ÉRETTSÉGI VIZSGA • 2011. május 3.

MATEMATIKA
ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI
ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

NEMZETI ERŐFORRÁS
MINISZTÉRIUM

Important Information

Formal requirements:

1. The papers must be assessed in pen and of different colour than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
2. The maximal score for each questions is printed in the first shaded rectangle next to the question. The score given by the examiner should be entered into the other rectangle.
3. In case of correct solutions it is enough to enter the maximal score into the corresponding rectangle.
4. In case of faulty or incomplete solutions, please indicate the corresponding partial score within the body mof the paper.
5. Nothing, apart from the diagrams, can be evaluated if written in pencil.

Substantial requirements:

6. In case of some problems there are more than one marking schemes given. However, if you happen to come accross with some solution different from those outlined here, please identify the parts equivalent to those in the solution provided in this booklet and do your marking accordingly.
7. The scores given in this booklet can be split further. Keep in mind, however, that any partial score can be an integer number only.
8. If the candidate's argument is clearly valid and the answer is correct then the maximal score can be given even if the actual solution is less detailed than the one in this booklet.
9. If there is a calculation error or any other flaw in the solution, then the score should be deducted for the actual item only where the error has occured. If the candidate is going on correctly working with the faulty intermediate result and the problem has not suffered substantial damage due to the error, then the subsequent partial scores should be awarded.
10. If there is a fatal error within an item (these are separated by double lines in this booklet), then no points should be given in this item, even for formally correct steps. If, however, the wrong result obtained by invalid argument is used correctly throughout subsequent steps, maximal scores should be given for the parts remaining, unless the problem has changed essentially due to the error.
11. If an additional remark or a measuring unit appears in brackets in this booklet then the solution is complete even if it does not appear in the candidate's solution.
12. If there are more than one correct attempts to solve a problem, it is the one indicated by the candidate that can be marked.
13. You should not give any bonus points (points beyond the maximal score for a solution or for some part of a solution).
14. You should not reduce the score for erroneous calculations or steps unless its results are used by the candidate in the actual course of the solution.
15. There are only 2 questions to be marked out of the 3 in part II./B of this examination. Hopefully, the candidate has entered the number of the question not to be marked int he square area provided for this. Accordingly, this question should not be assessed even if there is some kind of solution contained in the paper. Should there be any ambiguity about the student's request with respect to the question not to be considered, it is the last one in this problem set, by default, that should not be marked.

I.

1.		
$a\cdot\left(a^2+1\right)$	2 points	*These 2 points cannot be split further.*
Total:	**2 points**	

2.		
The price of the textbooks is 6300 Ft.	1 point	
The price of the notebooks is 2250 Ft.	1 point	
Total:	**2 points**	

3.		
The relative frequency of the M-size T-shirts is: $\left(\frac{238}{1116}\approx\right)0{,}2133$.	1 point	*Correct rounding to at least two decimal places may be accepted.*
The mode is the size L.	1 point	*This point should be given even if the answer is 322.*
186 shirts of each size should have been sold.	1 point	
Total:	**3 points**	

4.		
The codes of the true assertions are A and C, respectively.	2 points	*A single correct code is worth 1 point. If B is also indicated then the total is 0 point.*
Total:	**2 points**	

5.		
$x=12$;	1 point	
$y=9$.	1 point	
Total:	**2 points**	

6.		
The number of handshakes is 9.	2 points	*If the answer is 18 then 1 point should be given.*
Total:	**2 points**	

7.		
$X\cdot Y=24\cdot 10^{101}$	1 point	*This point is due even if nothing but the correct result is written down.*
$X\cdot Y=2{,}4\cdot 10^{102}$	1 point	
Total:	**2 points**	

8.		
$q=\frac{3}{4}$	1 point	
$a_5=a_3\cdot q^2$	1 point	
$a_5=\frac{27}{8}(=3.375)$	1 point	
Total:	**3 points**	

9.		
$f=\frac{3h-256}{10}$ or $182=\frac{10f+256}{3}$	1 point	
$f=\frac{3\cdot 182-256}{10}$	1 point	*If the answer is correct this point is also due.*
The forearm is $f=29$ cm long.	1 point	
Total:	**3 points**	

10.		
The price is scaled up by $1.2\cdot 1.3(=1{,}56)$ after the two years.	1 point	
The actual price after two years is hence $(23000\cdot 1.2\cdot 1.3=)35880$ Ft.	1 point	
The increment is 56%.	1 point	
Total:	**3 points**	

11.		
$b<0$	1 point	
$b=0$	1 point	
Total:	**2 points**	

12.		
A={1; 2; 3; 4; 6; 9; 12; 18; 36}	1 point	*This point can be given for a complete list only.*
B={1; 4; 16}	1 point	*This point can be given for a complete list only.*
$A\cap B$={1; 4}	1 point	
$A\setminus B$ = {2; 3; 6; 9; 12; 18; 36}	1 point	
Total:	**4 points**	

II/A.

13. a) **first solution**		
Since $(x-1)^2 = x^2 - 2x + 1$,	2 points	*These 2 points cannot be split further.*
the equation to be solved is $x^2 - (x^2 - 2x + 1) = 2$,	1 point	*This point is given for the recognition of the essential role of the use of brackets. It should also be given if the brackets are resolved correctly without writing down the details.*
that is $x^2 - x^2 + 2x - 1 = 2$.	1 point	
Hence $x = \frac{3}{2}$.	1 point	
Checking.	1 point	
Total:	**6 points**	

13. a) **second solution**		
For recognizing the relevant identity.	2 points	
$x^2 - (x-1)^2 = 2 \iff \left(x - (x-1)\right)\left(x + (x-1)\right) = 2$,	1 point	*These 1–1 points should also be given if the brackets are resolved correctly without writing down the details.*
yielding $(x - x + 1)(2x - 1) = 2$.	1 point	
Hence $x = \frac{3}{2}$.	1 point	
Checking.	1 point	
Total:	**6 points**	

13. b)		
For x greater than 1	1 point*	
$\lg x - \lg(x-1) = 2 \iff \lg\frac{x}{x-1} = 2$.	1 point	*This point is due for the correct application of the rule of logarithms.*
(By the definition of common logarithms) $\frac{x}{x-1} = 100$.	1 point	
Hence $x = 100(x-1)$,	1 point	
yielding $x = \frac{100}{99}$ $(\approx 1{,}01)$.	1 point	
Checking.	1 point	
Total:	**6 points**	
*The point marked by * can also be given if the candidate is checking the result by substituting it to the original equation.*		

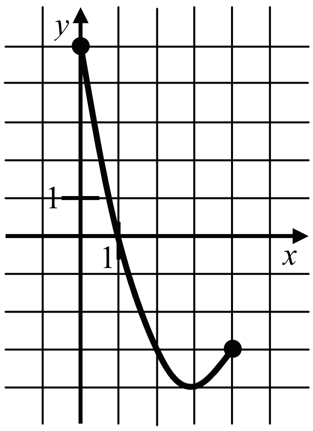
14.		
The phone number has 7 digits and it can be split into two parts: the digits of one part are from the column of the keyboard with 4 digits and those of the other part are from a column containing 3 digits.	1 point	*These 1–1 points are due even if these ideas are used in the solution without being explicietely stated.*
There are two different cases: the leading digit is either in the middle column or it is in one of the columns in either side.	1 point	
First case There are 3 possibilities for the leading digit to be in the middle column, since it cannot be zero.	1 point	
Hence there are $3 \cdot 3! = 18$ orders of the first 4 digits.	1 point	
The 3 digit number following the leading 4 digits is now composed from the digits of one of the two side columns. In both cases there are $3! = 6$ ways to press the keys.	1 point	
There are $3 \cdot 3! \cdot 3! \cdot 2 (= 216)$ 7-digit phone numbers obtained this way.	2 points	*Just 1 point may be given if the two options are not mentioned here.*
Second case The leading 3 digits of the phone number are from the first column. There are $3! = 6$ possible orders of them and for each order there are $4! = 24$ orders to press the 4 keys in the middle column.	1 point	
There are $3! \cdot 4! (= 144)$ phone numbers of this kind.	2 points	
Similarly, the same $(3! \cdot 4! = 144)$ number of phone numbers have their leading 3 digits from the third column.	1 point	
There are $216 + 144 + 144 = 504$ 7-digit phone numbers altogether, satisfying all the conditions.	1 point	
Total:	**12 points**	

15. a)

the function has maximal value only	the function has minimal value only	the function has both maximal and minimal values	the function has no extremal values
g	*j*	*f*	*h; m*

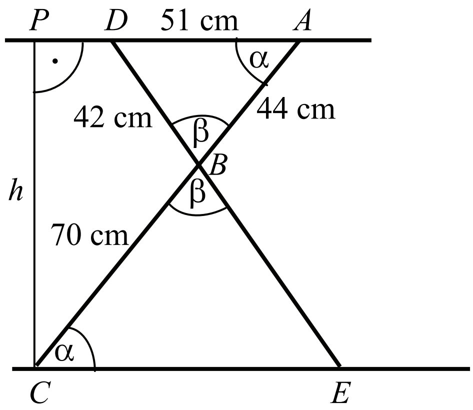
1 point is due for every correctly entered function as long as it appears only once.	5 points	
Total:	**5 points**	

15. b)

	3 points	*If the candidate knows that the curve is an arch of a parabola opened upwards: 1 point.* *The vertex is correct: 1 point.* *If the given domain is taken into account: 1 point.*
The range is $[-4;5]$.	2 points	*At most 1 point may be given if the candidate ignores the given domain.*
Among the roots (1 and 5) of the extended function it is just 1 lying in the domain therefore this is the zero of the function *k*.	2 points	*If the candidate states both roots of the extended function then at most 1 point may be given.*
Total:	**7 points**	

II/A.

16. a)



The triangles ABD and CBE are similar,	2 points	
because their angles are pairwise equal (opposite and parallel angles, respectively).	1 point	
Hence (the corresponding sides are in the same proportion and thus) $\frac{BE}{42}=\frac{70}{44}$.	2 points	
$BE = 42\cdot\frac{70}{44}\approx 66.8\,(\text{cm})$.	1 point	
The bearer bar DE is ≈109 cm long.	1 point	
Total:	**7 points**	

16. b)

The angle α can be calculated in the triangle ABD by the cosine rule.	1 point	*This point is due if this idea is clear from the solution.*
$42^2 = 51^2 + 44^2 - 2\cdot 51\cdot 44\cdot\cos\alpha$.	2 points	
Hence $\cos\alpha \approx 0.6179$,	2 points	
$\alpha \approx 51.8°$.	1 point	
In the right triangle APC one gets $h = AC\cdot\sin\alpha$,	2 points	
that is $h \approx 114\cdot\sin 51.8° \approx 89.6$ (cm).	1 point	
Therefore, the ironing surface is ≈ (90+3=) 93 cm high above the ground level.	1 point	
Total:	**10 points**	

The magnitudes of the remaining two angles of the triangle ABD are $\beta \approx 72.7°$ and $ADB\sphericalangle \approx 55.5°$, respectively.
Working with the length of the other bearer bar one gets $h \approx 109\cdot\sin 55.5° \approx 89.8$ cm which also yields 90 cm, when rounded properly.

17. a) first solution		
There are 6^3 sequences of length 3 (and they are all equally probable).	1 point	
a1) <u>The gain is 300 tokens:</u> Each score is even and there are 3^3 ways for this.	1 point	
The probability that the player wins 300 tokens is $\frac{3^3}{6^3}=\frac{1}{8}$	1 point	
a2) <u>The gain is 500 tokens:</u> The first score is 1, the second is odd and the third is even. There are $3\cdot 3$ ways for this.	1 point	
Similarly, after the first score is 1, there are the same 3·3 ways to have the second score be even and the third score be odd.	1 point	
There are $3\cdot 3+3\cdot 3\,(=18)$ favourable outcomes	1 point	
and thus the probability that the player wins 500 tokens is $\frac{2\cdot 3\cdot 3}{6^3}=\frac{1}{12}$.	1 point	
a3) <u>The gain is 800 tokens:</u> The first score is 3 and the other two scores are both odd. There are $3\cdot 3$ ways for this.	1 point	
(Since there are 6^3 outcomes altogether) $\frac{3\cdot 3}{6^3}=\frac{1}{24}$ is the probability that the player wins 800 tokens.	1 point	
a4) <u>The gain is 2000 tokens:</u> Since there is just 1 favourable outcome,	1 point	
the probability that the player wins 2000 tokens is $\frac{1}{6^3}=\frac{1}{216}$.	1 point	
Total:	**11 points**	

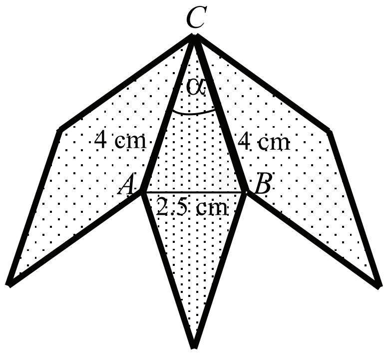
17. a) **second solution**		
a1) <u>The gain is 300 tokens:</u> Since (the three rolls are mutually independent and) the respective scores are even with the probability $\frac{1}{2}$,	1 point	
the probability that the player wins 300 tokens is $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{8}$	1 point	
a2) <u>The gain is 500 tokens:</u> $\frac{1}{6}$ is the probability that the first score is 1.	1 point	
$\frac{1}{2}\cdot\frac{1}{2}$ is the probability that the second score is even and the third score is odd.	1 point	
Similarly, one gets the same $\frac{1}{2}\cdot\frac{1}{2}$ probability for the second score be even and the third score be odd.	1 point	*Aliter: the parities of the second and the third scores can be even-even, even-odd, odd-even, odd-odd. These outcomes occur with the same probability, therefore the probability that the two scores have different parities is 0.5.*
Hence the probability that one of the scores is even and the other one is odd is $\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{2}$,	1 point	
therefore, the probability that the player wins 500 tokens is $\frac{1}{6}\cdot\frac{1}{2}=\frac{1}{12}$.	1 point	
a3) <u>The gain is 800 tokens:</u> $\frac{1}{6}$ is the probability that the first score is 3 and $\frac{1}{2}\cdot\frac{1}{2}$ is the probability that the remaining two scores are both odd,	1 point	
therefore the probability that the player wins 800 tokens is $\frac{1}{6}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{24}$.	1 point	
a4) <u>The gain is 2000 tokens:</u> In case of each of the three rolls the probability that the given score is 5 is $\frac{1}{6}$,	1 point	
(and since the rolls are mutually independent) the probability that the player wins 2000 tokens is $\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{216}$.	1 point	
Total:	**11 points**	

17. b)		
Not to win anything is complementary to winning something.	2 points	*These a 3 points are due even if the candidate is using these ideas without explicitely stating them.*
The sum of the probabilities of two complementary events is 1.	1 point	
The chance to win is $\frac{1}{8}+\frac{1}{12}+\frac{1}{24}+\frac{1}{216}$,	1 point	
that is $\frac{27}{216}+\frac{18}{216}+\frac{9}{216}+\frac{1}{216}=\frac{55}{216}$ (≈ 0.25).	1 point	
Hence the probability that a player does not win anything is $1-\frac{55}{216}=\frac{161}{216}$ (≈ 0.75).	1 point	
Total:	**6 points**	

18. a) first solution		
The number of cookies is an integer multiple of both 16 and 18.	1 point	*This point is due if this idea is clear from the solution.*
The least common multiple of 16 and 18 is 144,	2 points	
therefore the number of cookies is a multiple of 144.	1 point	*This point is due if this idea is clear from the solution.*
The only one among the multiples of 144 lying between 400 and 500 is $3\cdot 144$.	1 point	
The number of cookies is hence 432.	1 point	
Total:	**6 points**	

18. a) second solution		
If the girls were baking n cookies, each, and the boys got k cookies, each, then the total number of cookies can be calculated in two ways: either as $16n$ or as $18k$. Therefore $16n=18k$.	1 point	
(Hence $n=\frac{9k}{8}$. Since n and k are positive integers and $(9\,;8)=1$) k is divisible by 8.	2 points	
The actual number of cookies is between 400 and 500 yielding $400<18k<500$ that is $23\le k\le 27$.	1 point	
There is just one multiple of 8 in this range, namely 24. Hence $k=24$.	1 point	
The number of cookies is hence $18\cdot 24$, that is 432.	1 point	
Total:	**6 points**	

18. b)



For an appropriate diagram (of the view from the top, for example, showing the cookies in the required arrangement).	2 points	
In the isosceles triangle ABC $\sin\frac{\alpha}{2}=\frac{1.25}{4}=0.3125$.	2 points	
Hence $\alpha \approx 36.4°$ ($36.41^\circ < \alpha < 36.42^\circ$)	1 point	
(Since $9<\frac{360^\circ}{\alpha}<10$) Dani can arrange at most 9 cookies in the required manner.	1 point	
Total:	**6 points**	

18. c)

The area of the view from the top of the rhomb shaped cookies is $\approx 9{,}5$ cm^2.	2 points	*This area can also be computed by using the previous angle or by calculating the length of the other diagonal* (≈ 7.6 *cm;* 1 *point*) *and proceeding to the area as the product of the diagonals.* (1 *point*).
A area of the view from the top of the ring shaped cookies is $2^2\pi - x^2\pi$ (cm^2).	1 point	
$4\pi - x^2\pi \approx 9.5$	1 point	
$x \approx 0.99$. The inner radius of the ring shaped cookies is approximately 1 cm.	1 point	
Total:	**5 points**	