ÉRETTSÉGI VIZSGA ● 2013. október 15.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors, incomplete solutions, etc. in the conventional way.
2. The first one of the rectangles under each problem shows the maximum attainable score on that problem. The **points given by the examiner** are to be entered in the rectangle next to that.
3. **If the solution is perfect,** it is enough to enter the maximum scores in the appropriate rectangles.
4. If the solution is incomplete or incorrect, please indicate the individual **partial scores** in the body of the paper, too.
5. Do not assess anything except diagrams that is **written in pencil**.

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution by the candidate is different**, allocate the points by identifying the parts of their solution equivalent to those of the solution given in the markscheme.
2. The subtotals in the markscheme can be further **divided**, unless stated otherwise. The scores awarded should always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error is made. If the reasoning remains correct and the error has not changed the nature of the task to be completed, the points for the rest of the solution should be awarded.
4. **In the case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information obtained owing to the principal error is carried forward to the next section or in the next part of the problem and is used correctly, the maximum score is due for the next part, provided that the error has not changed the nature of the task to be completed.
5. Where the markscheme shows a remark or unit **in brackets**, the solution should be considered complete without that remark or unit as well.
6. If there are **more than one different approaches** to a problem, assess only the one indicated by the candidate.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
9. **Assess only two out of the three problems in Section II B.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I

1.		
$A \setminus B = \{-4; -3; -2; -1; 0\}$	2 points	*Award 1 point if there is one error, 0 points for more than one error.*
Total:	**2 points**	

2.		
$x_1 = -2$, $x_2 = 10$	1 + 1 points	
Total:	**2 points**	

3.		
$x_1 = \frac{\pi}{3}$, $x_2 = -\frac{\pi}{3}$	1 + 1 points	
Total:	**2 points**	

Remark. Award 1 point if the answer of the candidate is –60° and 60°. Award at most 1 point if the answer is given as a real number but the given interval is not considered.

4.		
A) false B) true C) false	2 points	*1 point for 2 correct answers, 0 points for 1 correct answer.*
Total:	**2 points**	

5. **solution 1**		
Let x denote the size of the voting age population. Then the given information means $x \cdot 0.635 \cdot 0.436 = 4\,152\,900$.	2 points	
There are $x = 15\,000\,000$ voting age people.	1 point	
Total:	**3 points**	

5. **solution 2**		
The number of those participating is $\frac{4\,152\,900}{0.436} =$	1 point	
= 9 525 000.	1 point	
The number of voting age people is $\frac{9\,525\,000}{0.635} = 15\,000\,000$.	1 point	
Total:	**3 points**	

6.		
$b = 140$	1 point	
$m = -20$	2 points	*Award 1 point for $m = 20$.*
Total:	**3 points**	

7.		
B) and D)	2 points	
Total:	**2 points**	

Award 1 point for 1 correct answer, or for 2 correct answers and 1 wrong answer. 0 points in any other case.

8.		
If *d* denotes the common difference of the arithmetic progression, $3d = -15$,	1 point	*Award this point for setting up the simultaneous equations* $\left.\begin{array}{l} a_1 + 5d = 15 \\ a_1 + 8d = 0 \end{array}\right\}$*, too.*
hence $d = -5$.	1 point	
The first term of the sequence is 40.	1 point	
Total:	**3 points**	

Remark. Award the 3 points, too, if the answer is determined by listing the first nine terms of the sequence.

9.		
A graph satisfying the conditions.	2 points	*Do not divide.*
Total:	**2 points**	

Remark. Multiple edges or loops are allowed.

10.		
The range of *f* is [0.5; 4].	1 point	
$a = 0.5$	2 points	*Award 1 point for setting up a correct equation (e.g.* $a^1 = 0.5$*).*
Total:	**3 points**	

11.		
All the numbers on a regular die are factors of 60.	2 points	
Thus the event in question (is the certain event, which) has a probability of 1.	1 point	
Total:	**3 points**	

12.		
The quantity of Jonathan apples is 36 (kg).	1 point	
The central angle of the sector representing Idared apples is 150 (degrees).	1 point	
Thus the quantity of Idared apples is 60 (kg).	1 point	
Total:	**3 points**	

II A

13. a)		
$(4x+21 \geq 0$ and $x+4 \geq 0)$ Squaring both sides: $x^2+8x+16=4x+21$.	2 points	
Rearranged: $x^2+4x-5=0$.	1 point	
$x_1=-5$, $x_2=1$.	1 point	
–5 is not a root,	1 point	*Using the restriction, or by substitution.*
1 is a root.	1 point	
Total:	**6 points**	

13. b) solution 1		
(By expression and substitution:) $y = 16 - 3x$	1 point	
$5x - 32 + 6x = 45$	1 point	
$11x = 77$	1 point	
$x = 7$	1 point	
$y = -5$	1 point	
Checking.	1 point	
Total:	**6 points**	

13. b) solution 2		
(Equal coefficients obtained by multiplying through the first equation by 2:) $\left.\begin{array}{l} 6x+2y=32 \\ 5x-2y=45 \end{array}\right\}$	2 points	
$11x = 77$	1 point	
$x = 7$	1 point	
$y = -5$	1 point	
Checking.	1 point	
Total:	**6 points**	

14. a) solution 1		
The length of the altitude drawn from vertex *C* of triangle *ADC* is $41 \cdot \sin 47° \approx$	1 point	
≈ 30 (mm).	1 point	
This is equal to the altitude drawn from vertex *C* of triangle *ABC*,	1 point	*This point is also due if the correct reasoning is revealed by the solution.*
so the area in question is $T \approx \frac{48 \cdot 30}{2} =$	1 point	
= 720 mm^2.	1 point	
Total:	**5 points**	

14. a) **solution 2**		
The area of triangle *ADC* is $\frac{24\cdot 41\cdot \sin 47°}{2}\approx$	1 point	
$\approx 360\,(\text{mm}^2)$.	1 point	
The median *CD* halves the area of triangle *ABC*,	2 points	*These 2 points are also due for calculating the area of triangle BCD, and also if the correct reasoning is only revealed by the solution.*
so the area in question is 720 mm^2.	1 pont	
Total:	**5 points**	

14. b)		
Angle *CDB* is 133°.	1 point	
$BC=\sqrt{24^2+41^2-2\cdot 24\cdot 41\cdot \cos 133°}$	2 points	*1 point for recognising that side BC can be calculated with the cosine rule.*
Therefore the length of side *BC* rounded to the required precision is 60 mm as stated.	1 point	
Total:	**4 points**	

14. c) **solution 1**		
Let angle *ABC* be β. The sine rule for triangle *BCD* states $\frac{\sin\beta}{\sin 133°}=\frac{41}{60}$.	1 point	
$\sin\beta\approx 0.4998$.	1 point	
It follows that $\beta\approx 30°$ (since the interior angle at vertex *D* of triangle *BCD* is obtuse).	1 point	
Total:	**3 points**	

14. c) **solution 2**		
Let angle *ABC* be β. The cosine rule for triangle *BCD* states $41^2=24^2+60^2-2\cdot 24\cdot 60\cdot\cos\beta$	1 point	
$\cos\beta=\frac{24^2+60^2-41^2}{2\cdot 24\cdot 60}$ (≈ 0.8663).	1 point	
Hence $\beta\approx 30°$.	1 point	
Total:	**3 points**	

15. a) solution 1		
Let x denote the number of those who own a diswasher. Then the number of microwave oven owners is $2x$.	1 point	
141 people own some kind of device: $2x + x - 63 = 141$,	2 points	
hence $x = 68$.	1 point	
$(150 - 2 \cdot 68 =)$ 14 of those questioned have no microwave ovens.	1 point	
They make about 9.3% of all those questioned.	1 point	
Total:	**6 points**	

15. a) solution 2		
Let y denote the number of those who own a dishwasher but do not own a microwave oven. Then the number of all dishwasher owners is $y + 63$.	1 point	
The number of those who have microwave ovens but do not have dishwashers is $2(y + 63) - 63 = 2y + 63$.	1 point	
The number of all students questioned is $y + (2y + 63) + 63 + 9 = 150$,	1 point	
hence $y = 5$.	1 point	
$(5 + 9 =)$ 14 of those questioned have no microwave ovens.	1 point	
They make about 9.3% of all those questioned.	1 point	
Total:	**6 points**	

15. b)		
The mean of the number of computers per household is $\frac{3 \cdot 0 + 94 \cdot 1 + 89 \cdot 2 + 14 \cdot 3}{200} =$	1 point	*This point is also due if the correct reasoning is revealed by the solution.*
= 1.57.	1 point	*Accept 1.6, too.*
The median is 2,	1 point	
the mode is 1.	1 point	
Total:	**4 points**	

15. c)		
The negative is formulated by C and D.	2 points	
Total:	**2 points**	

Remark. Award 1 point for 1 correct answer, or for 2 correct answers and 1 wrong answer. 0 points in any other case.

II B

16. a)		
The base radius of the cylinder is $2.5\cdot10^{-7}$ (m),	1 point	
its volume is $V=\left(2.5\cdot10^{-7}\right)^2\cdot\pi\cdot2\cdot10^{-6}$.	1 point	
Expressed in scientific notation: $V\approx3.9\cdot10^{-19}$ (m^3).	1 point	
The surface area of the cylinder is $A=2\cdot\left(2.5\cdot10^{-7}\right)^2\cdot\pi+5\cdot10^{-7}\cdot\pi\cdot2\cdot10^{-6}$.	1 point	
Expressed in scientific notation: $A\approx3.5\cdot10^{-12}$ (m^2).	1 point	
Total:	**5 points**	

16. b)		
During 1.5 hours, the number of *E. coli* bacteria doubles 6 times,	2 points	*These 2 points are also due if the correct reasoning is revealed by the solution.*
therefore in 1.5 hours the number of bacteria will be $3\,000\,000\cdot2^6=$	1 point	
= 192 million.	1 point	
Total:	**4 points**	

16. c)		
(The number of bacteria will be 600 million in x minutes.) The equation $3\cdot2^{\frac{x}{15}}=600$ needs to be solved.	2 points	
$2^{\frac{x}{15}}=200$	1 point	
$\frac{x}{15}=\log_2 200$	2 points	$\frac{x}{15}\cdot\lg2=\lg200$
$x=15\cdot\frac{\lg200}{\lg2}$	1 point	*This point is also due if the correct reasoning is revealed by the solution.*
Hence $x\approx115$, therefore	1 point	
the number of bacteria will reach 600 million in 115 minutes.	1 point	
Total:	**8 points**	

17. a)		
$\overrightarrow{AB}(6;2)$	1 point	
A normal vector of line e is $\mathbf{n}(1;-3)$,	1 point	
its equation is $x-3y=7-3\cdot(-1)$,	1 point	
$x-3y=10$.	1 point	
Total:	**4 points**	

17. b)		
$1^2+(-3)^2-6\cdot 1-2\cdot(-3)=10$, (that is, point *A* lies on circle *k*.)	1 point	
$7^2+(-1)^2-6\cdot 7-2\cdot(-1)=10$, (that is, point *B* lies on circle *k*.)	1 point	
The length of chord *AB*: $\sqrt{(7-1)^2+(-1+3)^2}=$	1 point	
$=\sqrt{40}(\approx 6.32)$.	1 point	
Total:	**4 points**	

17. c) solution 1		
A normal vector of line *f* is $\overrightarrow{AB}(6;2)$	1 point	
The equation of line *f* is $3x + y = 0$.	2 points	
The coordinates of the intersection are obtained by solving the simultaneous equations of the circle *k* and the line *f*.	1 point	*This point is also due if the correct reasoning is revealed by the solution.*
From the equation of line *f*, $y = -3x$.	1 point	
Substituted in the equation of the circle: $x^2+9x^2-6x-2\cdot(-3x)=10$.	1 point	
$x^2=1$	1 point	
The solution of this equation (different from 1) is $x=-1$.	1 point	
Therefore the point in question is *C*(–1; 3).	1 point	
Total:	**9 points**	

17. c) solution 2		
If *C* is the intersection different from *A*, it follows from the converse of Thales' theorem that chord *BC* is a diameter of circle *k*.	3 points	
Rearranging the equation of circle *k*: $(x-3)^2+(y-1)^2=20$.	2 points	
Thus the centre of the circle is the point *K*(3; 1).	1 point	
K bisects line segment *BC*, so the coordinates of point $C(x_C;y_C)$ satisfy $\frac{x_C+7}{2}=3$ and $\frac{y_C-1}{2}=1$,	2 points	
hence *C*(–1; 3).	1 point	
Total:	**9 points**	

Remark. Award 3 points if the candidate rearranges the equation of the circle to correctly determine the coordinates of the centre, and represents the circle correctly on the coordinate plane.
Award further 2 points if the candidate adds the graph of line f to the diagram, and correctly identifies the coordinates of intersection C without explanation or checking.
Award another 2 points if the candidate explains why the line drawn is perpendicular to the line segment AB, and another 2 points if the coordinates of point C obtained are substituted in the equation of the circle to check that the point lies on the circle.

18. a) solution 1		
Two cards are selected from 30. This can be done in $\binom{30}{2}=\frac{30\cdot 29}{2}$ (= 435) different ways (number of all cases).	2 points	
The number of favourable cases (when the numbers on the two cards are equal) is 15.	2 points	
The probability in question is $\frac{15}{435}\left(=\frac{1}{29}\approx 0.0345\right)$.	1 point	*This point is also due for a correct answer expressed as a percentage.*
Total:	**5 points**	

18. a) solution 2		
The card selected first may be any card.	2 points	*This point is also due if the correct reasoning is revealed by the solution.*
In the case of the second card, we need to select out of 29 cards (number of all cases)	1 point	
the only possible card identical to the first one.	1 point	
The probability of that is $\frac{1}{29}$ (≈ 0.0345).	1 point	*This point is also due for a correct answer expressed as a percentage.*
Total:	**5 points**	

18. b) solution 1		
There are 7 dominoes altogether that have the same number of dots on both halves.	2 points	
(If the two halves of the domino were distinguished,) the number of ways to place different numbers of dots on the two halves would be $7\cdot 6=42$,	2 points	
but in that case, each such domino would be counted twice, so the number of such dominoes is 21.	1 point	
The complete set consists of 28 dominoes.	1 point	
Total:	**6 points**	

18. b) **solution 2**		
Arrange each domino in a position such that there are at least as many dots on the left half as on the right half.	1 point	*This point is also due if the correct reasoning is revealed by the solution.*
There are 7 dominoes altogether that have 6 dots on the left half. (These are the 6-0, 6-1, 6-2, 6-3, 6-4 6-5 and 6-6 stones.)	1 point	
There are 7 further dominoes with 5 dots on the left half,	1 point	
and so on. Finally, there is a single domino that has no dots on either half (the 0-0 stone).	1 point	
That makes a total of 7 + 6 + 5 + 4 + 3 + 2 + 1 =	1 point	
= 28 dominoes in a complete set.	1 point	
Total:	**6 points**	

Award the 6 points for a correct solution obtained by listing all cases.

18. c)		
A player starting the game on the third roll may have rolled five different numbers in each of the first two rounds,	1 point	*These 2 ponts are also due if the correct reasoning is revealed by the solution.*
and must roll one particular number (a six) in the third round.	1 point	
Thus the number of favourable cases is $5 \cdot 5 \cdot 1$.	1 point	
The number of all cases is 6^3.	1 point	
The probability in question is $\frac{25}{216}$ (≈ 0.1157).	2 points	*These 2 points are also due for a correct answer expressed as a percentage.*
Total:	**6 points**	