ÉRETTSÉGI VIZSGA • 2007. október 25.

MATEMATIKA ANGOL NYELVEN

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI ÉS KULTURÁLIS MINISZTÉRIUM

Important Information

Formal requirements:

1. The papers must be assessed in **pen and of different colour** than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
2. The first one among the shaded rectangles next to each question contains the maximal score for that question. The **score** given by the examiner should be entered into the other **rectangle**.
3. **In case of correct solutions**, it is enough to enter the maximal score into the corresponding rectangle.
4. In case of faulty or incomplete solutions, please indicate the corresponding partial scores within the body of the paper.
5. Nothing, apart from the diagrams, can be evaluated if written in pencil.

Substantial requirements:

1. In case of some problems there are more than one marking schemes given. However, if you happen to come across with some **solution different** from those outlined here, please identify the parts equivalent to those in the solution provided here and do your marking accordingly.
2. The scores in this assessment **can be split further**. Keep in mind, however, that the number of points awarded for any item can be an integer number only.
3. In case of a correct answer and a valid argument the maximal score can be awarded even if the actual solution is **less detailed** than that in this booklet.
4. If there is a **calculation error** or any other flaw in the solution, then the score should be deducted for the actual item only where the error has occured. If the candidate is going on working with the faulty intermediate result and the problem has not suffered substantial damage due to the error, then the subsequent partial scores should be awarded.
5. If there is a **fatal error** within an item (these are separated by double lines in this booklet), then even formally correct steps should not be given any points, whatsoever. However, if the wrong result obtained by the invalid argument is used correctly throughout the subsequent steps, the candidate should be given the maximal score for the remaing parts, unless the problem has been changed essentially due to the error.
6. If an **additional remark** or a **measuring unit** occurs in brackets in this booklet, the solution is complete even if the candidate does not mention it.
7. If there are more than one correct attempts to solve a problem, it is the **one indicated by the candidate that can be marked**.
8. You should **not give any bonus points** (points beyond the maximal score for a solution or for some part of the solution).
9. You should not reduce the score for erroneous calculations or steps unless its results are actually used by the candidate in the course of the solution.
10. **There are only 2 questions to be marked out of the 3 in part II/B of this exam paper.** Hopefully, the candidate has entered the number of the question not to be marked in the square provided for this. Accordingly, this question should not be assessed even if there is some kind of solution contained in the paper. Should there be any ambiguity about the student's request with respect to the question not to be considered, it is the last one in this problem set, by default, that should not be marked.

I.

1.		
$A \cap B = \{5; 7; 9\}$	2 points	
Total:	**2 points**	*Writing down the sets A and/or B is not worth any points.*

2.		
$C = -2$	2 points	*Calculating the value of* $\frac{1}{C}$ *is worth 1 point.*
Total:	**2 points**	

3.		
$A = -1$, $B = -2$	1 point	
$A > B$	1 point	
Total:	**2 points**	

4.		
The number of blue marbles is 9.	1 point	
The number of red marbles is 11.	1 point	
$P = \frac{favourable\ outcomes}{all\ outcomes} = \frac{11}{20} = 0.55$	1 point	
Total:	**3 points**	*The correct answer by itself gets these 3 points. If the answer is something like „since the proportion of the red marbles is 55 %, therefore the probability is .,55", then this is still worth 3 marks.*

5.		
a) false	1 point	
b) true	1 point	
c) false	1 point	
Total:	**3 points**	

6.		
The set of positive real numbers.	2 points	*The answer* $x > 0$ *should be*

		accepted.
Total:	**2 points**	

7.		
$S_5 = \frac{a_1 + a_n}{2} \cdot n$	1 point*	***This point is due even if the formula is not stated but it is used correctly.**
$S_5 = \frac{60}{2} \cdot 5$	1 point	
$S_5 = 150$	1 point	
Total:	**3 points**	

8.		
$5 \cdot 4 \cdot 3 = 60$	2 points	*The 2 points are due even if the correct answer is given after having listed the possible cases.*
Total:	**2 points**	*If the list is not complete but there are at least 30 numbers written down then 1 point may be given.*

9.		
$x_1 = \frac{\pi}{6}$	1 point	
$x_2 = \frac{5\pi}{6}$	1 point	
Total:	**2 points**	*If there are multiple solutions indicated outside the given interval or the solution is given in angular measure then at most 1 point may be given.*

10.		
$\mathbf{c} = 2\mathbf{a} - \mathbf{b}$; $\mathbf{c} = 2(3\mathbf{i} - 2\mathbf{j}) - (-\mathbf{i} + 5\mathbf{j})$	1 point	
$\mathbf{c} = 6\mathbf{i} - 4\mathbf{j} + \mathbf{i} - 5\mathbf{j}$	1 point	
$\mathbf{c} = 7\mathbf{i} - 9\mathbf{j}$	1 point	
Total:	**3 points**	*The correct answer by itself gets these 3 points.*

11.		
Let the fifth term be x. Then $\frac{1+8+9+12+x}{5}=7$.	1 point	
$x = 5$	2 points	
Total:	**3 points**	

12.		
The minimum value of the function is 1,	1 point*	**These 1-1 points are due if the range is stated correctly.*
the values of the function at the endpoints of the interval are 5 and 10, respectively.	1 point*	
The range of the function is the interval [1; 10].	1 point	
Total:	**3 points**	*Any form of the correct range is worth 3 points.*

II./A

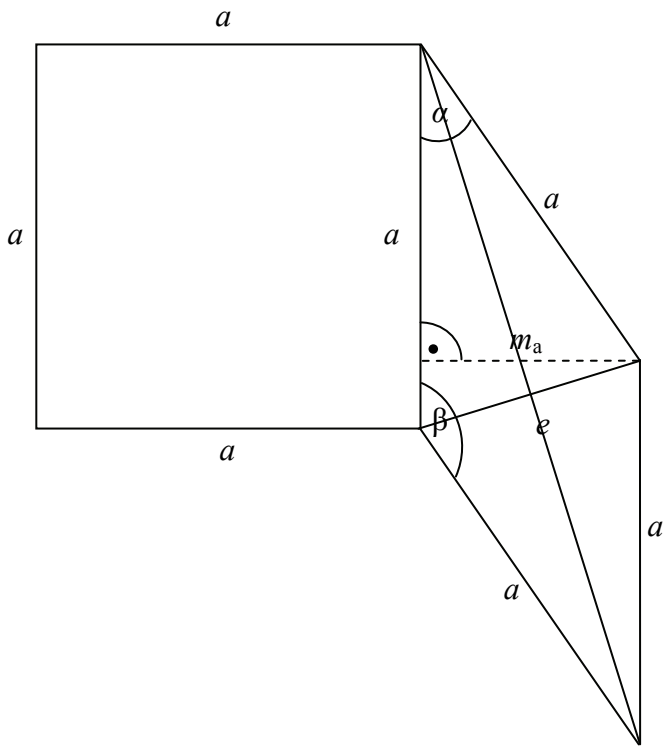
13. a)		
The 5-base exponential function is strictly increasing and hence	1 point	
$x - 2 < 13 - 2x$	1 point	
$x < 5$	1 point	
The solution set of the inequality is: {1; 2; 3; 4}	1 point	
Total:	**4 points**	*2 points may be given if the 4 solutions are listed and another 2 if the candidate shows that there are no more of them.*
13. b)		
$x \geq 0$	1 point*	
$3^{2\sqrt{x}} = 3^{x-3}$	1 point	
The 3-base exponential function is strictly monotone and hence $2\sqrt{x} = x - 3$.	1 point	*This point is due even if monotonicity is not mentioned.*
$4x = x^2 - 6x + 9$	1 point	
$x^2 - 10x + 9 = 0$	1 point	
$x_1 = 1 \qquad x_2 = 9$	1 point	
$x = 1$ is not a solution of the equation and thus the solution on the set of reals is $x = 9$.	2 points*	
Total:	**8 points**	**If the candidate checks the solutions of the quadratic by substitution or by comparing the resp. domains then full score should be given as long as the answer is correct.*

14. a)		
Denote the number of desks in the studio by x and the size of the class by y.	1 point	*This 1 point is due even if the meaning of the variables is clear from the correctly written equations.*
$2x + 8 = y$	1 point	
$3x - 7 = y$	1 point	
$x = 15$ and $y = 38$	1 point	
Checking the result	1 point	
There are 15 desks in the studio and 38 students in the class.	1 point	
Total:	**6 points**	

14. b)		
There are 12·4·10 possible settings, therefore,	2 points	
there are 480 „dates” can be adjusted.	1 point	
Total:	**3 points**	

14. c)		
There are 365 proper dates in a non leap year	1 point	
and any one of them will be set by the same probability*, hence the probability of a proper date is equal to $\frac{365}{480}$ (≈ 0.7604).	2 points	**If the answer is correct then this 2 points are due even if this idea is not stated in the argument.*
Total:	**3 points**	

15. a)



Correct diagram that shows the understanding of the relation of the given square and the rhombus.	1 point*	**If the solution is correct then this point should be given even if there is no diagram.*
$\left(A_{\text{square}} = a^2 \text{ and } A_{\text{rhombus}} = am_a\right)$ $\frac{a^2}{am_a} = \frac{2}{1}$	3 points	
The height of the rhombus is: $m_a = 6.5$ (cm)	1 point	
Total:	**5 points**	

15. b)		
$\sin\alpha = \frac{m_a}{a}$ (α is acute)	1 point	
$\alpha = 30°$	1 point	
$\beta = 150°$	1 point	
Total:	**3 points**	

15. c)		
One can proceed from any one of the right triangles of the diagram. If e is the longer diagonal then $\cos 15° = \frac{\frac{e}{2}}{13}$, for example. Hence	2 points	
$e = 2 \cdot 13 \cdot \cos 15°$	1 point	
$e \approx 25.11$ (cm)	1 point	
Total:	**4 points**	*Any other correct solution (using the cosine rule, for example) should be accepted.*

II./B

16. a)

Results of the first round	Question 1.	Question 2.	Question 3.	Question 4.
Ann's answer	correct	wrong	correct	***wrong***
Number of correct answers	7	10	***15***	8
Ann's score	***13***	***0***	5	0

Every correctly given piece of information is worth 1 point.

Total:	**4 points**	

16. b)

The column of the 2nd question now becomes: „correct" , 11, 9; Ann would get 9 points on this question.	1 point	
Her score in the first round would hence become 27, that is the 150 percent of her actual score.	1 point	
Therefore, her score would have increased by 50%.	1 point	*The candidate may also claim that the 9 points increment is the 50% of the original score.*
Total:	**3 points**	

16. c)

1st solution: There are $3^4 = 81$ equally probable ways to answer the 4 questions.	2 points	
There is only one case when every answer is correct and thus the probability is equal to $\frac{1}{81}$.	1 point	
Total:	**3 points**	
2nd solution: The probability of the correct answer is equal to $\frac{1}{3}$ in case of every question.	1 point	
Since the respective answers are independent, the probability is equal to $\left(\frac{1}{3}\right)^4 = \frac{1}{81}$.	2 points	
Total:	**3 points**	

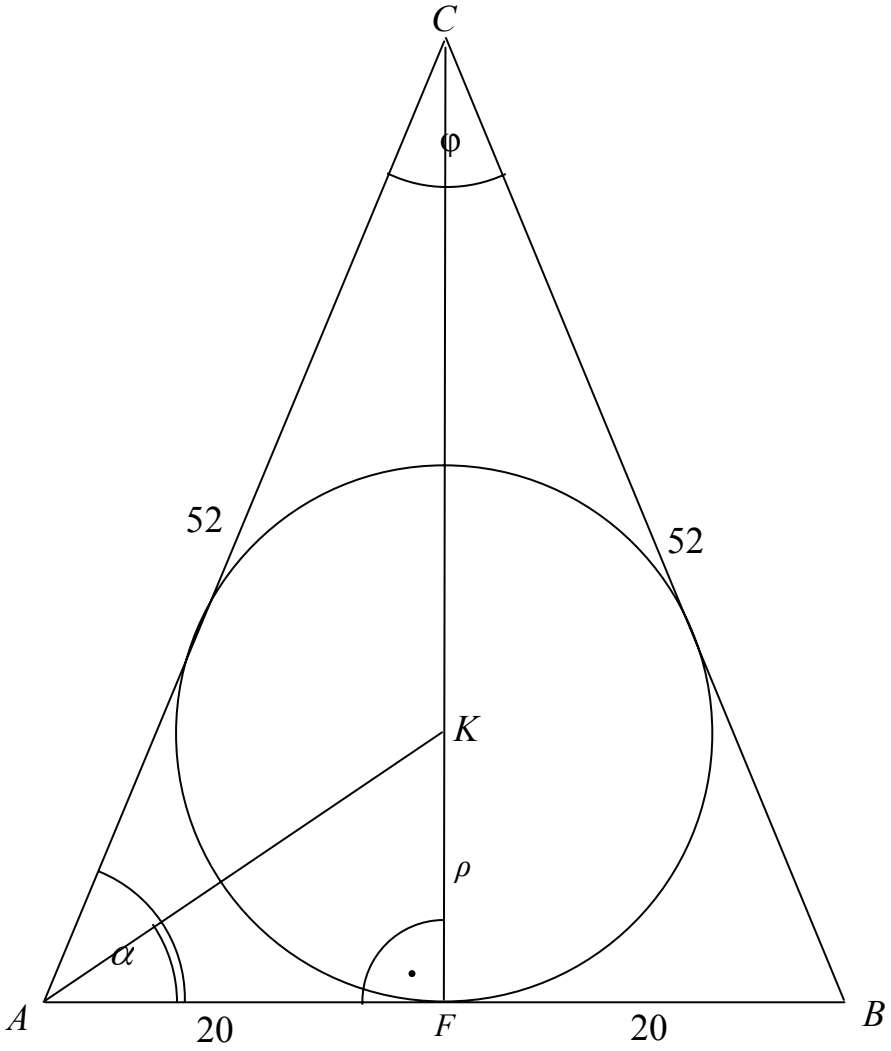
16. d)

If there are x correct answers for the given question then those who gave the correct answer are gaining $20 - x$ points each.	1 point	
The total score of the group is hence $x(20 - x)$.	2 points	

The maximum of the function $x \mapsto 20x - x^2$ is needed if x is a positive integer less than 20. The maximizing value of x is 10. (This can be found either graphically, or by completing the square, or by referring to the AM-GM inequality, or by simply listing the possible cases.)	3 points	
The total score of the group is maximal in case of ten participants' correct answers.	1 point	
Total:	**7 points**	

17. a)		
There are 5! possible orders	2 points	
and thus the grandchildren can receive their respective letters in 120 possible orders.	1 point	
Total:	**3 points**	
17. b)		
Anyone of the 5 grandchildren occurs at the last position with the same probability.	2 points	*The number of favourable outcomes is 4!, that of the total outcomes is 5!. This model also gets the 2 points.*
The probability in question is hence $\frac{1}{5}$.	1 point	
Total:	**3 points**	
17. c)		
The pieces knit on the consecutive days form a geometric progression.	1 point	*This point is due even if this idea is used in the solution but not stated.*
In this progression $g_1 = 8$, $r = 1.2$	2 points	
The total length of the scarf is the sum of the first n terms of the progression.	1 point	*These 2 points are due even if this observation is clear from the formulas used by the candidate.*
$S_n = g_1 \cdot \frac{r^n - 1}{r - 1}$	1 point	
$200 = 8 \cdot \frac{1.2^n - 1}{0.2}$	1 point	
$5 + 1 = 1.2^n$	1 point	
$n = \frac{\lg 6}{\lg 1.2}$	2 points	*These 3 points are due even if the value of n is found by repeated multiplication.*
$n \approx 9.83$	1 point	
The scarf is completed on the tenth day.	1 point	
Total:	**11 points**	

18. a)



Correct diagram indicating the data.	2 points	
If the opening angle of the cone is φ, then $\sin\frac{\varphi}{2}=\frac{20}{52}\approx 0.3846$	1 point	
Hence $\varphi \approx 45.24°$	1 point	
Total:	**4 points**	

18. b)

If h is the altitude of the cone then $h=\sqrt{2704-400}=48$	1 point	
$V=\frac{r^2\cdot\pi\cdot h}{3}=\frac{400\cdot\pi\cdot 48}{3}$	1 point	
$V\approx 20106.19$ (cm^3)	1 point	
Total:	**3 points**	

18. c)		
The radius of the sphere inscribed into the cone is equal to the radius ρ of the incircle of the isosceles triangle.	2 points	*These 2 points are due if this observation is clear from the diagram or from the calculations.*
The base angle of the triangle is $\alpha \approx 67.38°$.	1 point	
$\tan 33.69° = \frac{\rho}{20}$	1 point	
$\rho = 13.33$ (cm)	1 point	
The surface area of the sphere is $A \approx 2234.01$ (cm^2)	1 point	*Using ρ = 13.33 one gets A ≈ 223.90 (cm^2). This accuracy still merits the 1 point.*
Total:	**6 points**	
18. d)		
The length of the arc of the circular sector is $a = 2r\pi$, $a = 2 \cdot 20\ \pi \approx 125.66$ (cm)	2 points	
$A_{\text{lateral surface}} = \frac{a \cdot R}{2} = 2 \cdot 20 \cdot \pi \cdot 26$	1 point	
$A_{\text{lateral surface}} \approx 3267.26$ (cm^2)	1 point	*If the arc length a is rounded to two decimal places then the result is 3267.16 (cm^2): the 2 points are still due.*
Total:	**4 points**	