ÉRETTSÉGI VIZSGA ● 2007. május 8.

MATEMATIKA ANGOL NYELVEN MATHEMATICS

KÖZÉPSZINTŰ ÉRETTSÉGI VIZSGA STANDARD LEVEL FINAL EXAMINATION

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ MARKSCHEME

OKTATÁSI ÉS KULTURÁLIS MINISZTÉRIUM
MINISTRY OF EDUCATION AND CULTURE

Important Information

Formal requirements:

1. The papers must be assessed in **pen and of different colour** than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
2. The first one among the shaded rectangles next to each question contains the maximal score for that question. The **score** given by the examiner should be entered into the other **rectangle**.
3. **In case of correct solutions**, it is enough to enter the maximal score into the corresponding rectangle.
4. In case of faulty or incomplete solutions, please indicate the corresponding partial scores within the body of the paper.
5. Nothing, apart from the diagrams, can be evaluated if written in pencil.

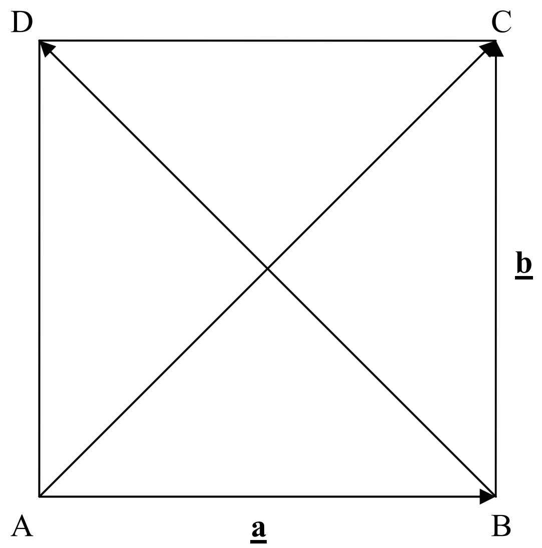
Substantial requirements:

1. In case of some problems there are more than one marking schemes given. However, if you happen to come across with some **solution different** from those outlined here, please identify the parts equivalent to those in the solution provided here and do your marking accordingly.
2. The scores in this assessment **can be split further**. Keep in mind, however, that the number of points awarded for any item can be an integer number only.
3. In case of a correct answer and a valid argument the maximal score can be awarded even if the actual solution is **less detailed** than that in this booklet.
4. If there is a **calculation error** or any other flaw in the solution, then the score should be deducted for the actual item only where the error has occured. If the candidate is going on working with the faulty intermediate result and the problem has not suffered substantial damage due to the error, then the subsequent partial scores should be awarded.
5. If there is a **fatal error** within an item (these are separated by double lines in this booklet), then even formally correct steps should not be given any points, whatsoever. However, if the wrong result obtained by the invalid argument is used correctly throughout the subsequent steps, the candidate should be given the maximal score for the remaing parts, unless the problem has been changed essentially due to the error.
6. If an **additional remark** or a **measuring unit** occurs in brackets in this booklet, the solution is complete even if the candidate does not mention it.
7. If there are more than one correct attempts to solve a problem, it is the **one indicated by the candidate that can be marked**.
8. You should **not give any bonus points** (points beyond the maximal score for a solution or for some part of the solution).
9. You should not reduce the score for erroneous calculations or steps unless its results are actually used by the candidate in the course of the solution.
10. **There are only 2 questions to be marked out of the 3 in part II/B of this exam paper.** Hopefully, the candidate has entered the number of the question not to be marked in the square provided for this. Accordingly, this question should not be assessed even if there is some kind of solution contained in the paper. Should there be any ambiguity about the student's request with respect to the question not to be considered, it is the last one in this problem set, by default, that should not be marked.

I.

1.		
$\frac{223650}{210000} = 1.065$	1 point	
The annual interest was 6.5 %.	1 point	
Total:	**2 points**	

2.

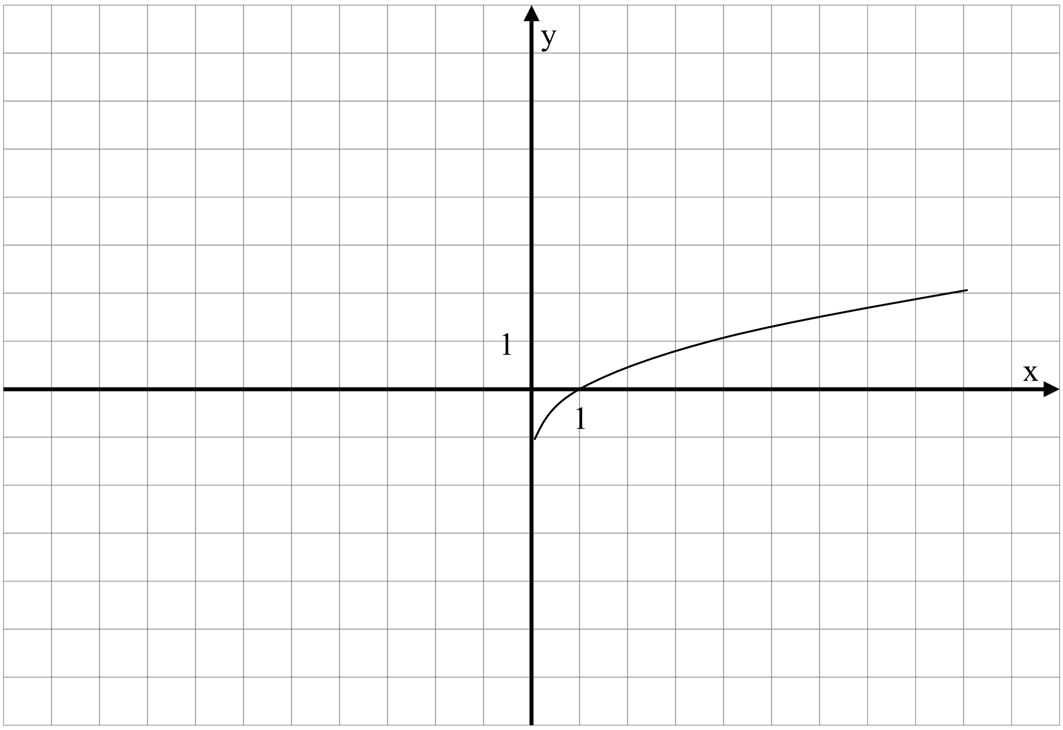


$\overrightarrow{AC} = \underline{\mathbf{a}} + \underline{\mathbf{b}}$	1 point	
$\overrightarrow{BD} = \underline{\mathbf{b}} - \underline{\mathbf{a}}$	1 point	
Total:	**2 points**	

3.		
Finding the roots with the quadratic formula: $x_1 = 7$ and $x_2 = –5$.	2 points	
Checking.	1 point	
Total:	**3 points**	

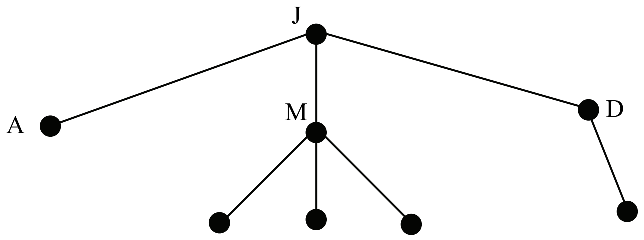
4.		
One hour ↔ 30°, thus the angle of the hands is 150º.	2 points	
Total:	**2 points**	

5.		
a) True.	1 point	
b) Cannot be known.	1 point	
Total:	**2 points**	

6.		
		
Graph.	2 points	*2 points for a correct graph obtained in any way. 1 point if the domain $x \geq 0$ is stated but there is no graph.*
$x = 1$	1 point	
Total:	**3 points**	

7.		
60º	1 point	*Award a maximum of 1 point if other angles are listed, too.*
240º	1 point	
Total:	**2 points**	

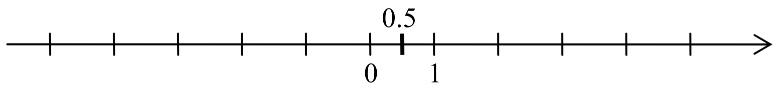
8.



The graph.	1 point	
The graph has 8 vertices.	1 point	
The graph has 7 edges.	1 point	
Total:	**3 points**	

9.

$z = 4^{-05} = \frac{1}{\sqrt{4}} = 0,5$	2 points	*Award 1 point if there is an intermediate result only. Award the 2 points for the correct answer without explanation.*



Position on the number line.	1 point	
Total:	**3 points**	

10.

The number of all cases: 6.	1 point	
The number of favourable cases: 2 (3 and 6).	1 point	
The probability is 2/6 = 1/3.	1 point	
Total:	**3 points**	

11.

Mode: 24º.	1 point	
Median: 23º.	1 point	
Total:	**2 points**	

12.

$V = r^2 \cdot \pi \cdot m = 11^2 \cdot \pi \cdot 25 \text{ cm}^3 = 9.5$ litres	3 points	*Formula, substitution and conversion are worth 1 point each.*
Total:	**3 points**	

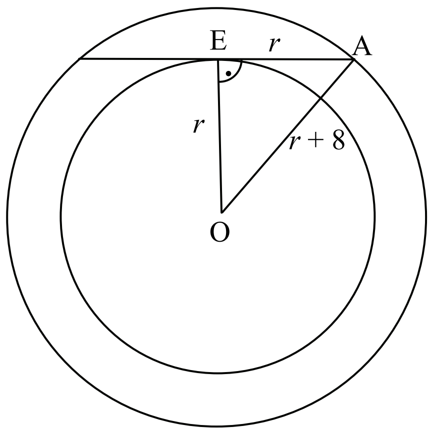
II/A

13. a)		
Stating the domain $x \neq 2$, or checking by substitution.	1 point	
$7 = -7 + 3.5x$	1 point	
$x = 4$, which is an integer.	1 point	
Total:	**3 points**	

13. b)		
The fraction is positive if (and only if) $2 - x > 0$.	1 point	
Hence $x < 2$, and x is an integer.	2 points	
Total:	**3 points**	*0 points for multiplying by (2 – x) without investigating the sign.*

13. c)		
The denominator must be a factor of 7.	2 points	*Also due if this idea only becomes clear from the way the solution is written down.*
Thus $2 - x = 1$ or 7,	1 point	
or $2 - x = -1$ or -7,	1 point	
Hence x may be: -5; 9; 1; 3.	2 points	
Total:	**6 points**	*Award a maximum of 4 points if only positive factors are considered.*

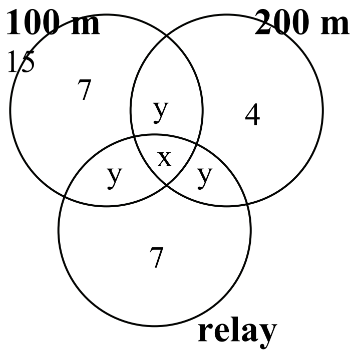
14. a)



Drawing the diagram. (The chord is perpendicular to the radius drawn to the point of contact, which may become clear from using the Pythagorean theorem.)	2 points	
Total:	**2 points**	

14. b)		
The radii of the circles in cm are r and $R = r + 8$	1 point	
The legs of the right-angled triangle *OAE* are r and r, and its hypotenuse is R.	1 point	*The point is also due if this idea is only shown in the diagram.*
The Pythagorean theorem applied to the right-angled triangle *OAE*: $(r+8)^2 = 2r^2$.	2 points	
$r^2 - 16r - 64 = 0$	2 points	
By substitution into the quadratic formula:	1 point	
the negative root $8(1-\sqrt{2})$ is not a solution,	1 point	
thus the lengths of the radii of the circles are $r = 8(1+\sqrt{2}) \approx 19.3$ cm, and	1 point	*The results should also be accepted if the approximate values and the unit are not stated.*
$R = r + 8 = 8(2+\sqrt{2}) \approx 27.3$ cm.	1 point	
Total:	**10 points**	

15. a)

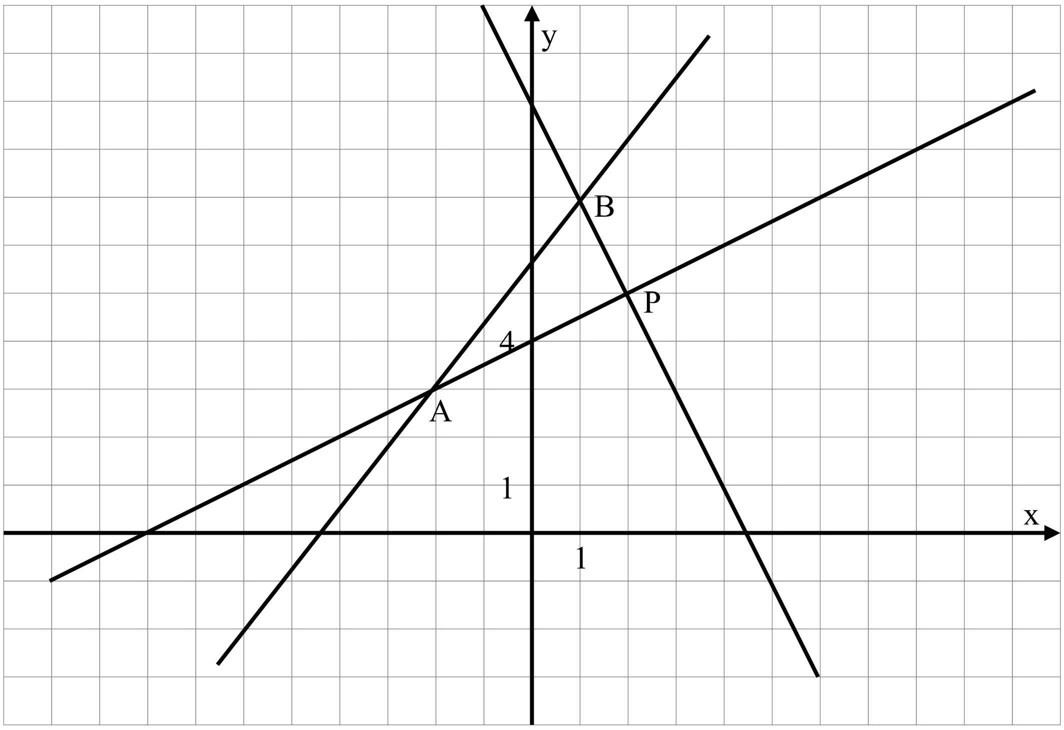


Set diagram.	2 points	*The 2 points are also due if the unknowns are not written in the diagram.*
Total:	**2 points**	

15. b)		
Let x be the number of runners in the intersection of the three sets, and let $x + y$ be the number of them training for any pair of races.	2 points	
From the number of 100-metre runners: $x + 2y = 8$	2 points	
For those outside the 100-m set: $4 + y + 7 = 14$.	2 points	
From the second equality: $y = 3$, and from the first one: $x = 2$.	3 point	
Thus there are 5 runners in the intersection of each pair of sets.	1 points	
Total:	**10 points**	

II/B

16. a)



Graph.	1 point	
$y = \frac{1}{2}x + 4$	1 point	
Total:	**2 point**	

16. b)

The point *P* lies on the line: $5 = \frac{1}{2} \cdot 2 + 4$.	1 point	
The slope of the perpendicular line is –2.	1 point	
$y = -2x + 9$	2 points	
Total:	**4 points**	*Award a maximum of 3 points for simply reading the answer from the diagram.*

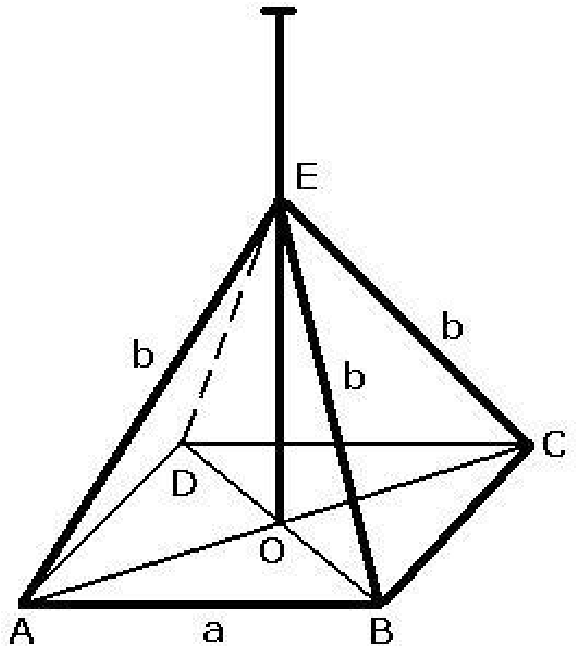
16. c)

$\left.\begin{array}{l} \frac{1}{2}x + 4 = y \\ 4x - 3y = -17 \end{array}\right\}$ Solution: $x = -2$; $y = 3$ $A(-2; 3)$	2 points	

$\left.\begin{array}{l}-2x+9=y\\ 4x-3y=-17\end{array}\right\}$ Solution: $x = 1$; $y = 7$ $B\,(1; 7)$	2 points	
Total:	**4 points**	

16. d)		
$PA=\sqrt{20}$; $PB=\sqrt{5}$	2 points	*It is also possible to calculate from the hypotenuse.*
The area of the triangle is $\frac{\sqrt{20}\cdot\sqrt{5}}{2}=5$ units.	2 points	
Total:	**4 points**	

16. e)		
The centre of the circle is the midpoint of the hypotenuse of the right-angled triangle.	1 point	
Its coordinates are (–0.5; 5)	2 points	
Total:	**3 points**	

17. a)		
 a = 10 m b = 14.5 m		*The aerial reaches beyond the apex: 1 point; the pyramid is shown: 1 point; and the aerial starts at the intersection of the diagonals: 1 point. (The first and last 1 points are also due if the appropriate information is conveyed by the calculations.)*
Drawing the diagram.	3 points	
Total:	**3 points**	

17. b)		
Each face of the "tent" is an isosceles triangle with sides of a, b, b. The altitude drawn to the base of the triangle is $m_o = \sqrt{14.5^2 - 5^2} \approx 13.61$ m	2 points	
The total area is $4 \cdot \frac{a \cdot m_o}{2}$. By substitution, this is ≈ 272 m^2.	2 points	*Award 1 point if the answer is not rounded to the nearest m^2.*
Total:	**4 points**	

17. c)		
The length of the diagonal of the square of side a is $a\sqrt{2} = 10\sqrt{2} \approx 14.1$ (m)	2 points	
In the right-angled triangle AOE, AO is half the diagonal: $5\sqrt{2}$	2 points	
The Pythagorean theorem applied to that triangle: $OE^2 = 14.5^2 - \left(5\sqrt{2}\right)^2 \approx 160.25$ (m^2)	3 points	
$OE \approx 12.66$ m	1 point	
The height of the aerial is $1.5 \cdot OE \approx 18.99$ m, which is about 190 dm.	2 points	*Award a maximum of 1 point if the answer is not given in dm or the rounding is wrong.*
Total:	**10 points**	

18. a)		
I learnt 8 + 11 + 14 + 17 + 20 = 70 words during the first week.	1 point	
I remembered 70·0.8 = 56 new words at the end of the week.	1 point	
Total:	**2 points**	

18. b)**		
An arithmetic progression is obtained, $a_1 = 56$, $d = 4$, $n = 13$.	3 points	*These points can be divided. 1 point for naming the sequence, the parameters may become clear from the subsequent calculations*
Total:	**3 points**	

18. c)**		
I remembered $a_{13} = a_1 + (n-1) \cdot d = 56 + 12 \cdot 4 = 104$ new words on the 13th week.	3 points*	*Formula, substitution and calculation are worth 1 point each.*
Total:	**3 points**	

18. d)**		
I learnt and remembered $S_{13}=\frac{a_1+a_{13}}{2}\cdot 13=\frac{56+104}{2}\cdot 13=1040$ words altogether during that one quarter of a year.	3 points*	*Formula, substitution and calculation are worth 1 point each.*
Total:	**3 points**	
* *Award full mark if the numbers of words learnt are listed or tabulated and correctly added, and the answers to the questions are correct.*		

18. e)		
I select two words out of 70, which can be done in $\binom{70}{2}$ different ways.	2 points	
The two words are to be selected from the 56 words remembered.	2 points	
The probability that I know both words is $\frac{\binom{56}{2}}{\binom{70}{2}}(\approx 0.638)$.	2 points	*The 2 points are also due for stating the ratio without calculating the decimal value.*
Total:	**6 points**	

**Remark: If the candidate considered the problem such that starting from the second week he learnt new words six days of the week, then the marking should be done according to the above system. This way:
For part b) the terms of the sequence are not integers, but the rounded values of the terms form a strictly increasing sequence.
For part c) the solution: On the second week he learnt 99 words, on the 13th week $99+11\cdot 6=165$ words. Hence he remembered $165\cdot 0{,}8=132$ new words.
The solution for part d): he learnt and remembered $\left(70+\frac{99+165}{2}\cdot 12\right)\cdot 0{,}8\approx 1323$ new words.