ÉRETTSÉGI VIZSGA ● 2012. május 8.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

NEMZETI ERŐFORRÁS MINISZTÉRIUM

Instructions to examiners

Formal requirements:

1. Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors, incomplete solutions, etc. in the conventional way.
2. The first one of the rectangles under each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to that.
3. **If the solution is perfect**, it is enough to enter the maximum scores in the appropriate rectangles.
4. If the solution is incomplete or incorrect, please indicate the individual **partial scores** in the body of the paper, too.
5. Do not assess anything, except diagrams, that is written in pencil.

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution by the candidate is different**, allocate the points by identifying the parts of the solution equivalent to those of the solution(s) given in the markscheme.
2. The subtotals in the markscheme can be **further divided**, but the scores awarded should always be whole numbers.
3. If it is clear that the reasoning and the final answer are both correct, you may award the maximum score even if the solution is **less detailed** than the one in the markscheme.
4. If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error occurs. If the reasoning remains correct and the error is carried forward, the points for the rest of the solution should be awarded.
5. In the case of a **principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information obtained owing to the principal error is carried forward to the next section or in the next part of the problem and is used correctly, the maximum score is due for the next part, provided that the error has not changed the nature of the task to be completed.
6. Where the markscheme shows a **unit** or a **remark** in brackets, the solution is considered complete without that unit or remark as well.
7. If there are more than one different approaches to a problem, **assess only the one indicated by the candidate**.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I.

1.

$x-3=20$	1 point	
$x=23$	1 point	*Award full mark for a correct answer without explanation.*
Total:	**2 points**	

2.

a + **b**	2 points	*Award 1 point if the answer does not make it clear that **a** and **b** are vectors.*
Total:	**2 points**	

3.

$x=-3$	2 points	
Total:	**2 points**	

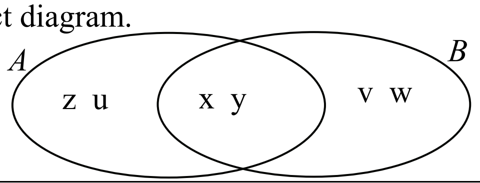
4.

The letter marking the graph of function g: B.	2 points	
The zero of the function: $(x=)-1$.	1 point	
Total:	**3 points**	

5.

There are 15 possibilities.	2 points	*Accept* $\binom{6}{4}$ *as well.*
Total:	**2 points**	

6.

Correct diagram. 	1 point	
$A\cap B=\{x; y\}$	1 point	
Total:	**2 points**	

7.

$t_2=t_0\cdot q^2$	1 point	*These two points are due if* $50000\cdot 1.1^2$ *is stated without referring to a formula. Award 1 mark if the value in one year is calculated correctly but the final result is wrong.*
$t_2=50000\cdot 1.1^2$	1 point	
The value of the investment paper is 60 500 forints.	1 point	
Total:	**3 points**	

8.		
The possible values of y are 1; 4; 7.	2 points	*1 point for one or two correct values. No point if any incorrect value is stated.*
Total:	**2 points**	

9.		
The maximum occurs at 6.	1 point	
The maximum value is 3.	1 point	
Total:	**2 points**	

10.		
The diagram contains exactly one point of degree three,	1 point	
exactly three points of degree two,	1 point	
exactly one point of degree one.	1 point	
Total:	**3 points**	*All three points are due for a correct diagram.*

11.		
$(x-2)^2+(y+1)^2=5$	2 points	*These 2 points are also due if the formulae given in the data tables book are used correctly.*
The centre is point $O(2; -1)$,	1 point	
the radius is $\sqrt{5}$.	1 point	
Total:	**4 points**	

12.		
A: false.	1 point	
B: false.	1 point	
C: true.	1 point	
Total:	**3 points**	

II. A

13. a)		
$1011_2=11$,	2 points	
Paul's statement is false.	1 point	
Total:	**3 points**	

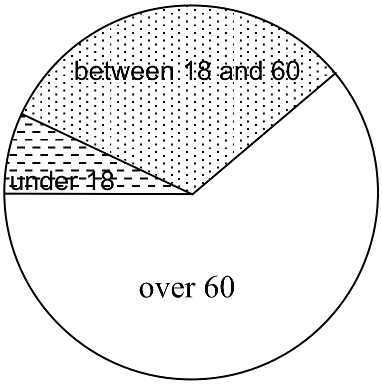
13. b)		
$10=a_1+36$	1 point	
$a_1=-26$	1 point	
Total:	**2 points**	

13. c) Solution 1		
$-26+(n-1)\cdot 4\geq 100$	2 points	*Award 1 point if a stricter relation is used.*
$n\geq 32.5$, therefore the index of the term is 33.	1 point	
The term in question is $a_{33}=102$.	1 point	
Total:	**4 points**	

13. c) Solution 2		
The sequence consists of numbers leaving a remainder of 2 when divided by 4.	1 point	
The smallest such number of three digits is 102.	1 point	
$10+k\cdot 4=102$; $k=23$	1 point	
Thus the index of the term in question is $10+23=33$.	1 point	
Total:	**4 points**	

13. d)		
The first two-digit positive term is $a_{10}=10$, and the last one is $a_{32}=98$,	2 points	
so the set has 22+1=23 elements.	1 point	
Total:	**3 points**	

14. a)		
$p=\frac{k}{n}\left(=\frac{\text{number of favourable cases}}{\text{number of all cases}}\right)$	1 point	*This point is also due if the idea is only reflected by the solution.*
$p=\frac{1978}{12320}\approx$	1 point	
≈ 0.16	1 point	*≈16,06%*
Total:	**3 points**	

14. b)		
		
The number of those over 60 treated in hospital is $1978-138-633=1207$.	1 point	
The 138 people under 18 correspond to a central angle of $\frac{138}{1978}\cdot 360°\approx 25°$.	1 point	*If the correct method of calculating the central angle is not shown at all, award only 1 point even for numerically correct answers. If one detailed calculation is shown only but all three results are correct, award 2 points.*
The 633 between 18 and 60 correspond to a central angle of $\left(\frac{633}{1978}\cdot 360°\approx\right)115°$.	1 point	
The 1207 over 60 correspond to a central angle of $\left(\frac{1207}{1978}\cdot 360°\approx\right)220°$.	1 point	
Correct pie chart (with correct angles by eye, sectors labelled).	1 point	
Total:	**5 points**	

14. c)		
$12320\cdot 0.24=$	1 point	
$=2956.8(\approx 2957)$ of those living in Ailington are over 60.	1 point	*Accept 2956, too.*
The number of those over 60 treated in hospital is 1207, so the probability in question is $\frac{1207}{2957}(\approx 0.41)$.	1 point	
The probability increased by $0.41-0.16=0.25$.	1 point	
Total:	**4 points**	

15.		
Applying the cosine rule to triangle *ABP*:	1 point	*This point is also due if the idea is only reflected by the solution.*
$BP^2 = 620^2 + 720^2 - 2 \cdot 620 \cdot 720 \cdot \cos 53°$,	1 point	
$BP \approx 605$	2 points *	
Angle *AQB* is 19º.	1 point	
Applying the sine rule to triangle *ABQ* (twice):	1 point	*This point is also due if the idea is only reflected by the solution.*
$\frac{620}{\sin 19°} = \frac{AQ}{\sin 108°}$,	1 point	
$AQ \approx 1811$	1 point *	
$PQ \approx 1811 - 720 = 1091$	1 point *	
$\frac{620}{\sin 19°} = \frac{BQ}{\sin 53°}$,	1 point	
$BQ \approx 1521$	1 point *	
The distances, rounded to the nearest metre, are *PQ* = 1091 m, *BQ* = 1521 m and *BP* = 605 m.	1 point *	*This point is awarded for the correct unit (m) in the answer.*
Total:	**12 points**	
*The points marked with * are also due if the results differ by at most 3m from those in the markscheme, provided that there is evidence for rounding correctly during the calculations.*		

II. B

16. a)

(In team A, each of the 7 players plays a game against each of the other 6. That means every game is counted twice.) There were $\frac{7 \cdot 6}{2} = 21$ games in team A.	1 point	
(Team B has n members,) the number of games played is $\frac{n \cdot (n-1)}{2} = 55$.	2 points	
This leads to the equation $n^2 - n - 110 = 0$.	1 point	
The positive root is 11 (the roots are –10 and 11).	2 points	
Team B has 11 members.	1 point	
Total:	**7 points**	

16. b)

Each of the 6 players from team A plays 8 games.	1 point	
That makes a total of 6·8 = 48 games played during the second week.	2 points	
Total:	**3 points**	

16. c)

(The classical model of probability can be applied.) $p = \frac{\text{number of favourable cases}}{\text{number of all cases}}$	1 point	*This point is also due if the idea is only reflected by the solution.*
The winners may be selected in $\binom{18}{4}$ ways.	1 point	
There are 7 ways to choose 1 out of the 7 members of team A,	1 point	*These points are also due if the candidate only expresses the number of favourable cases correctly.*
and $\binom{11}{3}$ ways to choose 3 out of the 11 members of team B.	1 point	
(The two selections are independent of each other.) The number of favourable cases is $7 \cdot \binom{11}{3}$.	1 point	
The probability in question is $p = \frac{7 \cdot \binom{11}{3}}{\binom{18}{4}} \quad \left(= \frac{7 \cdot 165}{3060}\right) \approx$	1 point	
$\approx 0.377 \approx 38\%$.	1 point	*Award the 1 point for the correct probability expressed in any form.*
Total:	**7 points**	

17. a)		
$2x-1>0$ and $2x-3>0$, therefore $x>1.5$	1 point	*This point is also due if the extraneous root is rejected at the end by substitution.*
By the identities of logarithms: $\lg(2x-1)(2x-3)=\lg 8$	1 point	
(The logarithm function is one-to-one,) therefore $(2x-1)(2x-3)=8$, that is	1 point	
$4x^2-8x-5=0$.	1 point	
The roots are $x_1=\frac{5}{2}$ and $x_2=-\frac{1}{2}$.	1 point	
Only $x_1=\frac{5}{2}$ is in the domain, and that is really a solution.	1 point	
Total:	**6 points**	

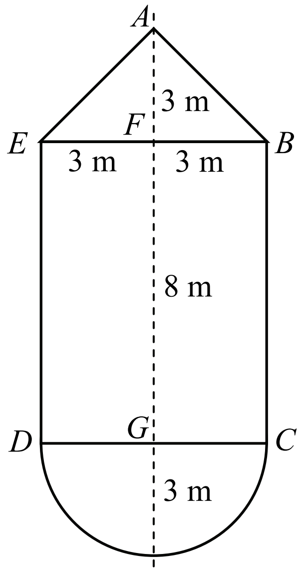
17. b)		
By solving the equation for cos *x*, the roots of the quadratic equation in **a)** are obtained. ($(\cos x)_1=\frac{5}{2}$ and $(\cos x)_2=-\frac{1}{2}$)	2 points	
$\cos x=\frac{5}{2}$ is not possible.	1 point	
The only angle *x* obtained from $\cos x=-\frac{1}{2}$ that may be an angle of a triangle is $x=120^\circ=\frac{2\pi}{3}$, and that is really a solution.	1 point	*Award the point for either correct value of the angle. Award no point if any other angle is listed.*
Total:	**4 points**	

17. c) Solution 1		
With the new variable $\sqrt{y}=z$ introduced,	1 point	
only solutions $0\le z$ are sought.	1 point	
The only non-negative root of the quadratic equation $4z^2-8z-5=0$ is $z=\frac{5}{2}$.	1 point	
It follows that the solution of the original equation is $y=\frac{25}{4}$, and that is really a solution.	1 point	
Total:	**4 points**	

17. c) **Solution 2**		
Squaring both sides: $16y^2 - 40y + 25 = 64y$	1 point	
The roots of the quadratic equation $16y^2 - 104y + 25 = 0$ are $y_1 = \frac{25}{4}$, $y_2 = \frac{1}{4}$.	2 points	
Checking by substitution, or considering the range of each side to show that only the first root satisfies the equation.	1 point	
Total:	**4 pont**	

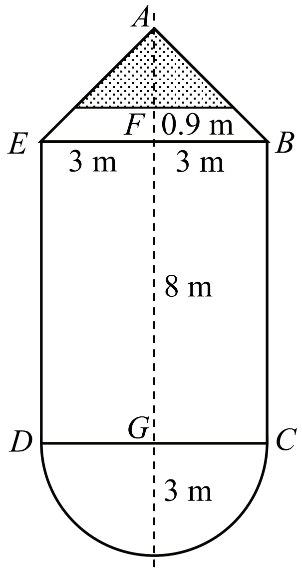
17. d)		
If the middle number is fixed,	1 point	*This point is also due if the idea is only reflected by the solution.*
the other six numbers can be ordered in 6! ways,	1 point	
thus there are 720 possible orders of the seven numbers.	1 point	
Total:	**3 points**	

18. a)



Understanding the problem.	1 point	
The surface of the lower part of the water tank (surface area of a hemisphere of radius r = 3 metres): $A_1 = \frac{4r^2\pi}{2} = 2r^2\pi = 2\cdot 3^2\cdot\pi = 18\pi\,(\approx 56.5)$	1 point	
The surface of the middle part (the lateral surface area of a circular cylinder of radius r = 3 metres and height m = 8 metres): $A_2 = 2r\pi m = 2\cdot 3\cdot\pi\cdot 8 = 48\pi\,(\approx 150.8)$	1 point	
The surface of the upper part of the tank (the lateral surface area of a right circular cone of radius r = 3 metres and height m = 3 metres): The slant height is $AB = a = \sqrt{2}r$	1 point	
$A_3 = ra\pi = 3\cdot 3\sqrt{2}\cdot\pi = 9\sqrt{2}\pi\,(\approx 40)$	1 point	
The total interior surface area is $A = 18\pi + 48\pi + 9\sqrt{2}\pi = \left(66+9\sqrt{2}\right)\pi \approx 247.33\ \text{m}^2$. Since the wording of the problem implies that the result should be rounded upwards to make sure that there is enough material for coating the entire surface, the correct answer is 248 m^2.	1 point	*Award this point for the mathematical rounding to 247 m^2 as well.*
Total:	**6 points**	

18. b)



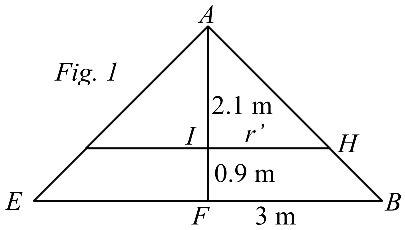


Fig. 1

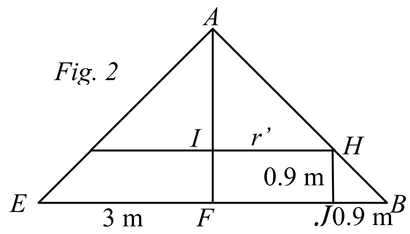


Fig. 2

The height of the water tank is $(3+8+3=)14$ metres. 85% of the height is $(14\cdot 0.85=)11.9$ metres,	1 point	
that is, the hemisphere and the cylinder are full, and the water level is 0.9 metres above the base of the cone.	1 point	
The volume of the lower part of the water tank (volume of a hemisphere of radius r = 3 metres): $V_1=\frac{1}{2}\cdot\frac{4r^3\pi}{3}\left(=\frac{2r^3\pi}{3}\right)=$	1 point	
$=\frac{2\cdot 3^3\cdot\pi}{3}(=18\pi\approx 56.5)$.	1 point	
The volume of the middle part (volume of a circular cylinder of radius r = 3 metres and height m = 8 metres): $V_2=r^2\pi m=$	1 pont	
$=3^2\cdot\pi\cdot 8(=72\pi\approx 226.2)$.	1 point	
The remaining volume in the upper part is the volume of a truncated cone. The radius of the top circle of the truncated cone can be calculated by using the theorem about line segments cut out of parallel lines by the arms of an angle: *(Figure 1)* $\left(\frac{IH}{FB}=\right)\frac{r'}{3}=\frac{2.1}{3}\left(=\frac{AI}{AF}\right)$,	1 point *	
$r'=2.1$.	1 point *	
$V_3=\frac{\pi}{3}m\left(r^2+r'^2+rr'\right)=$	1 point	
$=\frac{\pi}{3}\cdot 0.9\cdot\left(3^2+2.1^2+3\cdot 2.1\right)=(5.913\pi\approx 18.6)$.	1 point	
The total volume of the water in the tank is $A=18\pi+72\pi+5.913\pi=95.913\pi\approx 301\,\mathrm{m}^3$.	1 point	
Total:	**11 points**	

*Another way to calculate the results for the two points marked with *:*

The remaining volume in the upper part is the volume of a truncated cone. The radius of the top circle of the truncated cone can be calculated by observing that ΔAFB and ΔHJB both are isosceles right-angled triangles, *(Figure 2)*	1 point *	
thus $r' = \left(FB - JB = 3 - 0.9 =\right) 2.1$.	1 point *	