ÉRETTSÉGI VIZSGA ● 2006. május 9.

MATEMATIKA ANGOL NYELVEN
MATHEMATICS

KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA
INTERMEDIATE LEVEL WRITTEN EXAM

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ
KEY AND GUIDE FOR EVALUATION

OKTATÁSI MINISZTÉRIUM
MINISTRY OF EDUCATION

Important Information

Formal requirements:

- The papers must be assessed in **pen and of different colour** than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
- The first one among the shaded rectangles next to each question contains the maximal score for that question. The **score** given by the examiner should be entered into the other **rectangle**.
- **In case of correct solutions**, it is enough to enter the maximal score into the corresponding rectangle.
- In case of faulty or incomplete solutions, please indicate the corresponding partial scores within the body of the paper.

Substantial requirements:

- In case of some problems there are more than one solutions outlined with the corresponding marking schemes. However, if you happen to come across with some **solution different** from those in the assessment bulletin, please identify the parts equivalent to those in the solution provided here and do your marking accordingly.
- The scores in this assessment bulletin can be split further. Keep in mind, however, that the number of points awarded for any item can be an integer number only.
- In case of a correct answer and a valid argument the maximal score can be awarded even if the actual solution is **less detailed** than that in this bulletin.
- If there is a **calculation error** or any other flaw in the solution, then the score should be deducted for the actual item where the error has occured. If the candidate is going on working with the faulty intermediate result and the problem has not been changed essentially due to the error, then the subsequent partial scores should be given out.
- If there is a **fundamental error** within an item (these are separated by double lines in this bulletin), even formally correct steps should not be awarded by any points, whatsoever. However, if the candidate is using the wrong result obtained by the invalid argument throughout the subsequent steps correctly, they should be given the maximal score for the remaing parts if the problem has not been changed essentially due to the error.
- If a **measuring unit** occurs in braces in this bulletin, the solution is complete even if the candidate does not indicate this unit.
- If there are more than one attempts to solve a problem, there is **just one** of them (with the highest score) that can be considered in the final assesment.
- You should **not give out any bonus points** (points beyond the maximal score for a solution or for some part of the solution).
- You should not reduce the score for erroneous calculations or steps unless its results are actually used by the candidate in the course of the solution.
- **There are only 2 questions to be marked out of the 3 in part II/B of this exam paper.** Hopefully, the candidate has entered the number of the question not to be markad in the square provided for this purpose. Accordingly, this question should not be assessed even if there is some kind of solution written down in the paper. Should there be any ambiguity about the student's request with respect to the question not to be checked, it is the last one in this problem set, by default, that should not be marked.

I.

1.		
$A \cap B = \{12; 16; 20\}$	2 points	*If the candidate finds two of the elements correctly, 1 point may be given.*
Total:	**2 points**	*The elements of the sets A and B should not be given any credit.*

2.		
The leg is $3 \cdot \sin 42^\circ \approx 2.01$ cm.	2 points	*The leg: 1 point, rounding: 1 point.*
Total:	**2 points**	

3.		
a) true	1 point	
b) false	1 point	
c) true	1 point	
d) false	1 point	
Total:	**4 points**	

4.		
The mode is 174.	1 point	
The median is 173.	1 point	
Total:	**2 points**	

5.		
$3y - x = 3$ or $y = \frac{1}{3}x + 1$ $(x \in [-9; 9])$	3 points	*In case of partially correct answer both the slope and the y-intercept are worth 1 point each.*
Total:	**3 points**	*The 3 points are due even if the candidate gives the formula of the mapping insted of the equation of the graph.*

6.		

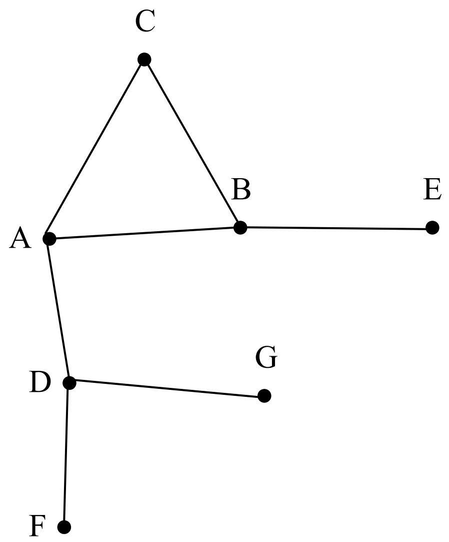



Diagram	1 point	*This point may be given for a correct diagram only.*
The sum of the degrees is 14.	1 point	
Total:	**2 points**	

7.		
Not every grandma is in fond of her grandchild. or: There is a grandma who does not like her grandchild.	2 points	*Any correct answer is worth 2 points.*
Total:	**2 points**	

8.		
The index is equal to $-\frac{1}{2}$.	2 points	*The index may be given in any correct form.*
Total:	**2 points**	*If the answer is* $10^{-\frac{1}{2}}$ *then 1 point should be given.*

9.		
The range is $-1 \le y \le 3$, y is a real number. or [–1, 3].	2 points	*Specifying y be real is not neccessary.*
Total:	**2 points**	

10.		
The number of possible arrangements is equal to 12 (= 3·2·1·2).	3 points	
Total:	**3 points**	*If the candidate is listing the arrangements and the list is incomplete but there are at least 6 correct items then 1 point may be given.*

11.		
The number of outcomes is 90.	1 point	
The number of favourable outcomes is 9.	1 point	
The probability is equal to $\frac{9}{90} = 0.1$.	1 point	
Total:	**3 points**	

12.		
The equation of the circle is $(x+2)^2 + (y-1)^2 = 25$.	1 point	
Substituting the cordinates of the point $P(1, -3)$ one gets 25 = 25,	1 point	*The candidate might calculate the distance of the point P and the centre of the circle.*
Therefore, the point P is lying on the circle.	1 point	
Total:	**3 points**	

II./A

13.		
By the respective domains of the logarithm and the square root one gets $x > \frac{2}{3}$ and $x > \frac{7}{4}$,	1 point*	
and thus the domain of the equation is the set $x > \frac{7}{4}$.	1 point*	
By the laws of logarithms $\lg\left(\sqrt{3x-2}\cdot\sqrt{4x-7}\right) = \lg 2$.	2 points	
The ten base logarithm function is strictly monotone and thus $\sqrt{3x-2}\cdot\sqrt{4x-7} = 2$.	1 point	*This point is due even if the explanation is missing.*
Squaring $(3x-2)\cdot(4x-7) = 4$.	1 point	
Multiplying and collecting the terms $12x^2 - 29x + 10 = 0$.	2 points	
The solutions of the equation are $x_1 = 2$; $x_2 = \frac{10}{24}\left(= \frac{5}{12}\right)$.	2 points	
Equality holds when $x_1 = 2$ is plugged.	1 point	
$x_2 = \frac{5}{12}$ does not satisfy the equation.	1 point	** If the domain is not stated but the check is correct then these 2 points should be given.*
Total:	**12 points**	

14. a)		
We write the cosine rule for the length AB of the umbrella: $AB^2 = 25^2 + 60^2 - 2\cdot 25\cdot 60\cdot\cos 120°$.	3 points	*2 points for realizing the relevance of the cosine rule and 1 more for correct substitution.*
$AB^2 = 5725$	1 point	
$AB = \sqrt{5725} \approx 76$ cm is the length of the umbrella.	1 point	
Total:	**5 points**	

14. b)		
If x is the length of the part of the string from the endpoint A, then that of the other part is $85 - x$.	1 point	*This point is due even if the coresponing division is clear from the Pithagorean equation.*
Writing down Pithagoras' theorem in the right triangle yields $x^2 + (85 - x)^2 = 5725$.	1 point	
$x^2 + 85^2 + x^2 - 170x = 5725$.	1 point	*For the squaring.*
$x^2 - 85x + 750 = 0$.	1 point	*For collecting the terms.*
The roots of this quadratic are 75 and 10.	2 points	
The distance of the right angle vertex from the endpoint A is either 75 cm or 10 cm.	1 point	
Total:	**7 points**	

15. a)

Total:	**4 points**	*Separating the age groups is worth 2 points, the labelling of the axes is worth 1 point and the chart is another 1 point.*

15. b)

The mean age of the team is $\frac{19+20+3\cdot 21+2\cdot 22+3\cdot 23+24+4\cdot 25+3\cdot 26+27+3\cdot 28}{22}=$ $=\frac{528}{22}=24$ years.	3 points	*In case of any computation error at most 2 points may be given.*
Total:	**3 points**	

15. c)

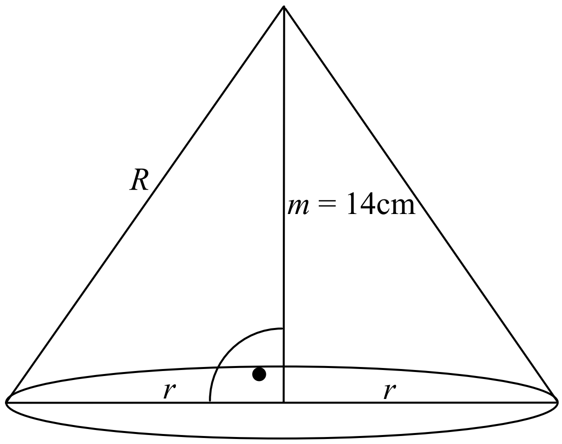
There are $\binom{4}{2}$ (= 6) ways to choose 2 out of the four 25 years old players and $\binom{3}{2}$ (= 3) ways to choose 2 out of the three 28 years old ones.	3 points	*The correct urn model is worth 1 point and the two cases are worth 1 point each. (If there are no combinatorial terms but the correct answer, full credit should still be given.)*
There are 6·3·1 = 18 ways to choose the 5 players.	2 points	
Total:	**5 points**	*There can be at most 2 points given if the explanation is missing.*

II./B

16. a)		
2.5% of 20 000 Ft is the comission.	1 point	
The exchange for 19 500 Ft is 19 500·146 = 2 847 000 lej.	2 points	*284.7 NEW LEJ may also be accepted.*
Total:	**3 points**	
16. b)		
300 NEW LEJ = 3 000 000 lej	1 point	
If this is the exchange for x Ft, one may write $x \cdot 0.975 \cdot 146 = 3\ 000\ 000$.	3 points	
Hence x = 21 075 Ft.	1 point	
Total:	**5 points**	*If there is a computation error then at most 4 points may be given.*
16. c)		
1 NEW LEJ = $\frac{10000}{146}$ Ft = 68.49 Ft	3 points	*Computation errors or rounding errors should be penalized by 1-1 point, respectively.*
Total:	**3 points**	
16. d)		
There are $\binom{8}{4}$ ways to select four out of the eight coins so the number of outcomes is 70.	1 point	*Stating that each selection is equally probable is not required.*
Given are the four denominations any favourable outcome is of the form 90 = 50 + 20 + 10 + 10.	1 point	
There is one way to pick the single 50, three ways to choose one out of the three 20-bani coins, finally there are six ways to choose the two 10-bani coins from a supply of four.	2 points	
There are 1·3·6 = 18 ways for the cashier to collect the 90 NEW BANI change altogether.	1 point	
The probability is hence $\frac{18}{70} \approx 0.2571$.	1 point	
Total:	**6 points**	

17. a)		
$a_3 = 5\cdot r^2$, $a_5 = 5\cdot r^4$.	2 points	
Total:	**2 points**	
17. b)		
$a_4 = 5 + 3d$, $a_{16} = 5 + 15d$.	2 point	
Total:	**2 points**	
17. c)		
$5\cdot r^2 = 5 + 3d$, $5\cdot r^4 = 5 + 15d$.	2 points	
Eliminating d one gets $r^4 - 5\cdot r^2 + 4 = 0$.	3 points	*Squaring the first equation the common ratio can also be eliminated. The corresponding equation is $d(d-5) = 0$.*
This is a quadratic in r^2. The quadratic formula yields	1 point	
$r^2 = 1$ or 4 .	2 points	
Hence r is either ± 1 or ± 2.	2 points	*At most 1 point may be given if the negative values are missing.*
Accordingly, d is either 0 or 5.	1 points	
Checking the answers against the text of the question.	2 points	
Total:	**13 points**	

18. a)		
The side of length 31.4 cm is the perimeter of the base of the cylinder: $31.4 = 2r\cdot\pi$.	1 point	
$r \approx 5$ (cm)	1 point	
$V_{cylinder} = r^2\cdot\pi\cdot 14$	1 point	
The volume of the cylinder is $\approx 1.1\ \text{dm}^3$.	1 point	
Total:	**4 points**	

18. b)		
		
Total:	**2 points**	

18. c)		
The length $R\cdot\pi$ of the semicircle is equal to the perimeter of the base of the cone: $R\cdot\pi = 2r\cdot\pi$;	1 point*	*This point is due even if there is no explanation.*
therefore $r = \frac{R}{2}$.	1 point	**These 1+1 points should be given if the candidate finds thet ratio as a result of any correct method.*
Writing down Pithagoras' theorem for the right triangle of sides $\frac{R}{2}$, 14 and R yields	1 point	
$\frac{R^2}{4} + 14^2 = R^2$.	1 point	
The solution is $R = \frac{28}{\sqrt{3}} \approx 16.2$ cm.	2 points	
Total:	**6 points**	

18. d)		
The area of the base circle is $r^2 \cdot \pi$.	1 point	*≈ 206 cm²* *(here r ≈ 8.1 cm)*
The area of the superficies is equal to $\frac{R^2\pi}{2}$.	1 point	*≈ 412 cm²*
The ratio of the areas is $\frac{r^2\pi}{0.5 \cdot R^2\pi} = \frac{2r^2}{R^2}$	1 point	
Plugging $r = \frac{R}{2}$	1 point*	*This step is not neccessary if the candidate is working with actual numbers.*
The ratio of the areas is equal to $\frac{1}{2}$.	1 point	** 1+1 points should be given for the correct value of the ratio no matter what the underlying method is.*
Total:	**5 points**	