ÉRETTSÉGI VIZSGA ● 2011. október 18.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

NEMZETI ERŐFORRÁS MINISZTÉRIUM

Instructions to examiners

Formal requirements:

1. Mark the paper **in ink, different in colour** from the one used by the candidate. Indicate errors and incomplete solutions, etc. in the conventional way.
2. The first one of the grey rectangles under each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to that.
3. **If the solution is perfect**, it is enough to enter the maximum scores in the appropriate rectangles.
4. If the solution is incomplete or incorrect, please indicate the individual **partial scores** in the body of the paper, too.
5. Do not assess anything written in pencil, except diagrams.

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution by the candidate is different**, allocate the points by identifying the parts of the solution equivalent to those of the one given in the markscheme.
2. The subtotals in the markscheme can be **further divided**, but the scores awarded should always be whole numbers.
3. If it is clear that the reasoning and the final answer are both correct, you may award the maximum score even if the solution is **less detailed** than the one given in the markscheme.
4. If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error occurs. If the reasoning remains correct and the error is carried forward without changing the nature of the task, the points for the rest of the solution should be awarded.
5. In the case of a **principal error**, no points should be awarded at all for that section of the solution, not even for formally correct steps. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information based on the principal error is carried forward to the next section or to the next part of the problem and is used correctly there, the maximum score is due for the next part, provided that the error has not changed the nature of the task.
6. Where the markscheme shows a **unit** or **remark** in brackets, the solution should be considered complete without that unit or remark as well.
7. If there are more than one different approaches to a problem, **assess only the one indicated by the candidate**.
8. **Do not give extra points** (i.e. more than the maximum score due for the problem or part of problem.
9. **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. If it is not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I

1.		
$420 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \left(= 2^2 \cdot 3 \cdot 5 \cdot 7\right)$.	2 points	*Do not divide.*
Total:	**2 points**	

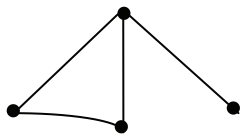
2.		
20 000 and 16 000.	2 points	*1 point for knowing that 36 000 is to be divided into nine equal parts.*
Total:	**2 points**	

3.		
During the 8 days, the number of cells (c) doubled 4 times.	1 point	*These 2 points are also due if the first four terms of the sequence are listed correctly.*
$c = 5000 \cdot 2^4$.	1 point	
$c = 80\,000$.	1 point	
Total:	**3 points**	

4.		
a) **N**;	1 point	*Award the points for a correct answer in any form.*
b) **Z**;	1 point	
c) ∅.	1 point	
Total:	**3 points**	

5.		
$a = 2$.	1 point	
$b = -3$.	1 point	
Total:	**2 points**	

6.		
The median is 7.	2 points	*Do not divide.*
Total:	**2 points**	

7.		
Correct graph, e.g. 	2 points	*Do not divide.*
Total:	**2 points**	

8.		
$d = -3$	1 point	
$a_{50} = a_1 + 49d$	1 point	
$a_1 = 176$	1 point	
Total:	**3 points**	

9.		
B)	2 points	*Do not divide the 2 points.*
Total:	**2 points**	

10.		
B)	2 points	*Do not divide the 2 points.*
Total:	**2 points**	

11.		
$2000 \cdot 1.06^x = 4024$.	1 point	*The point is also due if this idea is reflected by the calculation.*
Calculation of x: $\log 2000 + x \log 1.06 = \log 4024$ $x = \frac{\log 4024 - \log 2000}{\log 1.06} \approx 11.998$.	2 points	*It is also accepted as a correct solution to calculate the balance year by year with a calculator and obtain 12 years.*
12 whole years.	1 point	
Total:	**4 points**	

12.		
A pair (any pair) of face diagonals drawn from the same vertex can be completed to form a regular triangle with the third face diagonal connecting their endpoints,	2 points	*1 point for a correct face diagonal drawn.*
therefore the angle in question is 60°.	1 point	
Total:	**3 points**	

II A

13. a)

The value of the square root is non-negative: $x \le 5$,	1 point*	
and only non-negative numbers have square roots: $\lvert x \rvert \ge \sqrt{35.5}$.	1 point*	
Squaring: $x^2 - 10x + 25 = 2x^2 - 71$.	1 point	
Rearranged: $x^2 + 10x - 96 = 0$.	1 point	
There are two real roots: –16 and 6.	1 point	
The latter is against the first restriction, so it is not a solution of the equation. The only solution of the equation is –16 since it satisfies both conditions, and the transformations were equivalent on the appropriate domain.	1 point*	*The points marked with * are also due if the conditions are not formulated but the roots of the quadratic equation are substituted in the original equation, and the solution is correctly identified.*
Total:	**6 points**	

13. b)

Substituting $\sin^2 x = 1 - \cos^2 x$ on the left-hand side: $1 - \cos^2 x = 1 + 2\cos x$.	1 point	
$\cos^2 x + 2\cos x = 0$;	1 point	
$\cos x(\cos x + 2) = 0$.	1 point	
If $\cos x = 0$, then $x = \frac{\pi}{2} + k\pi$, where $k \in \mathbf{Z}$.	2 points	*Award 1 point if the solution is formulated as shown but without stating the set from which* k *is taken or with a period different from 2π. Award 1 point if the answer is given in the form* $x = 90° + k \cdot 180°$ $(k \in \mathbf{Z})$ *or with degrees and radians mixed. Award no points if the period is missing altogether (for example, if the answer is* $x = 90°$*).*
The equation $\cos x + 2 = 0$ has no solution (since $\cos x = -2$ is impossible).	1 point	
Total:	**6 points**	
Remark. Award full mark, too, if the quadratic is solved with the quadratic formula.		

14. a)		
The quoted answer was given by 18.75% of those at least 40 years old.	1 point	*This point is also due if this idea is reflected by the calculations.*
18.75% of 80 is $80 \cdot 0.1875$.	1 point	
That is, 15 participants of 40 years or older age answered, "less than 5 times".	1 point	
Total:	**3 points**	

14. b)		
Out of those under 40: $120 \cdot 0.35 = 42$,	1 point	
out of those at least 40 years old: $80 \cdot 0.375 = 30$,	1 point	
that is, a total of 72 persons go to the theatre 5 to 10 times a year.	1 point	
This is 36% of the participants.	1 point	
Total:	**4 points**	

14. c) **Solution 1.**		
There are $\binom{200}{2}$ (= 19 900) selections altogether.	1 point	
The number of cases with two persons under 40 selected is $\binom{120}{2}$ (= 7140).	1 point	
The probability of both of the selected participants being younger than 40 is $\frac{\binom{120}{2}}{\binom{200}{2}} \left(= \frac{7140}{19\,900} \approx 0.359 \right)$.	1 point	
The probability of the complementary event is $1 - \frac{\binom{120}{2}}{\binom{200}{2}} \left(= \frac{12\,760}{19\,900} \right)$.	1 point	
Thus the probability of at most one selected participant being younger than 40 years is 0.641.	1 point	*This point is not due if the answer is not rounded to three decimal places or the rounding is wrong.*
Total:	**5 points**	

14. c) **Solution 2.**		
The number of all possible selections is $\binom{200}{2}$ (= 19 900).	1 point	
In $\binom{80}{2}$ (= 3160) out of these cases, both randomly selected participants are at least 40 years old.	1 point	
They belong to different age groups in $80 \cdot 120$ (= 9600) cases.	1 point	
The probability of the event in question is $\frac{\binom{80}{2}+80\cdot 120}{\binom{200}{2}} \left(=\frac{12\ 760}{19\ 900}\right)$.	1 point	
Thus the probability of at most one selected participant being younger than 40 years is 0.641.	1 point	*This point is not due if the answer is not rounded to three decimal places or the rounding is wrong.*
Total:	**5 points**	

15. a) **Solution 1.**		
(The coordinates of *P* are obtained by solving the simultaneous equations.) From the first equation: $y = 2.5x + 7.25$.	1 point	
Substituted in the second equation and rearranged: $x = -1.5$.	1 point	
$y = 3.5$.	1 point	
Thus *P*(–1.5; 3.5).	1 point	
Total:	**4 points**	

15. a) **Solution 2.**		
(The coordinates of *P* are obtained by solving the simultaneous equations.) $10x - 4y = -29$ $10x + 25y = 72,5$ $29y = 101.5$	1 point	
$y = 3.5$	1 point	
$x = -1.5$.	1 point	
Thus *P*(–1.5; 3.5).	1 point	
Total:	**4 points**	

Remark. 1 point for each line graphed correctly. 1 point for correctly reading the coordinates (–1.5; 3.5) of the intersection of the correct lines. 1 point for checking the coordinates by substitution.

15. b) **Solution 1.**		
The normal vectors of the lines are $\mathbf{n}_e(5;-2)$ and	1 point	
$\mathbf{n}_f(2;5)$.	1 point	
The scalar product of the normal vectors is $\mathbf{n}_e \cdot \mathbf{n}_f = 5\cdot 2 + (-2)\cdot 5 = 10 - 10 = 0$.	1 point	*These 2 points are also due if the candidate refers to the two normal vectors being obtained from each other by 90° rotation.*
Therefore the two lines are perpendicular.	1 point	
Total:	**4 points**	

15. b) **Solution 2.**		
The slopes of the lines are $m_e = \frac{5}{2}$,	1 point	
$m_f = -\frac{2}{5}$.	1 point	
The product of the slopes is –1,	1 point	
therefore the two lines are perpendicular.	1 point	
Total:	**4 points**	

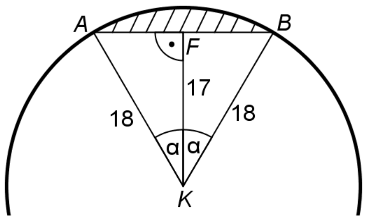
15. c)		
The slope of line *e* is 2.5, thus the tangent of its angle α enclosed with the *x*-axis is $\tan\alpha = 2.5$.	3 points	
Hence $\alpha \approx 68.2°$.	1 point	
Total:	**4 points**	

II B

16. a)		
$M = -4.42 + \frac{2}{3}\log_{10}\left(1.344 \cdot 10^{14}\right)$	1 point	
$M \approx 5$	2 points	*Any value between 4.9 and 5 is acceptable.*
Total:	**3 points**	

16. b)		
$9.3 = -4.42 + \frac{2}{3}\log_{10} E$.	1 point	
$\log_{10} E = 20.58$.	1 point	
Thus the energy released is about $E \approx 3.8 \cdot 10^{20}$ (J).	1 point	
Total:	**3 points**	

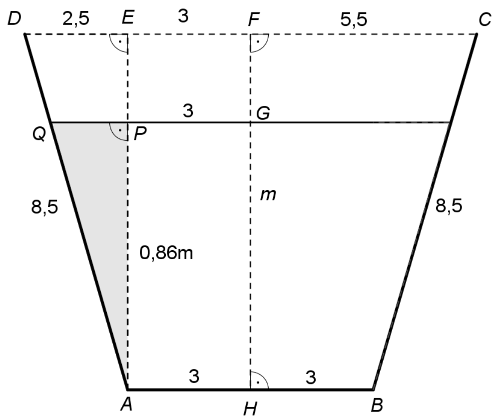
16. c)		
The Chile quake had a magnitude 2 greater than the Canada quake: $-4.42 + \frac{2}{3} \cdot \log_{10} E_C = -4.42 + \frac{2}{3} \cdot \log_{10} E_K + 2$.	1 point	
Rearranged: $\log_{10} E_C - \log_{10} E_K = 3$.	1 point	
(Using an identity of logarithms:) $\log_{10} \frac{E_C}{E_K} = 3$.	1 point	
Hence $\frac{E_C}{E_K} = 1000$.	1 point	
The energy released was greater by a factor of 1000.	1 point	
Total:	**5 points**	

16. d)		
The solution uses the notations of the figure.  In the right-angled triangle *AKF*, $\cos\alpha = \frac{17}{18}$,	1 point	
$\alpha \approx 19.2°$. ($2\alpha \approx 38.4°$.)	1 point	*α ≈ 19° is also accepted.*
The area of the triangle *AKB* is $A_{AKB\Delta} \approx \frac{18^2 \cdot \sin 38.4°}{2}$ (≈ 100.6 km^2).	1 point	*Award at most 1 point if the formula for the area of the circular segment from the data tables book is used incorrectly.*
The area of the circular sector is $A_{\text{sector}} \approx 18^2 \pi \cdot \frac{38.4°}{360°}$ (≈ 108.6 km^2).	1 point	
$A_{\text{circular segment}} \approx 108.6 - 100.6 = 8$ (km^2).	1 point	
The area devastated is about 8 km^2.	1 point	
Total:	**6 points**	

17. a)		
There are $7 \cdot 6 \cdot 5 \cdot 4$,	2 points	
that is 840 such four-digit numbers altogether.	1 point	
Total:	**3 points**	

17. b)		
Each of the first five digits may be any number out of 1, 2, 3, 4, 5, which makes $5^5 (= 3125)$ possibilities altogether.	2 points	
Because of the requirement of divisibility by 4, the last two digits may only be as follows: 12, 24, 32, 44, 52. This is 5 cases.	2 points	*Award 1 point if there are 4 correct cases (and no incorrect ones) listed or there is one wrong number listed along with all five correct numbers. Award no points for any other incorrect answer.*
Altogether, there are $5^5 \cdot 5$,	1 point	
that is 15 625 such seven-digit numbers.	1 point	
Total:	**6 points**	

17. c)		
Each of the digits 1, 2, 3, 4, 5 occurs in the six-digit number, and one of them occurs exactly twice.	1 point	*This point is also due if this idea is reflected by the solution.*
The digit occurring twice has to be 3	1 point	
since the sum of the digits needs to be divisible by 3,	1 point	
and $1+2+3+4+5=15$ (which is divisible by 3).	1 point	
There are $\binom{6}{2}$ ways to select the positions of the two digits of 3.	1 point	*The number of appropriate 6-digit numbers is the number of all permutations of the characters 1; 2; 3; 3; 4; 5, that is,* $\frac{6!}{2!}=360$.
The remaining 4 places may be filled with the other digits in 4! ways.	1 point	
There are $\binom{6}{2}\cdot 4!$,	1 point	
that is, 360 suitable six-digit numbers.	1 point	
Total:	**8 points**	

18 a)		
 Diagram with the given information shown.	1 point	*This point is also due if the solution is correct but there is no diagram. Do not award this point if the given diameters are used as radii.*
The height of the truncated cone is *m* cm. (Because of symmetry,) $ED = 2.5$ cm.	1 point	
In the right-angled triangle *AED,* ($AD = 8.5$; $AE = m$): $m^2 = 8.5^2 - 2.5^2$, $m \approx 8.1$.	1 point	
86% of this is $0.86m \approx 7.0$.	1 point	
The right-angled triangles *APQ* and *AED* are similar (because of the right angles and the acute angle they have in common);	1 point	
and the scale factor of the similitude is (the ratio of the lengths of corresponding sides:) 0.86.	1 point	*The point is also due if this idea is only reflected by the calculations.*
Hence $PQ = 0.86 \cdot DE$, that is, $PQ = 0.86 \cdot 2.5 = 2.15$.	1 point	*The rounding $PQ \approx 2.2$ is also acceptable.*
The radius of the cross-section is $GQ = 3 + 2.15 = 5.15$.	1 point	*$GQ \approx 5.2$ is also acceptable.*
The volume of the sour cream is $V \approx \frac{7.0 \cdot \pi}{3} \cdot \left(5.15^2 + 3^2 + 5.15 \cdot 3\right)$.	1 point	
$V \approx 372.9$ (cm^3).	1 point	
Rounded to the nearest ten cm^3, the volume of the sour cream is 370 cm^3.	1 point	*This point is also due if $GQ \approx 5.2$ is used and hence, with correct rounding, 380 cm^3 is obtained for the volume of the sour cream.*
Total:	**11 points**	

18. b) **Solution 1.**		
The complementary event is considered.	1 point	*The point is also due if this idea is only reflected by the calculations.*
The probability of selecting a defective tub is 0.03, so the probability of selecting an intact one is 0.97.	1 point	
The probability that the officer will not find a defective product is 0.97^{10},	2 points	
therefore the probability of finding a defective one is $1-0.97^{10}$ (≈ 0.2626).	1 point	
Rounded to two decimal places, the probability in question is 0.26.	1 point	*This point is also due if the probability is expressed as a percentage (26%, or 26,26%).*
Total:	**6 points**	

18. b) **Solution 2.**		
The probability of selecting a defective tub is 0.03, so the probability of selecting an intact one is 0.97.	1 point	
Let $P(k)$ denote the probability that there are k defective ones among the 10 tubs selected. $P(1)=\binom{10}{1}\cdot 0.03\cdot 0.97^{9}\approx 0.228$; $P(2)=\binom{10}{2}\cdot 0.03^{2}\cdot 0.97^{8}\approx 0.032$; $P(3)=\binom{10}{3}\cdot 0.03^{3}\cdot 0.97^{7}\approx 0.003$; $P(4)=\binom{10}{4}\cdot 0.03^{4}\cdot 0.97^{6}\approx 0.0001$. In each of the cases $5\le k\le 10$, the probability is less than 0.00001, thus they will not influence the result rounded to two decimal places.	3 points	*Award 1 point if the binomial distribution is applied correctly in at least one case (correct substitution in the formula). 1 point is due for realizing that there are 10 cases to investigate. The full score (3 points) is due if the indicated reasoning is applied or all 10 cases are calculated correctly.*
Therefore the probability in question is about $0.228+0.032+0.003=0.263$,	1 point	
Rounded to two decimal places: 0.26.	1 point	
Total:	**6 points**	