ÉRETTSÉGI VIZSGA • 2008. október 21.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI ÉS KULTURÁLIS MINISZTÉRIUM

Important Information

Formal requirements:

1. The papers must be assessed in **pen and of different colour** than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
2. The first one among the shaded rectangles next to each question contains the maximal score for that question. The **score** given by the examiner should be entered into the other **rectangle**.
3. **In case of correct solutions**, it is enough to enter the maximal score into the corresponding rectangle.
4. In case of faulty or incomplete solutions, please indicate the corresponding partial scores within the body of the paper.
5. Nothing, apart from the diagrams, can be evaluated if written in pencil.

Substantial requirements:

1. In case of some questions there are more than one marking schemes provided. However, if you happen to come across with some **solution different** from those outlined here, please identify the parts equivalent to those in the solution(s) given here and do your marking accordingly.
2. The scores in this assessment **can be split further**. Keep in mind, however, that the number of points awarded for any item can be an integer number only.
3. In case of a correct answer and a valid argument the maximal score can be awarded even if the actual solution is **less detailed** than that in this booklet.
4. If there is a **calculation error** or any other flaw in the solution, then the score should be deducted for the actual item only where the error has occured. If the candidate is going on working with the faulty intermediate result and the problem has not suffered substantial damage due to the error, then the subsequent partial scores should be awarded.
5. If there is a **fatal error** within an item (these are separated by double lines in this booklet), then even formally correct steps should not be given any points, whatsoever. However, if the wrong result obtained by the invalid argument is used correctly throughout the subsequent steps, the candidate should be given the maximal score for the remaing parts, unless the problem has been changed essentially due to the error.
6. If an **additional remark** or a **measuring unit** occurs in brackets in this booklet, the solution is complete even if the candidate does not mention it.
7. If there are more than one correct attempts to solve a problem, it is the **one indicated by the candidate that can be marked**.
8. You should **not give any bonus points** (points beyond the maximal score for a solution or for some part of the solution).
9. You should not reduce the score for erroneous calculations or steps unless its results are actually used by the candidate in the course of the solution.
10. **There are only 2 questions to be marked out of the 3 in part II/B of this exam paper.** Hopefully, the candidate has entered the number of the question not to be marked in the square provided for this. Accordingly, this question should not be assessed even if there is some kind of solution contained in the paper. Should there be any ambiguity about the student's request with respect to the question not to be considered, it is the last one in this problem set, by default, that should not be marked.

I.

1.		
The set in question is $\{1; 2; 3; 4; 6; 8\}$.	2 points	*If there is just one error then 1 point can be given. Also 1 point may be given only if every divisor is listed.*
Total:	**2 points**	

2.		
The area becomes $(3^2 =)9$ times larger .	2 points	
Total:	**2 points**	

3.		
$A_1 = \{1; 10\}$; $A_2 = \{1; 100\}$; $A_3 = \{10; 100\}$.	2 points	*1. Two correct subsets are worth 1 point only.* *2. The score should not be reduced for incorrect notations.*
Total:	**2 points**	

4.		
The vector is $\mathbf{r} = (12; -4)$.	2 points	*In case of calculation errors 1 point may be given for the right idea.*
Total:	**2 points**	

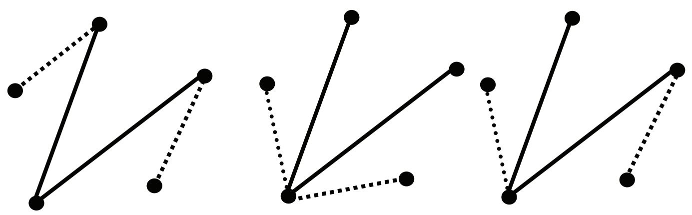
5.		
The acute angles are 23° and 67°, respectively.	2 points	*In case of erroneous rounding at most 1 point can be given.The correct trigonometric ratio is worth 1 point.*
Total:	**2 points**	

6.		
In case of choosing the median the end-of-year grade would be 4.	2 points	
Total:	**2 points**	*The score cannot be split further.*

7.		
Statement *A* is false.	1 point	
Statement *B* is true.	1 point	
Statement *C* true.	1 point	
Statement *D* is false.	1 point	
Total:	**4 points**	

8.		
The expression is undefined once $x = 90^\circ + n \cdot 180^\circ,\ n \in \mathbf{Z}$	3 points	*Stating that the denominator cannot be equal to zero is worth 1 point. Writing down a correct value of x is worth 1 point. If both the unit and the period is correct then it is worth 1 point.*
Total:	**3 points**	

9.		
The sum of the respective heights of the 16 students is (16 · 172 =) 2752 (cm).	2 points	
Total:	**2 points**	

10.		
Some correct solutions: 	2 points	
Total:	**2 points**	

11.

	YES	NO
$\underline{e}(\frac{1}{2};\frac{\sqrt{3}}{2})$		X
$\underline{e}(-\frac{\sqrt{3}}{2};\frac{1}{2})$		X
$\underline{e}(\frac{1}{2};-\frac{\sqrt{3}}{2})$	X	
$\underline{e}(\sin 30^\circ;-\cos 30^\circ)$	X	

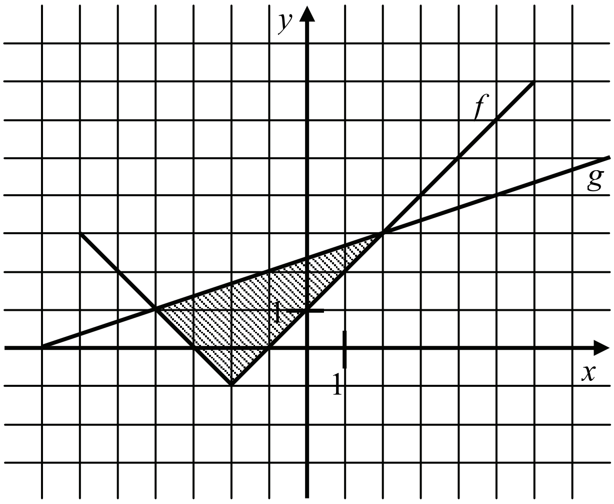
(table above)	4 points	*Each correct answer is worth 1 point.*
Total:	**4 points**	

12.		
The number of grades 5 is 30.	1 point	
The number of grades 4 is 50.	1 point	
The number of grades 3 is 40.	1 point	
Total:	**3 points**	

II/A

13.		
$x = \frac{600}{y}$.	1 point	
$xy + 5x - 10y = 650$.	2 points	
$600 + \frac{3000}{y} - 10y = 650$. $3000 - 10y^2 = 50y$.	1 point	*This 1 point should be given for the correct substitution.*
$y^2 + 5y - 300 = 0$.	2 points	*These 2 points are due even if the candidate does not simplify the equation.*
$y_1 = 15$; $y_2 = -20$.	2 points	
$x_1 = 40$; $x_2 = -30$.	2 points	
Checking the solutions.	2 points	
Total:	**12 points**	

14. a)		
Translating the graph of $f_0 = \lvert x \rvert$ by the vector $(-2;0)$,	1 point	*These 2 points are due even if the candidate gives the correct answer as a single translation.*
and then translating the result by the vector $(0;-1)$ yields the graph of the function *f*.	1 point	
[The graph consists of two line segments connected at the point $(-2;-1)$. The other endpoints of the segments are $(-6;3)$ and $(6;7)$, respectively.] Correct graph.	3 points	*1. These 3 points are due even if beyond the correct graph there is no textual description.* *2. If the domain is not correct, i. e. it is larger than the given interval then at most 2 points may be given.*
Total:	**5 points**	

14. b)		
		
The equation of the line *AB* is $x-3y=-7$.	3 points	*Correct direction vector $\overrightarrow{AB}(9\ ;\ 3)$, (normal-vector or slope) is worth 1 point, and the other 2 points are due if the equation is correct.*
One of the intersections is $A(-4;1)$.	2 points	*If the correct answers are gathered from the diagram then 1-1 points should be given, respectively, however, full score is due.if the candidate has checked the results by substitution.*
The other intersection point is $C(2;3)$.	2 points	
Total:	**7 points**	

15. a)		
Due to the 8% yearly interest Anna's capital is scaled up by 1.08 at the end of each subsequent year.	1 point	
There are 18 compounding steps by her 18th birthday,	1 point	
therefore the final balance is $C_{Anna}=500\ 000\cdot 1.08^{18}\approx 1998009.75$ forints.	2 points	*If the candidate is using the rounded value of 1.08^{18} then the result should be accepted.*
Accordingly, the sum to the nearest forint payed to Anna by the bank is 1 998 010 forint.	1 point	
Total:	**5 points**	

15. b)		
If the semi-annual interest rate is p% then Albert's capital is scaled up yearly by a scale factor of $\left(1+\frac{p}{100}\right)^2$	1 point	
in a period of 18 years.	1 point	
Thus, by his 18th birthday Albert's balance is $C_{Albert} = 400000 \cdot \left(1+\frac{p}{100}\right)^{36} = 2000000$ forints.	2 points	
Hence $\left(1+\frac{p}{100}\right)^{36} = 5$ that is $\left(1+\frac{p}{100}\right) = \sqrt[36]{5} \approx 1.04572$.	2 points	
The six-month interest rate is hence 4.57%.	1 point	
Total:	**7 points**	

1.) If the candidate makes a mistake while calculating the number of years, its score should be reduced by 2 points only, even if this error occurs more than once.
2.)An answer obtained without the use of the formula, e.g. the candidate calculates the balances year by year, should be accepted. However, full score should be given only, if the final result when rounded correctly is equal to the given figure.

II/B

16. a)

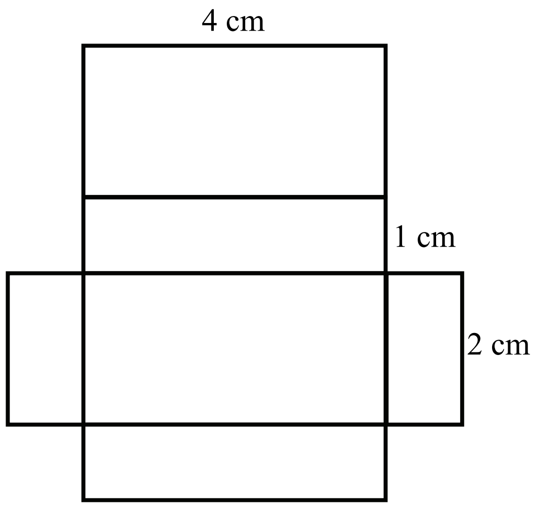
The block	The dimensions of the block (cm)	Surface area of the block (cm²)
basic block	$8\times4\times2$	112
block A	$16\times4\times2$	208
block B	$8\times8\times2$	192
block C	$8\times4\times4$	160

Each correct result is worth 1 point. — 4 points

Total: 4 points

16. b)

When reduced into the proportion 1:2, the lengths of the edges of the basic block are 4 cm, 2 cm and 1 cm, respectively.



The correct shape of the planar net.	3 points
The correct dimensions of the planar net.	1 point
Total:	**4 points**

16. c)		
The volume of the basic block is 64 cm³. Apart from that one, there are three different types of blocks in the set, however, they all have the same volume, namely $2 \cdot 64 = 128$ (cm³).	1 point	
The sum of the respective volumes of the four types of blocks is hence 448 cm³.	1 point	
The total volume of the blocks in the set is ten times bigger and thus it is 4480 cm³.	1 point	
Since the volume of a cube of edge 16 cm is 4096 cm³, the set would not fit in the box.	1 point	
Total:	**4 points**	

16. d) first solution		
There are 40 blocks in the set and since the blocks *B* and *C* are those of having square faces, there are 20 square based cuboids altogether in the set.	1 point	
Therefore, the probability of choosing a square based cuboid at first is $\frac{20}{40}$. After that there are one less blocks in the set and since there are also one less square based cuboids left, the probability that the first two selected bricks are both square based cuboids is $\frac{20}{40} \cdot \frac{19}{39}$,	1 point	
and so on. (Each favourable selection decreases both the number of blocks and the number of square based cuboids.) Therefore the probability of choosing five square based cuboids in a row is $\frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36} (\approx 0.02356)$.	2 points	
The probability that each of the five selected blocks is a square based cuboid is ≈ 0.024.	1 point	
Total:	**5 points**	

16. d) **second solution**		
There are 40 blocks in the set and since the blocks *B* and *C* are those that have square faces, there are 20 square based cuboids in the set.	1 point	
Among the equally probable selections of 5 blocks out of 40, the favourable outcomes are those in which each block is taken from the 20 element subset of the square based cuboids.	1 point	
The probability in question is hence $\frac{\binom{20}{5}}{\binom{40}{5}}$.	1 point	
Its value is $\frac{\binom{20}{5}}{\binom{40}{5}} = \frac{\frac{20!}{5!\cdot 15!}}{\frac{40!}{5!\cdot 35!}} = \frac{20!\cdot 35!}{15!\cdot 40!} = \frac{20\cdot 19\cdot 18\cdot 17\cdot 16}{40\cdot 39\cdot 38\cdot 37\cdot 36}$.	1 point	
The probability that each of the five selected blocks is a square based cuboid is ≈ 0.024.	1 points	
Total:	**5 point**	

17. a)		
The product on the l. h. s. is 0 if and only if one of the factors is 0.	1 point	*This point is due if this idea appears in the solution.*
If the first factor is 0 then $\log_2 x = 3$.	1 point	
Hence $x_1 = 2^3 = 8$.	1 point	
If the second factor is 0 then $\log_2 x^2 = -6$,	1 point	
Hence $x^2 = 2^{-6} = \frac{1}{64}$,	1 point	
and only $x_2 = \frac{1}{8}$ is positive.	1 point	*If x > 0 is not mentioned then at most 1 point may be given out of these 2 points.*
Both numbers satisfy the given equation.	1 point	
Total:	**7 points**	

17. b)		
$\sin\left(x-\frac{\pi}{6}\right)=\frac{1}{2}$ or $\sin\left(x-\frac{\pi}{6}\right)=-\frac{1}{2}$.	2 points	
$x-\frac{\pi}{6}=\frac{\pi}{6}+2n\pi$ or $x-\frac{\pi}{6}=-\frac{\pi}{6}+2n\pi$.	2 points	
$x-\frac{\pi}{6}=\frac{5\pi}{6}+2n\pi$ or $x-\frac{\pi}{6}=\frac{7\pi}{6}+2n\pi$.	2 points	
$x_1=\frac{\pi}{3}+2n\pi$; $x_2=2n\pi$; $x_3=\pi+2n\pi$; $x_4=\frac{4\pi}{3}+2n\pi$, $n\in\mathbf{Z}$.	4 points	
Total:	**10 points**	

18. a)		
There are 4 „lucky ones" among the 25 parking places: no. 7; no. 17; no. 14 and no. 21.	2 points	
The probability is hence $\frac{4}{25}\left(=0.16\right)$.	2 points	
Total:	**4 points**	

18. b)		
There are 9 free places left.	1 point	
There are $\binom{9}{2}$ ways to choose the positions of the 2 red cars and henceforth the green cars have no choice left.	3 points	
The number of possible arrangements is 36.	1 point	
Total:	**5 points**	

18. c)		
Consider those customers who have indicated the colour green among their preferences. There are 4 of them with no other option and another 10 for either a green or a red car. Since there are no more than 6 red ones among the cars, there are at least 4 out of these 10 customers who must be given a green car.	4 points	*These 4-4 points may be given for other concise argumenst, e. g.: There were 4+10=14 bookings for green or red cars but there are no more than 7+6=13 of them.*
Since there are but 7 green cars only, the group of customers asking for green colour cannot be satisfied, no matter how the cars are distributed.	4 points	
Total:	**8 points**	