ÉRETTSÉGI VIZSGA ● 2009. október 20.

MATEMATIKA
ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI
ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

OKTATÁSI ÉS KULTURÁLIS
MINISZTÉRIUM

Important Information

Formal requirements:

1. The papers must be assessed in **pen and of different colour** than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
2. The first one among the shaded rectangles next to each question contains the maximal score for that question. The **score** given by the examiner should be entered into the other **rectangle**.
3. **In case of correct solutions**, it is enough to enter the maximal score into the corresponding rectangle.
4. In case of faulty or incomplete solutions, please indicate the corresponding partial scores within the body of the paper.
5. Nothing, apart from the diagrams, can be evaluated if written in pencil.

Substantial requirements:

1. In case of some problems there are more than one marking schemes given. However, if you happen to come across with some **solution different** from those outlined here, please find the parts equivalent to those in the solution provided here and do your marking accordingly.
2. The scores in this assessment **can be split further**. Keep in mind, however, that the number of points awarded for any item can be an integer number only.
3. In case of a correct answer and a valid argument the maximal score can be awarded even if the actual solution is **less detailed** than that in this booklet.
4. If there is a **calculation error** or any other flaw in the solution, then the score should be deducted for the actual item only where the error has occured. If the candidate is going on working with the faulty intermediate result and the problem has not suffered substantial damage due to the error, then the subsequent partial scores should be awarded.
5. If there is a **fatal error** within an item (these are separated by double lines in this booklet), then even formally correct steps should not be given any points, whatsoever. However, if the wrong result obtained by the invalid argument is used correctly throughout the subsequent steps, the candidate should be given the maximal score for the remaing parts, unless the problem has been changed essentially due to the error.
6. If an **additional remark** or a **measuring unit** occurs in brackets in this booklet, the solution is complete even if the candidate does not mention it.
7. If there are more than one correct attempts to solve a problem, it is the **one indicated by the candidate that can be marked**.
8. You should **not give any bonus points** (points beyond the maximal score for a solution or for some part of the solution).
9. You should not reduce the score for erroneous calculations or steps unless its results are actually used by the candidate in the course of the solution.
10. **There are only 2 questions to be marked out of the 3 in part II/B of this exam paper.** Hopefully, the candidate has entered the number of the question not to be marked in the square provided for this. Accordingly, this question should not be assessed even if there is some kind of solution contained in the paper. Should there be any ambiguity about the student's request with respect to the question not to be considered, it is the last one in this problem set, by default, that should not be marked.

I.

1.		
The arithmetic mean is 73.	1 point	
The geometric mean is 55.	1 point	
Total:	**2 points**	

2.		
The set A is $\{2;3;5;7\}$.	1 point	
The set B is $\{6;12;18;24;30\}$.	1 point	
The set $A \cup B$ is $\{2;3;5;6;7;12;18;24;30\}$.	1 point	
Total:	**3 points**	

3.		
The number of black marbles is 12.	2 points	*If the answer is wrong but the candidate writes down a correct equation for the data in demand then 1 point may still be given.*
Total:	**2 points**	

4.		
The value of the expression is 25.	2 points	*The score cannot be split further.*
Total:	**2 points**	

5.		
$\alpha = 27^\circ$	2 points	*1 point may be given if there is a rounding error only.*
Total:	**2 points**	

6.		
$a_{11} = (-5)\cdot(-2)^{10}$	1 point	
$a_{11} = -5120$	1 point	
Total:	**1 point**	

7. **First solution.**		
The formula defining the function is $x \mapsto -\lvert x-1\rvert + 5$	3 points	*Each one of the three transformation steps is worth 1 point.*
Total:	**3 points**	

7. **Second solution.**		
The formula of the function is $x \mapsto \begin{cases} x+4, & if\ x \le 1 \\ -x+6, & if\ 1 < x \end{cases}$	3 points	*1-1 point for the correct formulas and a further point for the correct domains.*
Total:	**3 points**	

8.		
The code of the correct expression is F	3 points	*The score cannot be split.*
Total:	**3 points**	*Maximal score is due even if instead of its code the candidate enters the correct expression.*

9.		
The number to be cancelled is 5.	2 points	
Total:	**2 points**	

10.		
The scalar product of the two vectors is 0.	2 points	*The score cannot be split.*
The vectors are perpendicular, their angle is 90°.	1 point	
Total:	**3 points**	

11.		
The radius of the sphere with the largest surface area that fits the cube is 10 cm,	1 point	
and its surface area is $400\pi \approx 1256\left(cm^2\right)$.	1 point	
The ball would not fit in the box.	1 point	
Total:	**3 points**	*Maximal score is due if the candidate calculates the radius of the ball* $(r \approx 11.28\ \text{cm})$ *and states that the diameter is greater than 20 cm..*

12.		
$f\left(\frac{\pi}{3}\right) = 2 \cdot \sin\left(\frac{\pi}{3} - \frac{\pi}{2}\right) =$	1 point	
$= 2 \cdot \sin\left(-\frac{\pi}{6}\right) =$	1 point	
$= -1$	1 point	
Total:	**3 points**	

II/A

13. a)		
Removing the braces one gets $x^2+4x+4-90=2{,}5x-85$.	1 point	
$x^2+1{,}5x-1=0$.	1 point	
$x_1=0{,}5$, $x_2=-2$.	2 points	
The results are solutions on the domain of real numbers, indeed.	1 point	*This point is due for any kind of correct check (mentioning equivalence included).*
Total:	**5 points**	

13. b) **First solution**		
If $x>0$ then $3-x<14x$.	1 point	*At most 2 points may be given if the candidate is working incorrectly without separating the cases.*
$x>0.2$,	1 point	
which is x > 0.2 on the stated domain.	1 point	
If $x<0$ then $3-x>14x$.	1 point	
$x<0{,}2$,	1 point	
which becomes $x<0$ on the stated domain.	1 point	
The solution of the inequality is $]-\infty\,;0[\,\cup\,]0{,}2\,;\infty[$.	1 point	*Any correct form of the solution should be accepted.*
Total:	**7 points**	

13. b) **Second solution**		
$\frac{3-x}{7x}-2<0$.	1 point	
$\frac{3-15x}{7x}<0$.	1 point	
$3-15x>0$ and $7x<0$.	1 point	
$x<0$	1 point	
or $3-15x<0$ and $7x>0$.	1 point	
$x>0.2$.	1 point	
The solution of the inequality is $]-\infty\,;0[\,\cup\,]0{,}2\,;\infty[$.	1 point	*Any correct form of the solution should be accepted.*
Total:	**7 points**	

14. a)		
(The number of tiles per row are consecutive terms of the arithmetic progression for which) $a_1 = 8$, $d = 2$.	1 point	
$\frac{2a_1 + (n-1)d}{2}n =$	1 point	
$= 858$.	1 point	
$n^2 + 7n - 858 = 0$.	1 point	
$n_1 = 26$ és $n_2 = -33$. (The answer for the question is a positive integer: $n = 26$.)	1 point	
Angela laid 26 rows and this value satisfies the conditions.	1 point	
Total:	**6 points**	*If the candidate arrives to the result n = 26 by summing termwise and does not state clearly that there can be no other solution then 4 points can be given.*

14. b)		
The number of claret coloured tiles is 144.	2 points	*1-1 points are due for the correct calculation of percentages and takijng into account the number of packs, respectively.*
There were $a_{26} = a_1 + 25d = 8 + 50 = 58$ tiles laid in the 26th row.	1 point	
There were 8 + 58 + 2·24 = 114 bordering tiles all claret cloured.	1 point	
There were 30 claret coloured tiles remaining.	1 point	
The total overleft was 900 − 858 = 42 tiles, 12 of which are grey and 30 are claret coloured.	1 point	
Total:	**6 points**	

15. a)		
The square numbers that can be produced this way are 16, 25, 36 and 64.	1 point	
There are 36 different possible two-digit numbers.	1 point	
The probability is $p = \frac{1}{9} (\approx 0.111)$.	1 point	
Total:	**3 points**	

15. b)		
There are 6-6 possible scores independently on both the decimal and the unary position.	1 point	
There are 6 cases when the two digits are equal, these are the favourable outcomes.	1 point	
The probability is $\frac{1}{6}$.	1 point	
Total:	**3 points**	

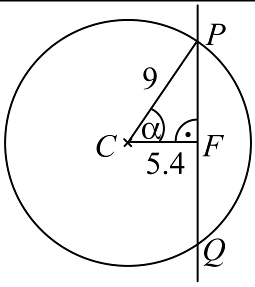
15. c) first solution		
The sum of the digits is at most 9 in case of the following numbers: 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 51, 52, 53, 54, 61, 62, 63.	4 points	*These 4 points should be given for any correct way of reasoning; the candidate is not expected to enumerate the actual two-digit numbers.*
There are 30 favourable outcomes altogether.	1 point	
The probability is 30/36 = 5/6.	1 point	
Total:	**6 points**	

15. c) second solution		
One can proceed by computing the probability of the complementary event (the sum is greater than 9).	2 points	*These 2 points are due even if this idea is not stated explicitely in the solution.*
This can happen in the following cases: 46, 55, 56, 64, 65, 66.	2 points	
There are 6 favourable outcomes, therefore the probability of the complementary event is 6/36 = 1/6.	1 point	
The probability is 1-1/6 = 5/6.	1 point	
Total:	**6 points**	

II/B

16. a)		
The simultaneous system $x^2+y^2-6x+8y-56=0 \quad \wedge \quad x=8.4$ should be solved.	1 point	
Substituting: $y^2+8y-35.84=0$,	1 point	
yielding $y=3.2$ or $y=-11.2$.	2 points	
There are two common points, $P_1(8.4\,;3.2)$, $P_2(8.4\,;-11.2)$.	2 points	
Total:	**6 points**	

16. b)		
Completing the square: $(x-3)^2+(y+4)^2=81$.	1 point	
The centre of the circle is $C(3;-4)$ (and its radius is 9).	1 point	
The straight line is parallel to the y-axis,	1 point	*This point is due if the candidate is effectively using this fact without stating it.*
therefore the foot of the perpendicular from the point $C(3;-4)$ to the line is $T(8.4\,;-4)$.	1 point	
The perpendicular distance of the centre of the circle from the line is $\overline{TC}=8.4-3=5.4$ units.	1 point	
Total:	**5 points**	

16. c) first solution		
		
Neat diagram.	1 point	*This point is also due for the correct use of the corresponding data.*
In the right triangle CFP $\cos\alpha = \frac{5,4}{9} = 0.6$,	1 point	
and hence $\alpha \approx 53.13°$.	1 point	
The central angle of the major arc PQ is $360^\circ - 2\alpha \approx 253.74°$.	1 point	
The length of the arc is $\frac{2\cdot 9\cdot \pi \cdot 253.74}{360} \approx 39.9$.	1 point	
The length of the major arc PQ is app. 39,9 cm.	1 point	
Total:	**6 points**	
The correct result should be accepted if the candidate uses radian measure when calculating the central angle (4,43) and applies the correct formula involving radians for the length (39,9 cm) of the arc.		

16. c) second solution		
The central angle 2α of the minor arc *PQ* can be calculated in the triangle *PCQ* by the cosine rule. By Pithagoras' theorem in the right triangle *CFP* $FP = 7.2$ (cm), therefore $PQ = 14.4$ (cm).	1 point	*This point for the calculation of the length PQ can be given only if it is clear from the solution that the candidate is planning to determine the central angle from the triangle PCQ.*
$\cos 2\alpha = \frac{2\cdot 9^2 - 14.4^2}{2\cdot 9^2} =$	1 point	
$= -0.28$,	1 point	
yielding $2\alpha \approx 106.26°$.	1 point	
The length of the minor arc is app. $\frac{2\cdot 9\cdot \pi \cdot 106.26}{360} \approx 16.7$ (cm)	1 point	
The length of the major arc *PQ* is app. 39,9 cm.	1 point	
Total:	**6 points**	

Due to multiple rounding and different calculations (e. g. the length of the second arc is obtained by subtraction or by applying the arc length formula a second time, or by simply using the rounded value for π in the computations) results differing from the given ones may be also accepted if the solution is flawless and the candidate applied the rounding rules correctly.

17. a) first solution		
The perimeter of the paralelogram on the map is 17.0 cm and the length of the cycle path is $17.0\cdot 1.25 = 21.25$ cm.	1 point	
The actual cycle path is hence $21.25\cdot 3\cdot 10^4$ cm that is	1 point	
6.375 km long.	1 point	
The length of the cycle path rounded to one decimal place is hence 6.4 km.	1 point	
Total:	**4 points**	

17. a) second solution		
The actual lengths of the sides of the parallelogram are $AB = 4.7\cdot 3\cdot 10^4$ cm=1.41 km and	1 point	
$AD = 3.8\cdot 3\cdot 10^4$ cm=1.14 km (and $BD = 0.99$ km).	1 point	
The perimeter of the parallelogram is 5,1 km and the length of the cycle path is $1.25\cdot 5.1 = 6.375$ km.	1 point	
The length of the cycle path rounded to one decimal place is hence 6.4 km.	1 point	
Total:	**4 points**	

17. b)		
The longest distance in the parallelogram is the diagonal AC.	1 point	*This point is due if the candidate is just using this without stating it.*
Writing down the cosine rule in the triangle ABD $3.3^2 = 4.7^2 + 3.8^2 - 2\cdot 4.7\cdot 3.8\cdot \cos BAD\sphericalangle$.	1 point	
Hence $\cos BAD\sphericalangle = \frac{4.7^2 + 3.8^2 - 3.3^2}{2\cdot 4.7\cdot 3.8} \approx$	1 point	
≈ 0.7178 (therefore $BAD\sphericalangle \approx 44.1°$ and thus $ABC\sphericalangle \approx 135.9°$).	1 point	
By the cosine rule in the triangle ABC $AC^2 = 4.7^2 + 3.8^2 - 2\cdot 4.7\cdot 3.8\cdot \cos ABC\sphericalangle$,	1 point	
yielding $AC \approx 7.9$ (cm).	1 point	
This corresponds to the actual distance 2.4 km (rounded to one decimal place.)	1 point	
Total:	**7 points**	

The candidate might also use the actual distances. Then (since $BD = 0{,}99$ *km):*

$$\cos BAD\sphericalangle = \frac{1.41^2 + 1.14^2 - 0.99^2}{2\cdot 1.41\cdot 1.14} \approx 0.7178 \text{ and } AC^2 \approx 1.41^2 + 1.14^2 + 2\cdot 1.41\cdot 1.14\cdot 0.7178$$

that is $AC^2 \approx 5.595$ *yielding AC= 2,4 km rounded to one decimal place.*

17. c)		
The area of the actual surface is $9 \cdot 10^8 \cdot 4.7 \cdot 3.8 \cdot \sin 44.1° \approx 1.119 \cdot 10^{10}$ (cm^2) (Hero's formula may also be used here)	2 points	*These 2 points may be split as follows: any correct method (the area formula of the parallelogram, Hero's formula, correct area formula of the triangle once having divided the parallelogram into two congruent triangles) yielding the area of the parallelogram (either the one on the map or the actual one) is worth 1 point and the correct calculation is worth another point.*
which is equal to $1.119 \cdot 10^6$ m^2.	1 point	*This point is due for the expression of the surface area in m^2. However, it should also be given if the candidate is using some other unit but still converts the value of the amount of water correctly into m^3.*
Accordingly, the increment of water in the reservoir is app. $1.119 \cdot 10^6 \cdot 0.15 \approx 1.679 \cdot 10^5$ m^3	2 points	*At most 1 point may be given if the candidate is using incompatible units..*
which is 168 thousand m^3 rounded to the nearest 1000 m^3 .	1 point	
Total:	**6 points**	

18. a)

x (mm)	0	0,3	0,6	1,2	1,5	2,1	3
$I(x)$ $\left(\frac{\text{watt}}{\text{m}^2}\right)$	800	713	635	505	450	357	253

(table above)	3 points	*1-1 point should be given for any 2 correct and appropriately rounded results, yielding the total of 3 point for six correct answers. At most 2 points may be given in case of a rounding error in any one of the six values, however, no further points can be deducted for further rounding errors.*
Total:	**3 points**	

18. b)

One should solve the equation $0.15 = 0.1^{\frac{x}{6}}$ (where x is the distance in millimeters.)	2 points	*These 2 points cannot be split further.*
$\lg 0.15 = \frac{x}{6} \cdot \lg 0.1$ $x = 6 \cdot \frac{\lg 0.15}{\lg 0.1}$	2 points	
$x \approx 4.9$	1 point	
The intensity of the laser beam is reduced to 15% of its original magnitude at app. 4,9 mm depth.	1 point	
Total:	**6 points**	

1) At most 3 points may be given if the candidate finds the correct answer by intelligent guessing, e.g. by experimenting on the calculator.
2) However, having found he answer 4,9 by trial and error and referring to the monotonicity of the function I is worth full score.

18. c) first solution

There are three possibilities for each star: it is blue, green or not drawn at all.	3 points	*These 3 points are due even if this idea is used in the solution without being stated.*
There are $3^4 = 81$ illumination plans, altogether.	4 points	
There is at least one star to be drawn, therefore the total is one less: 81-1=80.	1 point	
Total:	**8 points**	

At most 4 points may be given if the candidate is working with two states of the stars, only.

18. c) second solution		
There are $\binom{4}{1}\cdot 2 = 8$ ways to draw one star.	1 point	
There are $\binom{4}{2}\cdot(1+2+1) = 24$ ways to draw 2 stars. (There are 6 ways to choose the 2 drawn stars out of 4 and either both of them are drawn by the same colour or they are different)	2 points	
There are $\binom{4}{3}\cdot(1+3+3+1) = 32$ ways to draw 3 stars. (There are 4 ways to choose the 3 drawn stars out of 4. There are 2 ways to draw them with identical colours and 3 ways to draw 1 of them blue and 2 of them green. Switching the two colours yields another 3 possibilities.	2 points	
There are $2\cdot 1+2\cdot 4+\binom{4}{2}=16$ ways to draw all the 4 stars. (2 ways to draw each star with the same colour, 3 identical and the 4th one gets the other colour yields $2\cdot 4$ possibilities, 2-2 identically drawn stars yield $\binom{4}{2}$ possibilities.	2 points	
There are 8+24+32+16=80 illumination plans altogether.	1 point	
Total:	**8 points**	

18. c) third solution		
Each illuminated star is either blue or green.	2 points	*These 2 points are due even if this idea is used in the solution without being stated.*
There are $4\cdot 2=8$ ways to draw 1 star.	1 point	
There are $\binom{4}{2}\cdot 2^2 = 24$ ways to draw 2 stars. (There are 6 ways to choose the two illuminated stars and 2^2 ways to draw them.)	2 points	
There are $4\cdot 2^3=32$ ways to draw 3 stars.	1 point	
There are $2^4=16$ ways to draw 4 stars. (Each star can be illuminated by one of the 2 colours.)	1 point	
There are 8+24+32+16=80 illumination plans altogether.	1 point	
Total:	**8 points**	