ÉRETTSÉGI VIZSGA ● 2009. május 5.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI ÉS KULTURÁLIS MINISZTÉRIUM

Important Information

Formal requirements:

1. The papers must be assessed in **pen and of different colour** than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
2. The first one among the shaded rectangles next to each question contains the maximal score for that question. The **score** given by the examiner should be entered into the other **rectangle**.
3. **In case of correct solutions**, it is enough to enter the maximal score into the corresponding rectangle.
4. In case of faulty or incomplete solutions, please indicate the corresponding partial scores within the body of the paper.
5. Nothing, apart from the diagrams, can be evaluated if written in pencil.

Substantial requirements:

1. In case of some problems there are more than one marking schemes given. However, if you happen to come across with some **solution different** from those outlined here, please find the parts equivalent to those in the solution provided here and do your marking accordingly.
2. The scores in this assessment **can be split further**. Keep in mind, however, that the number of points awarded for any item can be an integer number only.
3. In case of a correct answer and a valid argument the maximal score can be awarded even if the actual solution is **less detailed** than that in this booklet.
4. If there is a **calculation error** or any other flaw in the solution, then the score should be deducted for the actual item only where the error has occured. If the candidate is going on working with the faulty intermediate result and the problem has not suffered substantial damage due to the error, then the subsequent partial scores should be awarded.
5. If there is a **fatal error** within an item (these are separated by double lines in this booklet), then even formally correct steps should not be given any points, whatsoever. However, if the wrong result obtained by the invalid argument is used correctly throughout the subsequent steps, the candidate should be given the maximal score for the remaing parts, unless the problem has been changed essentially due to the error.
6. If an **additional remark** or a **measuring unit** occurs in brackets in this booklet, the solution is complete even if the candidate does not mention it.
7. If there are more than one correct attempts to solve a problem, it is the **one indicated by the candidate that can be marked**.
8. You should **not give any bonus points** (points beyond the maximal score for a solution or for some part of the solution).
9. You should not reduce the score for erroneous calculations or steps unless its results are actually used by the candidate in the course of the solution.
10. **There are only 2 questions to be marked out of the 3 in part II/B of this exam paper.** Hopefully, the candidate has entered the number of the question not to be marked in the square provided for this. Accordingly, this question should not be assessed even if there is some kind of solution contained in the paper. Should there be any ambiguity about the student's request with respect to the question not to be considered, it is the last one in this problem set, by default, that should not be marked.

I.

1.		
The subsets containing even numbers only are: $\{6\}$; $\{28\}$; $\{6;28\}$.	2 points	*1 point can be given if the candidate writes only two correct subsets. Also 1 point can be given if the results are correct but the notation used is lousy. .*
Total:	**2 points**	

2.		
$t=\frac{(a^3)^5}{a^{-2}}=a^{17}$	2 points	*1 point may be given for the correct use of one of the identities.*
Total:	**2 points**	

3.		
The logical value of the statement is: TRUE.	1 point	
The converse of the statement is as follows: If a number is divisible by 12 then it is also divisible by 36.	1 point	
Total:	**2 points**	

4.		
The number of handshakes is 10.	2 points	
Total:	**2 points**	

5.		
For $c_3=50000\cdot 1.074^3$.	2 points	
The capital after three years is 61 942 forints.	1 point	*This point cannot be given in case of erroneous rounding.*
Total:	**3 points**	

6.		
The possible passwords are 2244; 2424; 2442; 4422; 4242; 4224.	3 points	*1-1 points should be given for every correct pair of passwords.*
Total:	**3 points**	

7.		
The domain is $\{x \in \mathbf{R} \mid x \le 0\}$.	2 points	*1. Any form of the correct answer is worth 2 points.* *2. If zero is missing, i. e. the domain is stated as the set of negative numbers then 1 point should be given.*
Total:	**2 points**	

8.		
The answer is –1.	2 points	*No point can be given for any other result.*
Total:	**2 points**	

9.		
The hypotenuse of the triangle is 13 cm.	1 point	*A correct diagram can be accepted for an explanation.*
The circumcenter of a right triangle is the midpoint of the hypotenuse.	1 point	
Therefore, the circumradius is 6.5 cm.	1 point	
Total:	**3 points**	

10.		
$g(x) = \sin\left(x - \frac{\pi}{2}\right) - 3$.	3 points	*The correct internal linear function is worth 2 points and 1 point should be given for the correct value of the external constant.*
Total:	**3 points**	

11.		
$H \cup G = \{A; B; C; E; I; K; L; N; Ó; T\}$	3 points	
Total:	**3 points**	
1) If the candidate writes down the sets H and/or G but the final answer is wrong then 1 point can be given for each correctly given set. *2) If every element of $H \cup G$ is present but there are elements occuring more than once in the list.then 1 point can be given*		

12.		
The equation of the straight line is $x - 2y = 8$.	3 points	*Any form of the correct equation is worth 3 points.*
Total:	**3 points**	*If the line is just parallel to the one to be found then 1 point can be given.*

II/A

13. a)		
There are powers of 3 occuring on both sides of the equation because $9 = 3^2$.	1 point	*This point is due if this idea is present in the solution.*
The 3-base exponential is strictly monotone and thus the exponents are also equal.	1 point	
$x^2 - 3x - 10 = 0$.	1 point	
$x_1 = 5$ and $x_2 = -2$.	2 points	
Both numbers are satisfying the starting equation and thus its solutions are $x_1 = 5$ és $x_2 = -2$.	1 point	
Total:	**6 points**	

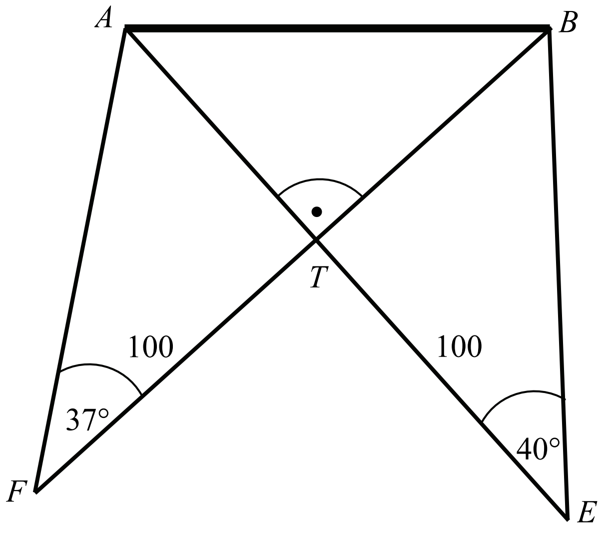
13. b)		
The solution of the first inequality is $x < 2$.	2 points	
The solution of the second inequality is $x \geq -2$.	2 points	
The integer numbers satisfying both inequalities are the elements of the set {–2; –1; 0; 1}.	2 points	
Total:	**6 points**	*If the answer is plainly $-2 \leq x < 2$ then the score should be reduced by 1 point.*

14. a)		
The integers between 645 and 654 should be checked.	1 point	
Since the number of students has to be a multiple of 11,	2 points	*These 3 points are due even if the candidate is checking every single number in the given range.*
the actual value is 649.	2 points	
Total:	**5 points**	

14. b)		
There are 56 students being at least 180 cm tall.	1 point	
$56 \cdot 0.75 = 42$ (among them) are playing basketball.	1 point	
There is a total of $\frac{42}{70} \cdot 100 = 60$ students of the school playing basketball.	2 points	
Total:	**4 points**	

14. c)		
There are 568 students being at most 180 cm tall.	1 point	
The probability that such a student is going to win the 1st prize is $p = \frac{568}{616} \approx 0.922$.	2 points	
Total:	**3 points**	

15.



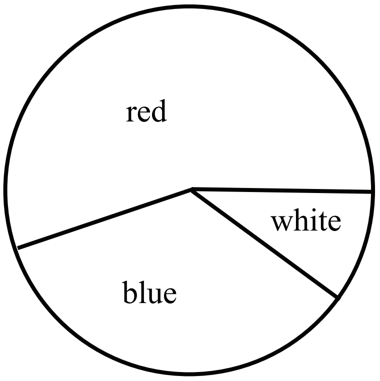
A sketchy map with correct notations that shows the understanding of the problem.	3 points	*The sketch is not expected to be proportional.*
Applying the tangent-function in the right triangles *TBE* and *TAF* yields	1 point	*This 1 point is due even if the candidate simply applies the tangent function without stating it.*
$\tan 40^\circ = \frac{TB}{100}$, that is	1 point	
$TB = 100 \cdot \tan 40^\circ \left(\approx 83.91\right)$ and, similarly	1 point	
$\tan 37^\circ = \frac{TA}{100}$, that is	1 point	
$TA = 100 \cdot \tan 37^\circ \left(\approx 75.36\right)$.	1 point	
Applying Pythagoras' theorem in the right triangle *ABT*: $AB^2 = TB^2 + TA^2$.	2 points	*These 2 points are due even if the candidate does not state Pythagoras' theorem explicitely just using it correctly.*
Substituting the values of TB and TA one gets $AB \approx \sqrt{12720} = 112.78$.	1 point	
The distance of the trees is 113m, correct to the nearest meter.	1 point	
Total:	**12 points**	*The score should be reduced only once if the candidate is rounding erroneously or uses inaccurate intermediate values in the calculations.*

II/B

16. first solution		
If the geometric and the arithmetic progressions of the problem are $\{a_n\}$ and $\{b_n\}$, respectively, then the conditions about the corresponding three terms of these two progressions can be written as $a_1 = b_1$; $a_2 = b_4$; $a_3 = b_{16}$.	1 point	*These points are due even if there are no detailed explanations but the candidate is correctly using the relevant relations.*
Denote the common difference of the AP $\{b_n\}$ by d.	1 point	
The corresponding terms of this AP are $b_1 = 5$; $b_4 = 5+3d$; $b_{16} = 5+15d$, respectively.	2 points	
Applying the geometric mean property in the geometric progression yields $a_2^{\,2} = a_1 \cdot a_3$	2 points	
Substituting the corresponding expressions for the terms b_i one gets $5\cdot(5+15d) = (5+3d)^2$.	2 points	
Collecting the terms: $9d^2 - 45d = 0$.	2 points	
Hence $d_1 = 0$ and $d_2 = 5$.	2 points	
If $d_1 = 0$ then the fifth term of the arithmetic progression is 5 and the sum of the first five terms of the geometric progression is 25.	2 points	
If $d_2 = 5$ then the fifth term of the progression is 25	1 point	
(the corresponding terms of the GP are : 5, 20, 80, respectively, therefore) $q = 4$.	1 point	
$s_5 = 5\cdot\frac{4^5-1}{3} = 1705$.	1 point	
Total:	**17 points**	

16. second solution		
If the geometric and the arithmetic progressions of the problem are $\{a_n\}$ and $\{b_n\}$, respectively, then the conditions about the corresponding three terms of these two progressions can be written as $a_1 = b_1; \quad a_2 = b_4; \quad a_3 = b_{16}$.	1 point	*These points are due even if there are no detailed explanations but the candidate is correctly using the relevant relations..*
Denote the common ratio of the GP $\{a_n\}$ by q, the corresponding terms of this GP are : $a_1 = 5; \quad a_2 = 5q; \quad a_3 = 5q^2$.	2 points	
If d is the common difference of the AP, then $b_4 - b_1 = 3d$ and $b_{16} - b_4 = 12d$.	2 points	
Combining these two equations one gets $4(b_4 - b_1) = b_{16} - b_4$.	1 point	
Substituting the corresponding expressions for a_i yields $4 \cdot (5q - 5) = 5q^2 - 5q$.	2 points	
Collecting the terms: $q^2 - 5q + 4 = 0$.	2 points	
Hence $q_1 = 1$ and $q_2 = 4$.	2 points	
If $q = 1$ then the fifth term of the arithmetic progression is 5 and the sum of the first five terms of the geometric progression is 25.	2 points	
If $q = 4$ ((the corresponding terms of the GP are : 5, 20, 80, respectively, therefore) $q = 4$.), $d = 5$ in the arithmetic progression,	1 point	
and the fifth term is 25.	1 point	
The sum of the first five terms in the geometric progression is $s_5 = 5 + 20 + 80 + 320 + 1280 = 1705$.	1 point	
Total:	**17 points**	

17. a)



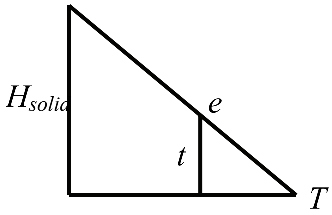
<table>
<tr><td>Correct chart:</td><td>2 points</td><td></td></tr>
<tr><td>The respective central angles are:
<table>
<tr><th></th><th>white</th><th>blue</th><th>red</th></tr>
<tr><td>in degrees</td><td>36</td><td>126</td><td>198</td></tr>
<tr><td>in radians</td><td>0,2π
(≈0.6283)</td><td>0,7π
(≈2.1991)</td><td>1,1π
(≈3.45581)</td></tr>
</table>
The calculation of the central angles is worth 1 points per unit.</td><td>2 points</td><td></td></tr>
<tr><td>Total:</td><td>4 points</td><td></td></tr>
</table>

17. b)

The number of favourable outcomes is 54.	1 point	
$p=\frac{54}{99}\approx 0.545$.	2 points	
Total:	**3 points**	

17. c)		
The probability of drawing any one of the labelled marbles is the same and thus one can apply the classical model.		
The total number of outcomes is $n = 10^4$.	1 point	
24 can be written as a 4-factor product of the given numbers as *a*) 1, 1, 3, 8 *b*) 1, 1, 4, 6 *c*) 1, 2, 2, 6 *d*) 1, 2, 3, 4 *e*) 2, 2, 2, 3	5 points	
There are 12 possible orders of the factors in each of the cases *a*), *b*) and *c*),	1 point	*This 1 point is due even if one of these cases is skipped.*
24 of them in *d*)	1 point	
and, finally, there are 4 of them in case *e*).	1 point	
The probability is hence $\frac{64}{10000} = 0{,}0064$.	1 point	
Total:	**10 points**	

18. a)		
The total area of the sheet is the sum of 6 congruent isosceles triangles.	1 point	*This point is due if this idea is clear from the computations.*
Denote the height of such a triangle by h_o; By Pythagoras' theorem : $h_o = \sqrt{{h_{solid}}^2 + {h_b}^2}$, where h_b is the height of a central triangle of the base.	3 points	*Finding a suitable triangle is worth 2 points and applying Pythagoras is worth 1 point.*
$h_o = \sqrt{256 + \frac{3}{4} \cdot 144} = \sqrt{364} (\approx 19.08)$.	1 point	
$A = 6 \cdot \frac{12}{2} \cdot \sqrt{364} (\approx 686{,}87)$.	1 point	
The surface area of the sheet is 687 m^2.	1 point	
Total:	**7 points**	

18. b)		
The length of a slant edge by Pythagoras' theorem is $e=\sqrt{16^2+12^2}=20$.	2 points	
Applying a central similarity of scale factor $\frac{1}{3}$ from *T* the length *t* of a small rod is equal to $t=\frac{1}{3}\cdot 16=\frac{16}{3}$,  	2 points	
The total length of the rods is hence : $H_{solid}+6\cdot e+6\cdot t=$	1 point	
=168 meter.	1 point	
Total:	**6 points**	

18. c)		
The rope when stretched determines a planar section of the pyramid parallel to its base. The distance of this plane from the apex is equal to $\frac{2}{3}H_{solid}$.	2 points	*Any correct explanation is worth 2 points.*
Therefore, the planar section is a regular hexagon of side 8 m	1 point	
and thus the total length of the stretched rope is 48 meters.	1 point	
Total:	**4 points**	