ÉRETTSÉGI VIZSGA • 2010. október 19.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

NEMZETI ERŐFORRÁS MINISZTÉRIUM

Instructions to examiners

Formal requirements:

1. Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors, incomplete solutions, etc. in the conventional way.
2. The first one of the grey rectangles under each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to that.
3. **If the solution is perfect**, it is enough to enter the maximum scores in the appropriate rectangles.
4. If the solution is incomplete or incorrect, please indicate the individual **partial scores** in the body of the paper, too.
5. Do not assess anything that is written in pencil, except diagrams.

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution by the candidate is different**, allocate the points by identifying the parts of the solution equivalent to those of the one given in the markscheme.
2. The subtotals in the markscheme can be **further divided**, but the scores awarded should always be whole numbers.
3. If it is clear that the reasoning and the final answer are both correct, you may award the maximum score even if the solution is **less detailed** than the one in the markscheme.
4. If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error occurs. If the reasoning remains correct and the error is carried forward without changing the nature of the task, the points for the rest of the solution should be awarded.
5. In the case of a **principal error**, no points should be awarded at all for that section of the solution, not even for formally correct steps. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information based on the principal error is carried forward to the next section or to the next part of the problem and is used correctly there, the maximum score is due for the next part, provided that the error has not changed the nature of the task.
6. Where the markscheme shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.
7. If there are more than one different approaches to a problem, **assess only the one indicated by the candidate**.
8. **Do not give extra points** (i.e. more than the maximum score due for the problem or part of problem).
9. **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. If it is not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I.

1.		
$A \cap B = \{a; b; d\}$,	1 point	*The points are only due if there is no error.*
$A \cup B = \{a; b; c; d; e; f\}$	1 point	
Total:	**2 points**	

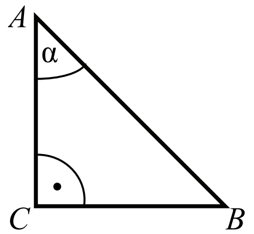
2.		
The group has 12 members.	1 point	
132 SMS texts were sent altogether.	1 point	
Total:	**2 points**	*Award the 2 points for a bald statement of the correct answer.*

3.		
$a = -2$	1 point	
$b = \frac{1}{2}$	2 points	
Total:	**3 points**	

4.		
The expression is meaningful if $x > -3.5$.	2 points	*1 point at most if equality is allowed or rearrangement is wrong.*
Total:	**2 points**	

5.		
$a > 1$	2 points	*1 point for* $a \geq 1$.
Total:	**2 points**	

6.		
The solutions of the equation in the set A are −1 and 0.	2 points	*1 point for each correct value. Take off 1 mark for every incorrect answer. (Deductions should not result in a negative score.)*
Total:	**2 points**	

7.		
		
(By definition of trigonometric functions,) $BC = \sin\alpha$,	1 point	*AC = cos α (by def.)*
$AC = BC$,	1 point	*cos α = sin α*
therefore $\alpha = 45°$.	1 point	
Total:	**3 points**	

8.		
I. false;	1 point	
II. true;	1 point	
III. true;	1 point	
IV. false.	1 point	
Total:	**4 points**	

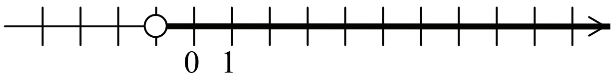
9.		
$b=\sqrt[3]{\frac{c}{d}}$ or $b=\left(\frac{c}{d}\right)^{\frac{1}{3}}$.	2 points	*1 point may be awarded if one identity is used incorrectly. 0 points for more than one error.*
Total:	**2 points**	

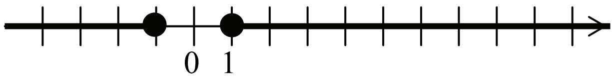
10.		
Correct formula.	2 points	*0 points for a graph without a formula.*
Correct maximum point(s).	1 point	
Total:	**3 points**	

11.		
Appropriate graph drawn.	2 points	
Total:	**2 points**	

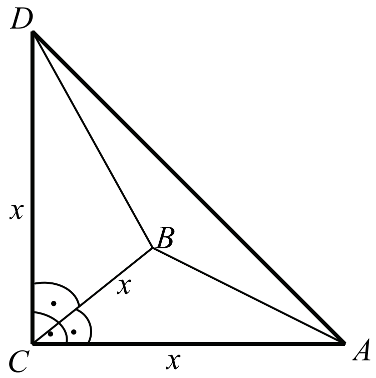
12.		
The centre lies on the perpendicular bisector of the chord,	1 point	*A correct representation of these conditions in the diagram is accepted as explanation.*
so its first coordinate is 4.	1 point	
The centre is $O(4; 4)$.	1 point	
Total:	**3 points**	
Stating u = v: 1 point; *setting up equations* $(1-u)^2+(-u)^2=r^2$ *and* $(7-u)^2+(-u)^2=r^2$ *: 1 point;* *solving the equations to get u=4 and O(4; 4): 1 point.*		

II/A.

13. a)		
$12x-6\cdot(x-1)>3\cdot(x-3)-4\cdot(x-2)$	1 point	
$12x-6x+6>3x-9-4x+8$	1 point	
$6x+6>-x-1$	1 point	
$7x>-7$ that is $x>-1$	1 point	
	1 point	
Total:	**5 points**	

13. b)		
$-3x^2\le -3$	1 point	
$x^2\ge 1$	2 points	*These 2 points cannot be divided further.*
(The set of solutions of the inequality is the set of numbers x, such that) $x\ge 1$,	1 point	
or $x\le -1$.	1 point	
	2 points	*The 1 point for each part is only due if the endpoint is correct.*
Total:	**7 points**	

14. a)



2.88 dl = 288 cm^3.	1 point	
The base area of the tetrahedron (pyramid) is $T_b = \frac{x^2}{2}$, (the height is x,)	1 point	*These 2 points are also due if the correct volume of the pyramid is obtained from a different reasoning.*
and its volume is $V = \frac{x^3}{6}$.	1 point	
$288 = \frac{x^3}{6}$, hence	1 point	
$x^3 = 1728;\ x = 12$.	1 point	
The sides of triangle *ABD* are all equal,	1 point	
and their length is $x \cdot \sqrt{2} \approx 16.97 \approx 17$ cm.	1 point	
The edges of the tetrahedron (pyramid) are 12 cm and 17 cm long.	1 point	
Total:	**8 points**	*Award at most 6 points if the result is wrong owing to incorrect conversion of units.*

14. b)

The area of each of the congruent right-angled triangles is $T_1 = \frac{144}{2} = 72$ (cm^2).	1 point	
The area of the fourth face is $T_2 = \frac{2x^2 \cdot \sqrt{3}}{4} \approx$	1 point	
≈ 124.7 (cm^2).	1 point	
The surface area of the carton is $A = 3T_1 + T_2 = 340.7 \approx 341$ cm^2.	1 point	*Calculating with the rounded value of 17 cm, $T_2 = 125.1$ cm^2, and the surface area is $A \approx 341$ cm^2.*
Total:	**4 points**	

15. a) **Solution 1.**		
(Every outcome of the pairs of rolls is equally probable, so the classical model is applicable.) There are $6^2 = 36$ outcomes for a round altogether.	2 points	*The 2 points are also due if these ideas are only reflected by the solution.*
There are 2 ways to roll the first time and 4 ways the second time,	1 point	
thus there are $2 \cdot 4 = 8$ “favourable” pairs of rolls,	1 point	
and $\frac{8}{36}\left(=\frac{2}{9}\approx 0.22\right)$ is the probability of scoring 1 point in a round, and scoring it in the first roll.	1 point	
Total:	**5 points**	

15. a) **Solution 2.**		
(The first and second rolls are independent.)		
The probability of scoring a point in the first roll is $\frac{2}{6}$,	1 point	
and the probability of not scoring in the second roll is $\frac{4}{6}$.	1 point	
The probability in question is $\frac{2}{6}\cdot\frac{4}{6}$,	2 points	
that is $\frac{8}{36}=\left(\frac{2}{9}=0.22...\right)$.	1 point	
Total:	**5 points**	

15. b)		
Exactly one point may be scored by scoring in the first roll and not scoring in the second roll, or the other way round.	2 points	*The 2 points are also due if this idea is only reflected by the solution.*
This is $2 \cdot 2 \cdot 4 = 16$ cases altogether.	1 point	
2 points are scored in $2 \cdot 2 = 4$ cases.	1 point	
Thus the probability of scoring at least one point in a round is $\frac{20}{36}=\frac{5}{9}$.	1 point	*At least one point is scored in 20 out of the 36 possible cases.*
The probability of not scoring any point is $1-\frac{5}{9}=\frac{4}{9}$,	1 point	*No points are scored in 16 cases.*
therefore the first event is more probable.	1 point	
Total:	**7 points**	

15. a) and b), another method

The **first row of the table shows** the possible outcomes of the **first roll,** and **the first column shows** those of **the second roll**. The fields of the table represent the total scores for the **round**. There are 36 equally probable cases, the combinatorial model is applicable.

	1	2	3	4	5	6
1	**0**	**0**	**0**	**1**	**1**	**0**
2	**0**	**0**	**0**	**1**	**1**	**0**
3	**0**	**0**	**0**	**1**	**1**	**0**
4	**1**	**1**	**1**	**2**	**2**	**1**
5	**1**	**1**	**1**	**2**	**2**	**1**
6	**0**	**0**	**0**	**1**	**1**	**0**

Table filled out correctly.	6 points	
marks the fields representing the event a): the probability in question is $\frac{8}{36}$.	2 points	
b) The probability of not scoring any point (fields marked) is $\frac{16}{36}$. This is less than $\frac{1}{2}$, therefore the probability of scoring at least one point is larger.	4 points	
Total:	**12 points**	

II/B.

16. a)		
$a_8 = a_1 + 7d$, where d is the common difference of the sequence. $14 = -7 + 7d$	1 point	
$d = 3$.	1 point	
$660 \geq S_n$	1 point	*These 7 points are also due if the candidate does not state (and manipulate) an inequality but explains that the solutions are the positive integers not greater than 24.*
$S_n = \frac{2a_1 + (n-1)\cdot d}{2}\cdot n = \frac{-14 + 3\cdot(n-1)}{2}\cdot n$	1 point	
$3n^2 - 17n - 1320 \leq 0$.	1 point	
The quadratic expression on the left-hand side has a minimum ($a = 3 > 0$, or reference to a graph, etc.),	1 point	
its zeros are 24 and $-\frac{55}{3}$ (which is negative).	1 point	
$\left(-\frac{55}{3} < 0 <\right) n \leq 24$	1 point	
Since in this problem n is a positive integer, the possible values of n are 1, 2, …, 23, 24.	1 point	
Total:	**9 points**	
A correct answer based on investigating S_1, S_2, …, S_{24}, S_{25} is also worth full mark. *Award 7 points if S_{25} is not considered or there is no reference to monotonicity.* *Award 4 points if only an equation is used and the answer is $n = 24$.*		

16. b)		
$a_4 = a_1 \cdot q^3$, where q is the common ratio of the sequence. $-189 = -7\cdot q^3$	1 point	
$q = 3$.	1 point	
$S_n = a_1\frac{q^n - 1}{q-1} = -7\cdot\frac{3^n - 1}{2}$	1 point	
$-68887 = -7\cdot\frac{3^n - 1}{2}$	1 point	
$3^n = 19\,683$	2 points	
The exponential function is one-to-one / strictly monotonic,	1 point	*Accept any other valid explanation.*
$n = 9$.	1 point	
Total:	**8 points**	

17. a)		
The area of the regular triangle of side a is $t_1=\frac{a^2\sqrt{3}}{4}\approx 2.7\,(\text{cm}^2)$.	1 point	
The region above the regular triangle is a circular segment intercepted by a central angle of 60° of the circle.	1 point	
Its area is $t_2=\frac{a^2\pi}{6}-\frac{a^2\sqrt{3}}{4}=\frac{a^2}{2}\cdot\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)\approx 0.6\,(\text{cm}^2)$.	1 point	
The uppermost region is a "crescent", its area is obtained by subtracting the area of the circular segment from that of the semicircle of radius $\frac{a}{2}$.	1 point	*The 1 point is also due if this idea is only reflected by the solution.*
$t_3=\frac{1}{2}\cdot\left(\frac{a}{2}\right)^2\pi-t_2=\frac{a^2\pi}{8}-\frac{a^2}{2}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)=.$	1 point	
$=\frac{a^2}{2}\left(\frac{\pi}{4}-\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)\approx 1.9\,(\text{cm}^2)$.	1 point	
Total:	**6 points**	

17. b) Solution 1.		
If condition (1) is considered only, the crescent may have four different colours,	1 point	
then, also because of (1), the circular segment may only have three colours,	1 point	
and the regular triangle may also have three colours since it may be any colour different from that of the circular segment.	1 point	
Thus there are $4\cdot 3\cdot 3=36$ different ways to meet condition (1).	1 point	
From these 36 cases, the number of cases violating condition (2) should be subtracted.	1 point	
The number of cases when three colours are used and a red region lies next to a yellow region is $4\cdot 2=8$,	2 points	
since there are 4 ways to place the red and yellow regions next to each other, and the third region may get two colours in each case.	1 point	
There are two ways to use red and yellow only.	2 ponts	
Thus the number of ways to meet both conditions is $36-(8+2)=26$.	1 point	
Total:	**11 points**	

17. b) **Solution 2.**		
If red and yellow are both used in the colouring, then it follows from (2) that they must be applied to the crescent and the regular triangle.	1 point	*Award 1 point for a correct answer without an explanation.*
Then the circular segment may be green or blue. That is $2 \cdot 2 = 4$ possibilities.	1 point	
If red is not used at all, then there are two cases: 1. The remaining three colours are all used. Then the number of colourings is $3! = 6$.	1 point	
2. Only two out of the remaining three colours are used. These two colours may be selected in three ways,	1 point	
and it follows from (1) that two different badges can be made with the two colours chosen.	1 point	*Award 1 point for a correct answer without an explanation.*
Thus the number of possibilities in this case is $3 \cdot 2 = 6$.	1 point	
Altogether, the number of colourings not containing red is therefore $6 + 6 = 12$.	1 point	*Award 3 point out of these 4 if the candidate does not consider the cases counted twice.*
The number of colourings not containing yellow is also 12.	1 point	
These include two that do not contain either of the colours red and yellow.	1 point	
Those two cases have been counted above, so the number of new cases not using yellow is 10.	1 point	
Therefore the number of all cases that meet both conditions is $4 + 12 + 10 = 26$.	1 point	
Total:	**11 points**	

17. b) **Solution 3**		
If condition (1) is considered only, there are $4 \cdot 3 \cdot 2 = 24$ ways to colour the badge with exactly three of the four colours.	2 points	
If condition (1) is considered only, there are $\binom{4}{2} \cdot 2 = 12$ ways to colour it with exactly two colours.	2 points	
This is 36 cases altogether. The number of cases not meeting condition (2) should be subtracted.	1 point	
The number of ways to use three colours with a red region lying next to a yellow region is $4 \cdot 2 = 8$,	2 points	
since there are four ways to place the red and yellow regions, next to each other, and the third region may get two colours in each case.	1 point	
There are two ways to use red and yellow only.	2 points	
Thus the number of ways to meet both conditions is $36 - (8 + 2) = 26$.	1 point	
Total:	**11 points**	

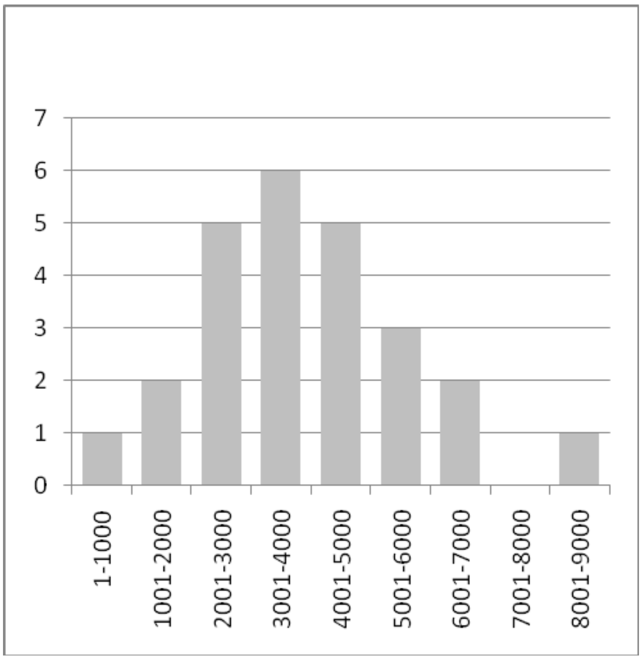
Remark. If the solution is sought by listing the individual cases:

- *11 points for a systematic list of all cases;*
- *award at most 9 points if the candidate lists all 26 colourings in some way but the list does not make it clear that there are no further colourings possible;*
- *at most 3 points if one of the conditions is ignored;*
- *at most 5 points if the cases listed are all good but the list is incomplete.*

18. a)		
The sum of the elements of the sample of 25 is 101 400.	1 point	
The mean is $\frac{101\,400}{25} =$	1 point	
$= 4056$ (forints).	1 point	
Total:	**3 points**	

18. b)		
The frequency table of the classes of range 1000 forints:	3 points	*2 points are due if 1 or 2 entries are wrong,* *1 point is due for 3 or 4 errors,* *no points for more than 4 errors.*

Monthly expenses in forints	Number of families
1-1000	1
1001-2000	2
2001-3000	5
3001-4000	6
4001-5000	5
5001-6000	3
6001-7000	2
7001-8000	0
8001-9000	1



(diagram above)	2 points	*A correct diagram with the axes interchanged is also accepted.* *The 2 points are also due if a correct graph (correct axes, correct units on axes) is made with the wrong data carried forward.*
Total:	**5 points**	

18. c)		
The new mean with the two extremes omitted is $\frac{91\,900}{23} \approx$	1 point	
≈ 3996 (forints).	1 point	
Since $\frac{3996}{4056} \approx 0.9852$,	1 point	
the mean decreased by $\approx 1.48\%$.	1 point	*Accept 1.49%, too.*
The smallest item of the new list of data is 1200 forints and the largest item is 6800 forints,	1 point	
thus the range is 5600 forints.	1 point	
Total:	**6 points**	

18. d)		
The new mean is $\frac{25 \cdot 4056 + (4056 - 1000) + (4056 + 1000)}{27} =$	2 point	*Correct numerator: 1 point, correct denominator: 1 point.*
$= \frac{27 \cdot 4056}{27} = 4056$.	1 point	
Total:	**3 points**	