ÉRETTSÉGI VIZSGA ● 2012. október 16.

MATEMATIKA
ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI
ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

EMBERI ERŐFORRÁSOK
MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors and incomplete solutions in the conventional way.
2. The first one of the grey rectangles under each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to that.
3. **If the solution is perfect,** it is enough to enter the maximum scores in the appropriate rectangles.
4. If the solution is incomplete or incorrect, please indicate the individual **partial scores** in the body of the paper, too.
5. Do not assess anything except diagrams that is written in pencil.

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution by the candidate is different**, allocate the points by identifying the parts of their solution equivalent to those of the solution given in the markscheme.
2. The subtotals in the markscheme can be **further divided**, unless stated otherwise. The scores awarded should always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error occurs. If the reasoning remains correct and the error is carried forward, the points for the rest of the solution should be awarded.
4. **In the case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information obtained owing to the principal error is carried forward to the next section or to the next part of the problem and is used correctly there, the maximum score is due for the next part, provided that the error has not changed the nature of the task to be completed.
5. Where the markscheme shows a **remark** or **unit** in brackets, the solution should be considered complete without that remark or unit as well.
6. If there are more than one different approaches to a problem, **assess only the one indicated by the candidate**.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
9. **Assess only two out of the three problems in Section II B.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I

1.		
$a_{26} = 104$.	2 points	*Award 1 point for substitution in the right formula and making an error in the calculation.*
Total:	**2 points**	

2.		
$A = \{1;2;4;5\}$	1 point	*Award 1 point if the lists of the elements are missing or wrong but there is a correct Venn.diagram.*
$B = \{2;3;5;6\}$	1 point	
Total:	**2 points**	

3.		
$x = 16$	2 points	*1 point for stating* $\sqrt{x} = 4$.
Total:	**2 points**	

4.		
The central angle of the sector representing male boarders is 45°.	1 point	
This is $\frac{1}{8}$ of 360°.	1 point	
The number of male boarders is 60.	1 point	
Total:	**3 points**	

5.		
The number of students to be selected is 5.	2 points	*Do not divide.*
Total:	**2 points**	

6.		
If x denotes the number in question, $\frac{5}{6}$ of the number is $\frac{5}{6}x$.	1 point	
$\frac{5}{6}x \cdot 0.2 = 31$	1 point	
$x = 186$	1 point	
Total:	**3 points**	

7.		
A) true B) false C) true D) false	1 point each	
Total:	**4 points**	

8.		
Appropriate graph.	2 points	*1 point if the graph drawn meets only two of the three conditions.*
Total:	**2 points**	

9.		
The range of *f*: $[-2;2]$	1 point	*The points are due for the correct ranges given in any form.*
The range of *g*: $[-1;1]$	1 point	
Total:	**2 points**	

10.		
The length of vector **a** + **b** is 4 cm.	2 points	*1 point is due if a diagram reflects that the candidate knows how vectors are added.*
Total:	**2 points**	

11. solution 1		
The sum of the interior angles of a (regular) dodecagon is $(12-2)\cdot 180° =$	1 point	
$=1800°$,	1 point	
thus one interior angle is $150°$.	1 point	
Total:	**3 points**	

11. solution 2		
The line segments drawn from the centre of a regular dodecagon to two adjacent vertices enclose a 30° angle.	1 point	
The base angles of the resulting isosceles triangle are 75°.	1 point	
The interior angle of the regular dodecagon is the double of this: $150°$.	1 point	
Total:	**3 points**	

11. **solution 3**		
The sum of the exterior angles of a convex polygon is $360°$,	1 point	
so one exterior angle of a regular dodecagon is 30°,	1 point	
that is, the interior angle is $150°$.	1 point	
Total:	**3 points**	

12.		
$94.5 = b_1 \cdot \frac{2^6 - 1}{2 - 1}$	1 point	*No points are due for the formula only (without substitution).*
$94.5 = b_1 \cdot 63$	1 point	
$b_1 = 1.5$	1 point	
Total:	**3 points**	

II A

13. a)		
A direction vector of line *BC* is the vector $\overrightarrow{BC}(-12;9)$.	1 point	*A normal vector of line BC is, for example, the vector (9; 12).*
Hence the equation of the line is $9x+12y=9\cdot 9+12\cdot(-3)$,	1 point	
that is, $9x+12y=45\ \ (3x+4y=15)$.	1 point	
Total:	**3 points**	

13. b) solution 1		
The midline parallel to side *BC* is the line segment connecting the midpoints of sides *AB* and *AC*.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
The midpoint of side *AB* is $M_{AB}(3.5;-2)$, The midpoint of side *AC* is $M_{AC}(-2.5;2.5)$.	1 point	
The length of the midline is $\sqrt{6^2+(-4.5)^2}=7.5$.	1 point	
Total:	**3 points**	

13. b) solution 2		
The midline parallel to side *BC* is half as long as side *BC*.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
The length of side *BC* is: $\sqrt{12^2+(-9)^2}=15$.	1 point	
The length of the midline is 7.5.	1 point	
Total:	**3 points**	

13. c) solution 1		
The lengths of the sides of triangle *ABC* are $AB=\sqrt{125}$, $BC=15$, $AC=\sqrt{50}$.	2 points	*Award 1 point if only two lengths are correct.*
Let γ denote the interior angle at vertex *C*. From the cosine rule:	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
$125=225+50-2\cdot 15\cdot\sqrt{50}\cdot\cos\gamma$	1 point	
$\cos\gamma=\frac{\sqrt{2}}{2}(\approx 0.7071)$	1 point	
(Since $0°<\gamma<180°$, it follows that) $\gamma=45°$.	1 point	
Total:	**6 points**	

13. c) **solution 2**		
$\overrightarrow{CB}(12;-9)$, $\overrightarrow{CA}(1;-7)$	1 point	
The lengths of the vectors are $\left\|\overrightarrow{CB}\right\| = 15$, $\left\|\overrightarrow{CA}\right\| = \sqrt{50}$.	1 point	
(By definition of the scalar product:) $\overrightarrow{CB}\cdot\overrightarrow{CA} = 15\cdot\sqrt{50}\cdot\cos\gamma$.	1 point	
But also $\overrightarrow{CB}\cdot\overrightarrow{CA} = 12\cdot 1 + (-9)\cdot(-7) = 75$.	1 point	
Hence $\cos\gamma = \frac{1}{\sqrt{2}} (\approx 0{,}7071)$.	1 point	
(Since $0° < \gamma < 180°$, it follows that) $\gamma = 45°$.	1 point	
Total:	**6 points**	

14. a)		
If three colours are to be used, the fields of the pin will all have different colours.	1 point	*These 2 points are also due if the correct reasoning is reflected by the solution.*
One field (e.g. the innermost one) can be coloured in 5 ways, the adjacent field in 4 ways, and the last one in 3 ways.	1 point	
Thus there are $5\cdot 4\cdot 3 = 60$ different pins with three colours.	1 point	
Total:	**3 points**	

14. b) **solution 1**		
There number of ways to select two colours out of five is $\binom{5}{2} =$	1 point	
$= 10$.	1 point	
There are three ways to select the two fields out of three that will have the same colour,	1 point	*Award 1 point if the candidate only finds those cases when the fields of the same colour are not adjacent fields.*
and there are two possible colourings in each case, which makes 6 possibilities.	1 point	
The number of pins with two colours is therefore $10\cdot 6 = 60$.	1 point	
Total:	**5 points**	

14. b) **solution 2**		
The pin can be coloured in one, two or three colours.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
Each field can be coloured in 5 ways, which makes $5\cdot5\cdot5=125$ different arrangements of colours altogether.	1 point	
There are 5 different pins with a single colour.	1 point	
The number of pins with two colours is obtained by subtracting from the number of all possible pins the numbers of pins with three colours or one colour.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
Thus the number of pins with two colours is $125-5-60=60$.	1 point	
Total:	**5 points**	

14. c)		
Each field can be coloured in 5 ways, which makes $5\cdot5\cdot5=125$ different arrangements of colours altogether.	1 point	
The three given colours occur on $3\cdot2\cdot1=6$ pins.	1 point	
Hence the probability in question is $p=\dfrac{\text{number of favourable cases}}{\text{number of all cases}}=$	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
$=\dfrac{6}{125}(=0.048)$.	1 point	
Total:	**4 points**	

15. a)		
$f(3)=20.25$	1 point	
$x^2+2x+3.5=2.5$	1 point	
$x=-1$	1 point	
Total:	**3 points**	

15. b)		
Transforming the rule of assignment: $x^2+2x+3.5=(x+1)^2+2.5$.	1 point	
The minimum of the function is 2.5.	1 point	*This point is also due if the minimum is revealed by a correct statement of the range.*
The range: $[2.5;\infty[$	1 point	*This point is also due for a correct range given in any other form.*
Total:	**3 points**	

15. c)		
Rearranged: $x^2 - 3x - 1.75 < 0$.	1 point	
The roots of the equation $x^2 - 3x - 1.75 = 0$ are $x_1 = -\frac{1}{2}$ and $x_2 = \frac{7}{2}$.	2 points	
Since the leading coefficient of the quadratic expression is positive,	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
the solution of the inequality is $-\frac{1}{2} < x < \frac{7}{2}$.	2 points	*Award at most 1 point if the endpoints of the interval are included in the solution set.*
Total:	**6 points**	

II B

16. a) solution 1		
Let x denote the number of peak minutes ($0 < x < 120$), and let y denote the peak rate in forints per minute ($25 < y$).	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
The following simultaneous equations can be set up: $\left.\begin{array}{l} xy = 2000 \\ (120 - x)(y - 25) = 2000 \end{array}\right\}$.	2 points	
Eliminating the brackets: $120y - xy - 25 \cdot 120 + 25x = 2000$.	1 point*	
Expressing one of the unknowns: $y = \frac{2000}{x}$.	1 point*	
Substituted: $120 \cdot \frac{2000}{x} + 25x = 7000$.	1 point*	
Rearranged: $25x^2 - 7000x + 240\,000 = 0$.	1 point*	
The two roots of the quadratic equation are $x_1 = 40$ and $x_2 = 240$.	1 point*	
240 is not a solution of the problem since Stefi talked 120 minutes altogether.	1 point*	
Stefi talked 40 minutes in peak hours during the time period in question.	1 point	*Peak rate: 50 forints per minute, Off-peak rate: 25 forints per minute.*
Checking against the wording.	1 point	
Total:	**11 points**	

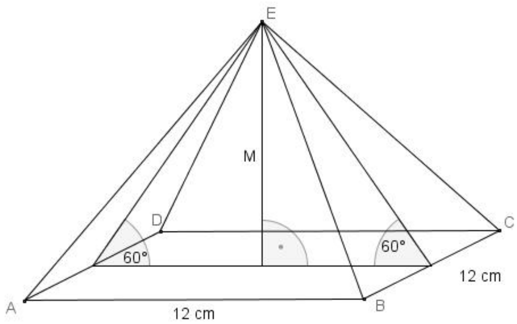
*The 6 points marked with * are also due for the following reasoning:*

Eliminating the brackets: $120y - xy - 25 \cdot 120 + 25x = 2000$.	1 point	
$xy = 2000$ substituted and rearranged: $24y + 5x = 1400$	1 point	
Expressing x and substituting in the first equation: $(280 - 4.8y)y = 2000$.	1 point	
Rearranged: $4.8y^2 - 280y + 2000 = 0$.	1 point	
The two roots of the quadratic equation are $y_1 = 50$ and $y_2 = \frac{25}{3}$	1 point	
$\frac{25}{3}$ is not a solution of the problem (since that would make $y - 25 < 0$).	1 point	

16. a) solution 2		
Let x denote the number of peak minutes ($0 < x < 120$), then the number of off-peak minutes is $(120 - x)$.	1 point	*These 3 points are also due if the correct reasoning is reflected by the solution.*
Since it is given that Stefi talked for 2000 forints in both peak and off-peak hours, the peak rate per minute was $\frac{2000}{x}$ forints, and	1 point	
the off-peak rate per minute was $\frac{2000}{120 - x}$ forints.	1 point	
According to the problem, $\frac{2000}{x} - 25 = \frac{2000}{120 - x}$.	2 points	
Multiplying each side by $x \cdot (120 - x)$: $2000(120 - x) - 25x(120 - x) = 2000x$.	1 point	
Rearranged: $25x^2 - 7000x + 240\ 000 = 0$	1 point	
The two roots of the quadratic equation are $x_1 = 40$ and $x_2 = 240$.	1 point	
240 is not a solution of the problem since she talked 120 minutes altogether.	1 point	
Stefi talked 40 minutes in peak hours during the time period in question.	1 point	*Peak rate: 50 forints per minute, Off-peak rate: 25 forints per minute.*
Checking against the wording.	1 point	
Total:	**11 points**	

16. b)		
If the number of new subscribers reached 20 000 n months after the first month, then $10000 \cdot 1.075^n = 20000$.	1 point	
(Since the decimal logarithm function is strictly increasing,)	1 point	*This point is also due if this idea is reflected by the solution.*
$n \cdot \lg 1.075 = \lg 2$	1 point	
$n \approx 9.58$	1 point	
It is in the 10th month after launching the plan,	1 point	
that is, in November that the number of new subscribers is going to reach 20 000.	1 point	
Total:	**6 points**	

Remarks.
1. Award the 6 points if the candidate correctly calculates the number of new subscribers month by month (possibly with reasonable rounding), and thus arrives at the correct answer.
2. The appropriate points are also due if the candidate calculates with an inequality instead of an equation.

17. a)		
Correct diagram that shows the given information. 	1 point	*This point is also due if there is no diagram but the candidate uses the correct data.*
The height of the pyramid is $M = 12 \cdot \frac{\sqrt{3}}{2} (= 6 \cdot \sqrt{3} \approx 10.39)$ (cm).	1 point	
The altitude of the lateral face drawn to the 12-cm side is also 12 cm.	1 point	
The surface area of the pyramid is $A = 12^2 + 4 \cdot \frac{12^2}{2} =$	1 point	
$= 432\ \text{cm}^2$.	1 point	
The volume of the pyramid is $V = \frac{12^2 \cdot 6\sqrt{3}}{3} \approx$	1 point	
$\approx 499\ \text{cm}^3$.	1 point	
Total:	**7 points**	

Remark: If there is no rounding in an answer or the rounding is wrong, take off at most 1 point on this problem.

17. b) **solution 1**		
The given plane cuts the pyramid into a truncated pyramid and a pyramid similar to the original one.	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
The scale factor of the similitude is $\lambda = \frac{2}{3}$.	1 point	
The ratio of similar bodies is $\frac{V_{\text{pyramid cut off}}}{V_{\text{original pyramid}}} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$,	1 point	
Thus the ratio of the volumes of the truncated pyramid and the original pyramid is 19:27,	1 point	
and the ratio of the volumes of the resulting solids is 8:19.	1 point	
Total:	**5 points**	

17. b) **solution 2**		
(It follows from the properties of central similitude that) the base edge of the pyramid cut off is $12 \cdot \frac{2}{3} = 8$ (cm), its height is $6\sqrt{3} \cdot \frac{2}{3} = 4\sqrt{3} (\approx 6.93\text{ cm})$,	1 point	
and its volume is $V = \frac{8^2 \cdot 4\sqrt{3}}{3} (\approx 147.8\text{ cm}^3)$.	1 point	
$\frac{V_{\text{pyramid cut off}}}{V_{\text{original pyramid}}} = \frac{8^2 \cdot 4}{12^2 \cdot 6} = \frac{8}{27}$,	1 point	
Thus the ratio of the volumes of the truncated pyramid and the original pyramid is 19:27,	1 point	
and the ratio of the volumes of the resulting solids is 8:19.	1 point	
Total:	**5 points**	

17. c) **solution 1**		
(It follows from the properties of central similitude that) the side of the top face of the truncated pyramid is $12 \cdot \frac{2}{3} = 8$ (cm). The base edge is 12 cm.	1 point	
The height of a lateral face is $12 \cdot \frac{1}{3} = 4$ (cm).	1 point	
The area of one lateral face is $T = \frac{12+8}{2} \cdot 4 = 40$ (cm^2).	1 point	
The area of the lateral face is $A = 12^2 + 8^2 + 4 \cdot 40 =$	1 point	
$= 368\text{ cm}^2$.	1 point	
Total:	**5 points**	

17. c) **solution 2**		
(It follows from the properties of central similitude that) the side of the top face of the truncated pyramid (same as the base edge of the small pyramid cut off) is $12 \cdot \frac{2}{3} = 8$ (cm).	1 point	
(With the result from part a)), the sum of the surface areas of the truncated pyramid and the small pyramid is $432 + 2 \cdot 8^2 = 560\,(\text{cm}^2)$.	1 point	
The small pyramid is similar to the large one and the scale factor is $\frac{2}{3}$,	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
so the surface area of the small pyramid is $432 \cdot \left(\frac{2}{3}\right)^2 = 192\,(\text{cm}^2)$.	1 point	
The surface area of the truncated pyramid is $560 - 192 = 368\ \text{cm}^2$.	1 point	
Total:	**5 points**	

18. a)		
The mean of the ages is $\frac{17 \cdot 2 + 18 + 19 + ... + 25 + 26 + 31}{13} =$	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
$= \frac{289}{13} (\approx 22.23\ \text{years})$.	1 point	*Any other correctly and reasonably rounded value (e.g. 22 years) is accepted.*
Total:	**2 points**	

18. b)		
(Since 9 players of the 13 are older than 20 years,) the number of cases with no player younger than 20 years among the 7 players selected is $\binom{9}{7}$.	1 point	
The number of cases when one player is younger than 20 (and 6 are older) is $\binom{4}{1} \cdot \binom{9}{6}$.	2 points	
The number of favourable cases regarding event A is the sum of these two results:	1 point	*This point is also due if the correct reasoning is reflected by the solution.*
$\binom{9}{7} + 4 \cdot \binom{9}{6} = 36 + 336 = 372$.	1 point	

The number of all cases is $\binom{13}{7}$.	1 point	
The probability in question is $P(A)=\dfrac{\binom{9}{7}+4\cdot\binom{9}{6}}{\binom{13}{7}}=$	1 point	
$=\dfrac{372}{1716}(\approx 0.2168)$.	1 point	
Total:	**8 points**	

18. c)		
(There is only one way to have an age difference of 12 years: if) the oldest player is $(a_6=)\,31$,	1 point	
and the youngest is $(a_1=)\,19$ years old.	1 point	
It follows from the mode that there are two players (a_2 and a_3) aged 22.	1 point	
Since there are 6 players, the median is the arithmetic mean of a_3 and a_4. That is, one player must be $(a_4=)\,24$ years old. (There are such players in the team.)	2 points	
It follows from the mean that $\dfrac{118+a_5}{6}=24$,	1 point	
thus the player in question is $(a_5=)\,26$ years old. (There exists such a player in the team.)	1 point	
Total:	**7 points**	

Remark. If the candidate lists the ages of the six players without explanation and checking, 2 points may be awarded. (1 point if there is one error, no points for more than one error). 3 further points may be awarded if the candidate checks the data against the given conditions.