ÉRETTSÉGI VIZSGA • 2006. október 25.

MATEMATIKA
ANGOL NYELVEN
MATHEMATICS

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA
INTERMEDIATE LEVEL WRITTEN EXAM

KEY AND GUIDE FOR EVALUATION
JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI ÉS KULTURÁLIS MINISZTÉRIUM
MINISTRY OF EDUCATION AND CULTURE

Important Information

Formal requirements:

1. The papers must be assessed in **pen and of different colour** than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
2. The first one among the shaded rectangles next to each question contains the maximal score for that question. The **score** given by the examiner should be entered into the other **rectangle**.
3. **In case of correct solutions**, it is enough to enter the maximal score into the corresponding rectangle.
4. In case of faulty or incomplete solutions, please indicate the corresponding partial scores within the body of the paper.
5. Anything, apart from the diagram when written in pencil cannot be evaluated.

Substantial requirements:

1. In case of some problems there are more than one marking schemes are included in this booklet. However, if you happen to come across with some **solution different** from those given here, please identify the parts equivalent to those in the solution provided here and do your marking accordingly.
2. The scores in this assessment **can be split further**. Keep in mind, however, that the number of points awarded for any item can be an integer number only.
3. In case of a correct answer and a valid argument the maximal score can be awarded even if the actual solution is **less detailed** than that in this booklet.
4. If there is a **calculation error** or any other flaw in the solution, then the score should be deducted for the actual item only where the error has occured. If the candidate is going on working with the faulty intermediate result and the problem has not suffered some essential change due to the error, then the subsequent partial scores should be awarded.
5. If there is a **fundamental error** within an item (these are separated by double lines in this bulletin), even formally correct steps should not be awarded by any points, whatsoever. However, if the wrong result obtained by the invalid argument is used correctly throughout the subsequent steps, the candidate should be awarded the maximal score for the remaing parts, if the problem has not been changed essentially due to the error.
6. If an **additional remark** or a **measuring unit** occurs in brackets in this booklet, the solution is complete even if the candidate does not mention it.
7. If there are more than one correct attempts to solve a problem, it is the **one indicated by the candidate that can be marked**.
8. You should **not give out any bonus points** (points beyond the maximal score for a solution or for some part of the solution).
9. You should not reduce the score for erroneous calculations or steps unless its results are actually used by the candidate in the course of the solution.
10. **There are only 2 questions to be marked out of the 3 in part II/B of this exam paper.** Hopefully, the candidate has entered the number of the question not to be marked in the square provided for this purpose. Accordingly, this question should not be assessed even if there is some kind of solution contained in the paper. Should there be any ambiguity about the student's request with respect to the question not to be considered, it is the last one in this problem set, by default, that should not be marked.

I.

1.		
$H = \{16; 25; 36; 49; 64; 81\}$	2 points	*In case of more than one errors there can be no points given. 1 point is due if there is one error or one number missing.*
Total:	**2 points**	

2.		
The intersection point is: $\left(0, -\frac{2}{3}\right)$	2 points	*For the form x=0; y=$-\frac{2}{3}$ 2 points should be given too. There can be at most 1 point given if there are no two coordinates in the answer.*
Total:	**2 points**	

3.		
The number of matches to be played is 30.	3 points	*If the model used is faulty and hence the answer is 15 or 60, or the model is correct but the number of cases is not, then at most 1 point can be given.*
Total:	**3 points**	

4.		
A possible data set is : –2; –1; 0; 1; 7 (it is consistent with both statisctics).	4 points	*If the five numbers satisfy only one of the conditions then 2 points, if there are separate data sets listed to satisfy each of the two conditions then 3 points may be given*
Total:	**4 points**	

5.		
The arclength is: $\frac{3\pi}{2}$.	2 points	*An approximation can be also accepted (4,712), as long as it is correct to at least one decimal place. If the arclength is given as a function of the radius 1 point may be given.*
Total:	**2 points**	

6.		
The numbers in question are: 570; 750; 705.	2 points	*There can be no points given if there are more than one errors; 1 point may be given in case of one error or a missing number.*
Total:	**2 points**	

7.		
The length of the space diagonal is $\sqrt{2a^2+b^2}$.	3 points	*1 point should be given if the answer is the square of the diagonal.*
Total:	**3 pont**	

8.		
The probability of the event B is $\frac{1}{2}$.	2 points	*50% is also accepted. These 2 points cannot be split further.*
Total:	**2 points**	

9.		
The cardinality of the set $A \cap B$ is 27.	2 points	*If the answer is wrong but the diagram is correct then 1 point should be given.*
Total:	**2 points**	

10.		
The diagonal vectors are perpendicular and thus	1 point	*This 1 point can be given for presenting this idea in any possible way.*
the value of the scalar product is 0.	2 points	
Total:	**3 points**	*If the scalar product is given by the candidate as* $12 \cdot 20 \cdot \cos\varphi$*, and there is no further progress then 1 point should be given.*

11.		
The logical value of B is FALSE.	1 point	
The satetement C is: If a quadrilateral is a rectangle then two of its opposite angles are right angles.	1 point	*It is not required to formulate the statement C in the form „if..., then...”.*
The logical value of C is TRUE.	1 point	
Total:	**3 points**	

12.		
There are $\binom{7}{3}$ =	1 point	*This point should be given even if nothing else but the correct result is written down.*
= 35 ways to do the shopping.	1 point	
Total:	**2 points**	*1 point may be given if the order is distinguished in the computation (and the result is hence* $7 \cdot 6 \cdot 5 = 210$*).*

II/A

13. a)		
Drawing the correct graph.	2 points	*Not considering the domain 1 point may be given. If the whole arc of the parabol is not drawn but the description of it is correct 2 points should be given.*
Total:	**2 points**	

13. b)		
The value of x yielding the minimum is $x = 1.5$.	1 point	
The value of the minimum is 0.75 .	1 point	
Total:	**2 points**	

13. c)		
Squaring both sides one gets $x^2 - 3x + 3 = 1 - 4x + 4x^2$	2 points	*These 2 points cannot be split further.*
clearing the terms one gets $3x^2 - x - 2 = 0$.	1 point	
The roots of this equation are $x_1 = 1$ and $x_2 = -\frac{2}{3}$.	2 points	
$x = 1$ is not a solution.	1 point	*Either by substitution or by checking the domain.*
If $x = -\frac{2}{3}$ then both sides are equal $\frac{7}{3}$ therefore, this number is a solution of the equation.	2 points	*When substituting $x = -\frac{2}{3}$, the candidate may also use rounded values. One can prove that the solution is correct by checking the range and the domain.*
Total:	**8 points**	

14. a)

No. of contestant	I.	II.	III.	Total score	percentual achievement
1.	28	16	40	84	56
2.	31	35	44	110	73
3.	32	28	56	116	77
4.	40	42	49	131	87
5.	35	48	52	135	90
6.	12	30	28	70	47
7.	29	32	45	106	71
8.	40	48	41	129	86

For the correct completed first column	2 points	*If there are more than two wrong entries in a column, then 0 points should be given instead of 2. In case of one or two errors 1 point may be given.*
For the correct completed second column	2 points	
1st place : contestant no. 5.; 2nd place: contestant no. 4. ; 3rd place: contestant no. 8 .	1 point	
Total:	**5 points**	

14. b)

Since there were 4 ones scoring above 75% among the 8 papers, the probability in question is: $\frac{4}{8} = 0.5\,(50\%)$.	2 points	*Stating nothing else but the correct answer is worth 1 point only.*
Total:	**2 points**	

14. c)

The median of the scores for problem I. is 31,5 (32 when rounded),	1 point	
the arithmetic mean of the scores for problem II. is 279/8 = 34.875 (35 when rounded),	1 point	
90% of the 60 points for problem III. is 54 points.	1 point	
Rounding and summing one gets $32 + 35 + 54 = 121$ points,	1 point	
which would have resulted the 4th position.	1 point	
Total:	**5 points**	*If roundings are not performed at all or they are incorrect then 1 point should be deducted from the 5 points.*

15. a)

The following table is containing the information concerning the three plots.

	no. of rows	no. of trees in a row	total	
pines	x	y	$x \cdot y$	
oaks	$x-4$	$y-5$	$(x-4)\cdot(y-5)$	$x \cdot y - 360$
planes	$x+3$	$y+2$	$(x+3)\cdot(y+2)$	$x \cdot y + 228$

Correct interpretation of the text.	3 points*	*1) It is enough to enter one of the results in the „total" column.* *2) The 3 points can be split both according to the logic of the rows or the columns of the above table.* *3) Any other consistent interpretation of the unknowns is worth 1-1 point.*
Counting the total number of oaks and planes in two different ways yields the following equations: $(x-4)\cdot(y-5)=x\cdot y-360$	1 point*	
$(x+3)\cdot(y+2)=x\cdot y+228$	1 point*	
Clearing the equal terms one gets $\left.\begin{matrix}5x+4y=380\\2x+3y=222\end{matrix}\right\}$	2 points	
Therefore, $x=36$ and $y=50$.	2 points	
There are 36 rows int he plot of the pine-trees and there are 50 trees in a row.	1 point	
Total:	**10 points**	

** If the candidate does not state clearly the meaning of the unknowns, it may get at most 4 points instead of 3+1+1 = 5.*

15. b)

There are 39 rows in the plot of the planes and there are 52 trees in each row.	1 point	
There are 2028 plane-trees altogether.	1 point	
Total:	**2 points**	

II/B

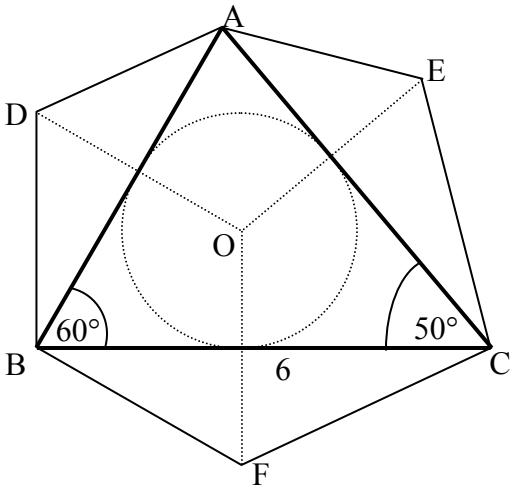
16. a)		
The problem is about an arithmetic progression: $a_1 = 220$; $d = 10$. $a_{11} = a_1 + 10 \cdot d =$	2 points	*If this idea shows up in the calculations later, then these 2 points should be given.*
$= 220 + 10 \cdot 10 = 320$. 320 meters were asphalted on the 11th working day.	1 point	
Total:	**3 points**	*The right answer may be accepted if there is any kind of correct reasoning.*

16. b)		
$S_n \geq 7100$; $n = ?$, where n is a positive integer.	1 point	*This point should be given if this idea appears later.*
$S_n = \frac{2a_1 + (n-1) \cdot d}{2} \cdot n$ $7100 = \frac{2 \cdot 220 + (n-1) \cdot 10}{2} \cdot n$	2 points	*If only the formula for S_n is written down, then there should be no points given.*
$1420 = (44 + n - 1) \cdot n$ $n^2 + 43n - 1420 = 0$	2 points	
There is one positive solution ($n \approx 21.88$),	1 point	
and it is not an integer.	1 point	
The asphalting was completed on the 22th working day.	1 point	
Total:	**8 points**	*At most 5 points may be given if the changing of the measure is forgotten.*

16. c)		
$S_{21} = \frac{2 \cdot 220 + (21-1) \cdot 10}{2} \cdot 21$	1 point	
$S_{21} = 6720$	1 point	
There were $7100 - 6720 = 380$ meters asphalted on the last working day.	1 point	
Total:	**3 points**	

16. d)		
In case of direct proportionality threre should be 440 meters asphalted on the 21th day.	1 point	
$a_{21} = 220 + 20 \cdot 10 = 420$.	1 point	
There is no direct proportionality.	1 point	
Total:	**3 points**	

17. a)



The third angle of the triangle is $BAC\angle = 70^\circ$.	1 point*	
The centre O of the inscribed circle is the intersection of the internal angle bisectors.	1 point*	
Therefore, when reflecting, the double of the half of each internal angle is added to the respective angles	1 point*	
and thus the angles of the hexagon are $DAE\angle = 140^\circ$; $ECF\angle = 100^\circ$; $FBD\angle = 120^\circ$.	1 point	
Because of reflection, the angles of the hexagon at the vertices D, E and F are equal to the angles formed by the angle bisectors of the triangle ABC at O. Hence	1 point*	
$BDA\angle = 115^\circ$; $AEC\angle = 120^\circ$; $CFB\angle = 125^\circ$.	1 point	
Total:	**6 points**	

17. b)

By symmetry $BO = BD = BF$. Therefore, it is enough to compute the length of the internal segment $x=BO$.	2 points*	
By the sine rule in the triangle BOC : $\frac{x}{6} = \frac{\sin 25^\circ}{\sin 125^\circ}$,	2 points	
yielding $x \approx 3.1$ cm, hence the two sides of the hexagon are both 3,1 cm long.	1 point	
Total:	**5 points**	

17. c)		
Because of the reflection the area of the hexagon is the double of that of the triangle.	1 point*	
For the side $AB = c$ of the triangle: $\frac{c}{6} = \frac{\sin 50°}{\sin 70°}$,	1 point	
and thus $c \approx 4.9$ (cm).	1 point	
The area of the triangle is : $\frac{6\,c \sin 60°}{2} \approx 12.7\ (cm^2)$.	2 points	
The area of the hexagon is : $2 \cdot 12.7 = 25.4$ (cm^2)	1 point	*25,5 cm^2 may also be accepted (It is the order of roundings…)*
Total:	**6 points**	

*1)The points marked by * should also be given if the corresponding ideas appear on a neat diagram or they are clearly visible in the line of calculations.*
2) There should be 1 point deducted from the 17 if there are erroneous roundings.

18. a)		
Substituting the given value of $G = 1090$ in the formula for E: $E_{2005} = 75{,}5 - 5 \cdot 10^{\frac{6000-1090}{6090}}$	2 points	
$E_{2005} \approx 75{,}5 - 5 \cdot 10^{0{,}8062}$	1 point	
Hence the life expectancy in the year 2005 is 43.5 years.	1 point	
Total:	**4 points**	*The 4 points may be given for the correct use of the formula and the correct result.*

18. b)		
$3 \cdot 1090 = 3270$ yields the new value of G.	1 point	
Substituting in the formula for E $E_{2020} = 75{,}5 - 5 \cdot 10^{\frac{6000-3270}{6090}} \approx 75{,}5 - 5 \cdot 10^{0{,}4483} \approx 61{,}5$.	3 points	
The change of the life expectancy is hence $E_{2020} - E_{2005} = 61{,}5 - 43{,}5 = 18$ (years).	1 point	
Total:	**5 points**	

18. c)		
Substituting the value $E = 68$ in the formula: $E_{2005} = 68 = 75{,}5 - 5 \cdot 10^{\frac{6000-G}{6090}}$	1 point	
Rearranging the terms one gets $10^{\frac{6000-G}{6090}} = 1.5$.	2 points	
(Using logarithms for the computations:) $\frac{6000-G}{6090} = \lg 1.5 \approx 0.17609$	3 points	
Hence by rearrangement one gets the value of the GDP in 2005 as $G = 4928$ dollars.	2 points	
Total:	**8 points**	
If in the solution the roundings are correct the maximal number of points may be given.		