrong value, 0 points for more than one wrong value.*
Total:	**2 points**	

9.

A) true	1 point	
B) false	1 point	
C) true	1 point	
Total:	**3 points**	

10.

The radius of the sphere is the half of the diagonal of the cube.	1 point	*The 1 point is also due if this idea is only reflected by the calculation.*
The length of the diagonal of the cube: $7 \cdot \sqrt{3}$ (≈ 12.1)	1 point	*The 1 point is also due if an appropriate approximate value is used.*
Thus the radius of the sphere is $\frac{7 \cdot \sqrt{3}}{2} \approx 6.1$	1 point	*Do not award this point if rounding is wrong.*
Total:	**3 points**	

11.

B)	2 points	
Total:	**2 points**	

12.

(Diagonal *AC* bisects angle *BCD*.) Angle *ACD* is 60º,	1 point	*Award 1 point for a correct diagram drawn according to the given information.*
and triangle *ACD* is isosceles, therefore equilateral.	1 point	
So the length of the diagonal in question is 6 cm.	1 point	
Total:	**3 points**	

II. A

13. a)		
Domain of definition: $x > 0$.	1 point	*Award this point, too if the domain is not investigated but the answer is checked, e.g. by substitution.*
Correct application of the appropriate logarithm identity.	1 point	
(The logarithm function is one-to one.) $\frac{7x+18}{x} = 9$ (or $7x+18 = 9x$)	1 point	
$x = 9$	1 point	
Checking by substitution, or, if $x > 0$ is stated, reference to the equivalence of the transformations.	1 point	
Total:	**5 ponts**	

13. b)		
Substituting $a = \cos x$ (where $-1 \le a \le 1$): $2a^2 - 7a - 4 = 0$.	1 point	*The point for correct rearrangement is also due if the candidate does not introduce a new variable.*
The roots of the equation are $a_1 = 4$,	1 point	
and $a_2 = -\frac{1}{2}$.	1 point	
$a = \cos x = 4$ is not a solution (since $\cos x \le 1$.)	1 point	
The solutions of the equation $\cos x = -\frac{1}{2}$ on the interval [0; 2π] are $x_1 = \frac{2\pi}{3}$,	1 point*	*Award 1 point if both roots are correct but expressed in degrees ($x_1 = 120°$ and $x_2 = 240°$).*
$x_2 = \frac{4\pi}{3}$.	1 point*	
Checking (e.g. by substitution).	1 point	
Total:	**7 points**	

*Award only one out of the two points marked with * if the roots are expressed correctly but the given domain is disregarded (e.g. in the case of infinitely many roots or a negative root).*

14. a)		
The mean of the data: $\frac{83\cdot 2+76\cdot 4+69\cdot 2+...+58\cdot 4+56\cdot 4+55}{28}=$	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
$=\frac{1816}{28}\approx 64.86$.	1 point	
Since the number of data is even, the median is the arithmetic mean of the two data items in the middle of a descending order of data:	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
$\frac{61+65}{2}=63$.	1 point	
The answer is yes, the mean and the median differ by at least 1 point.	1 point	
Total:	**5 points**	

14. b)		
6 classes are rated „excellent”, 13 are rated „very good”, 9 are rated „good”.	2 points	*1 point for two correctly calculated answers, 0 points for less than that.*
The bar chart: 	2 points	*Accept any principally correct representation (e.g. axes interchanged, touching bars). The 2 points are due if (1) the scale on the vertical axis is correct; (2) the individual bars are clearly identified; (3) the correct data are shown.*
Total:	**4 points**	

14. c) Solution 1		
The number of favourable cases: $2\cdot 4(=8)$.	1 point	
The number of all cases: $6\cdot 5(=30)$.	1 point	
The probability in question: $P=\frac{8}{30}(=0.2\dot{6})$.	1 point	*Accept the answer given as a percentage or in any correctly rounded form.*
Total:	**3 points**	

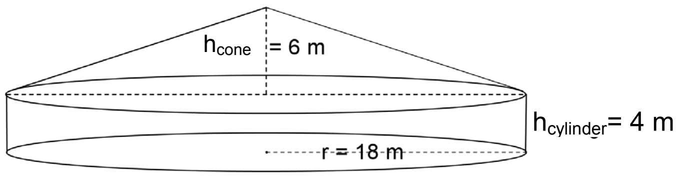
14. c) Solution 2		
The probability of an 83-point paper lying on top: $\frac{2}{6}$.	1 point	
The probability of a 67-point paper lying directly below: $\frac{4}{5}$.	1 point	
The probability in question: $P = \left(\frac{2}{6}\cdot\frac{4}{5} =\right) \frac{8}{30}\ (= 0.2\dot{6})$.	1 point	*Accept the answer given as a percentage or in any correctly rounded form.*
Total:	**3 points**	

15. a)		
The distance in question: $d_{AB} = \sqrt{(8-12)^2 + (9-1)^2} =$	1 point	*No point is due for the formula itself (without substitution). Do not award the second point if the exact value is not stated and the rounding is wrong.*
$= \sqrt{80} (\approx 8.944)$ (units).	1 point	
Total:	**2 points**	

15. b)		
Vector $\mathbf{n}_e(4;3)$ is a normal vector of the line.	1 point	
Thus the equation of the line is $4x + 3y = 4\cdot 4 + 3\cdot 3$,	1 point	
that is $4x + 3y = 25$.	1 point	
Total:	**3 points**	

15. c)		
Vector $\overrightarrow{AB}(4;–8)$ is a direction vector of line *f*.	1 point	
Thus the equation of the line is $-8x - 4y = (-8)\cdot 8 - 4\cdot 9$.	1 point	
The equation of line *f*: $2x + y = 25$.	1 point	
(The coordinates of the intersection are obtained as the solution of the following simultaneous equations:) $\left.\begin{array}{l} 4x + 3y = 25 \\ 2x + y = 25 \end{array}\right\}$	1 point	*Award 1 point if the coordinates of the intersection are correctly read from a correct graph. Award all 4 points if the intersection is also checked by substitution into the equations of both lines.*
The solution of the equations: $x = 25$ and $y = -25$.	2 points	
The intersection: $M(25;–25)$.	1 point	
Total:	**7 points**	

II. B

16. a)		
Diagram reflecting a correct understanding of the problem (the height of the cone is 6 metres): 	1 point	*This point is also due if there is no diagram but the candidate uses the correct data*
The volume of the cylinder: $V_{cyl} = 18^2 \cdot 4 \cdot \pi \approx$	1 point	*No point is due for the formulae (without substitution). Award the appropriate points for correct calculation with 3.14.*
≈ 4071.5 (m^3).	1 point	
The volume of the cone: $V_{cone} = \frac{1}{3} \cdot 18^2 \cdot 6 \cdot \pi \approx$	1 point	
≈ 2035.8 (m^3).	1 point	
$\frac{V_{cyl} + V_{cone}}{6} \left(= \frac{4071.5 + 2035.8}{6} \approx 1017.9 \right)$	1 point*	
The maximum number of spectators allowed in this tent is 1017.	1 point*	*This point is not due for a value rounded upwards.*
Total:	**7 points**	

*Award the two points marked with * if the volume of the cylinder is rounded to 4072 m^3, the volume of the cone is rounded to 2036 m^3, and the maximum number of spectators stated is 1018.*

16. b) Solution 1		
Let x denote the number of children's tickets sold. Then the number of adult tickets is $1000 - x$.	1 point	*This point is also due if the idea is only reflected by the solution.*
A children's ticket costs $800 \cdot 0.75 = 600$ forints.	1 point	
$600x + 800 \cdot (1000 - x) = 665\,800$	1 point	
The solution of the equation: $x = 671$.	1 point	
671 children's tickets and 329 adult tickets were sold.	1 point	
Checking against the wording of the problem.	1 point	
Total:	**6 points**	

16. b) Solution 2		
The number of children's tickets sold is obtained by subtracting the actual income from the income that could be achieved by selling 1000 adult tickets, and dividing by the reduction per children's ticket.	2 points	*These 2 points are also due if the idea is only reflected by the solution.*
There is a reduction of $800 \cdot 0.25 = 200$ forints on a children's ticket.	1 point	
$\frac{800\,000 - 665\,800}{200} = 671$	2 points	
671 children's tickets and 329 adult tickets were sold.	1 point	
Total:	**6 points**	

16. c)		
The 4 acrobats on the lowermost level can stand up in $4!(=24)$ different ways in a line.	1 point	
The 3 acrobats standing on their shoulders can be ordered in $3!(=6)$ ways,	1 point	
the 2 standing on the shoulders of these in 2 ways.	1 point	
The number of all cases is (the product of these results): $4! \cdot 3! \cdot 2! (= 288)$.	1 point	
Total:	**4 points**	

17. a)		
The numbers in question are terms of an arithmetic progression in which the first term is 2,	1 point	*These points are also due if the idea is only reflected by the solution.*
and the common difference is 3.	1 point	
The 25th term of the sequence: $a_{25} = 2 + 24 \cdot 3 = 74$.	1 point	
Total:	**3 points**	

The 3 points are also due if the correct answer is obtained by listing the terms of the sequence.

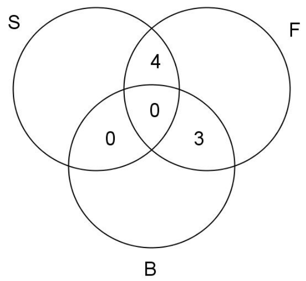
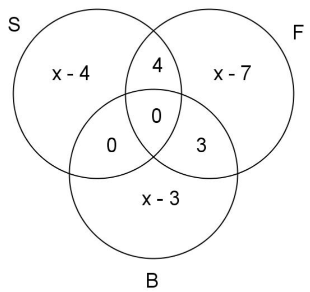
17. b)		
$S_n = \frac{2a_1 + (n-1)d}{2} \cdot n$	1 point	*This point is also due if the idea is only reflected by the solution.*
The task is to solve the equation $8475 = \frac{2 \cdot 2 + (n-1) \cdot 3}{2} \cdot n$ (on the set of positive integers).	1 point	
Rearranged: $3n^2 + n - 16950 = 0$.	2 points	
The roots of this equation are $n_1 = 75$ and $n_2 = -75,\dot{3}$.	1 point	
The (positive integer) solution of the problem is $n = 75$.	1 point	
Total:	**6 points**	

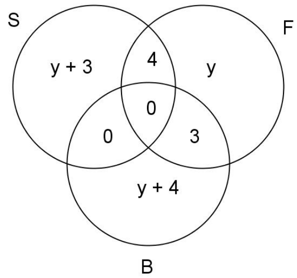
The 6 points are also due if the correct answer is obtained by listing the terms of the sequence.

17. c)		
The positive integers divisible by 5 and leaving a remainder of 2 on division by 3 form an arithmetic progression	1 point	*These points are also due if the idea is only reflected by the solution.*
with a common difference of 15.	1 point	
The smallest such three-digit number is 110,	2 points	*Do not divide.*
and the largest such three-digit number is 995.	2 points	*Do not divide.*
$995 = 110 + (n-1) \cdot 15$	1 point	*Do not award these 2 points if* $\frac{995-110}{15} = 59$ *is stated as answer.*
$n = 60$, the sequence has 60 terms that are three-digit numbers divisible by 5.	1 point	
Total:	**8 points**	

The 8 points are also due if the correct answer is obtained by listing the terms of the sequence.

18. a)		
7 of the 32 students chose two colours, thus the number of those voting for a single colour is 25.	1 point	
$P = \dfrac{\text{number of favourable cases}}{\text{number of all cases}}$	1 point	*This point is also due if the idea is only reflected by the solution.*
The probability in question: $P = \dfrac{25}{32} (= 0.78125)$.	1 point	*This 1 point is also due for a correct answer expressed as a percentage or a correctly rounded decimal.*
Total:	**3 points**	

<table>
<tr><th colspan="3">18. b) Solution 1</th></tr>
<tr><td>A Venn diagram correctly representing the number of elements in the appropriate intersections.

</td><td>2 points</td><td></td></tr>
<tr><td>(Let x denote the number of students selecting a particular colour.)

</td><td>2 points</td><td></td></tr>
<tr><td>$3x - 7 = 32$</td><td>2 points</td><td></td></tr>
<tr><td>$x = 13$</td><td>1 point</td><td></td></tr>
<tr><td>(White was chosen by 13 students altogether. Out of these, $13 - 7 =$) 6 students chose white only.</td><td>1 point</td><td></td></tr>
<tr><td>Total:</td><td>8 points</td><td></td></tr>
</table>

18. b) Solution 2		
Venn-diagram, as with Solution 1.	2 points	
(Let y denote the number of those marking white only.) 	2 points	
$3y+14=32$	2 points	
$y=6$	1 point	
6 students marked white only.	1 point	
Total:	**8 points**	

18. b) Solution 3		
Let Y denote the set of those selecting yellow, let W be the set of those selecting white, and let R be the set of those selecting wine red: $\lvert Y\cap W\rvert=4$ and $\lvert R\cap W\rvert=3$, furthermore $\lvert Y\cap R\rvert=0$ (and $\lvert Y\cap R\cap W\rvert=0$).	2 points	
$\lvert Y\rvert=\lvert W\rvert=\lvert R\rvert=x$	2 points	
(Applying the formula for the number of elements in a union, i.e. a logical sieve:) $32=x+x+x-(4+3)$	2 points	
$x=13$	1 point	
(White was chosen by 13 students altogether. Out of these, $13-7=$) 6 students chose white only.	1 point	
Total:	**8 points**	

18. c) Solution 1		
Two possible cases need to be investigated: 2 boys and 1 girl, or 1 boy and 2 girls will get the flowers.	1 point	*This point is also due if the idea is only reflected by the solution.*
There are $\binom{5}{2}(=10)$, ways to select 2 out of 5 boys, and 2 ways to select one out of 2 girls,	1 point	
that is, there are $10 \cdot 2 = 20$ possibilities in the first case.	1 point	
There are 5 ways to select one out of 5 boys, and there is one way to select two out of 2 girls,	1 point	
that is, there are 5 possibilities in the second case.	1 point	
(The number of all possible ways is the sum of these), so there are $20 + 5 = 25$ selections altogether.	1 point	
Total:	**6 points**	

18. c) Solution 2		
The number of selections satisfying the conditions is obtained by subtracting the number of wrong selections from the number of all possible selections.	1 point	*This point is also due if the idea is only reflected by the solution.*
There are $\binom{7}{3}(=35)$ ways to select 3 out of the 7 friends.	1 point	
Selections that consist of boys only are wrong.	2 points	*These 2 points are also due if the idea is only reflected by the solution. Do not divide.*
The number of selections to be rejected is $\binom{5}{3}(=10)$.	1 point	
Thus the number of suitable selections is $35 - 10 = 25$.	1 point	
Total:	**6 points**	

The 6 points are also due if the correct result is obtained by systematically listing the possible selections. In the case of a list of selections, take off 1 point for each wrong selection listed or each suitable selection missing. (Take off at most 6 points.)