ÉRETTSÉGI VIZSGA ● 2005. május 10.

MATEMATIKA ANGOL NYELVEN MATHEMATICS

KÖZÉPSZINTŰ ÉRETTSÉGI VIZSGA STANDARD LEVEL FINAL EXAMINATION

Az írásbeli vizsga időtartama: 180 perc
Time allowed for the examination: 180 minutes

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ MARKSCHEME

OKTATÁSI MINISZTÉRIUM
MINISTRY OF EDUCATION

Instructions to examiners

Formal requirements:

- Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors, incomplete solutions, etc. in the conventional way.
- The first one of the rectangles under each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to that.
- **If the solution is perfect,** it is enough to enter the maximum scores in the appropriate rectangles.
- If the solution is incomplete or incorrect, please indicate the individual **subtotals** on the paper, too.

Assessment of content:

- The markscheme contains more than one solution for some of the problems. If the solution by the candidate is different, allocate the points by identifying the parts of the solution equivalent to those of the one given in the markscheme.
- The subtotals in the markscheme can be **further divided**, but the scores awarded should always be whole numbers.
- If it is clear that the reasoning and the final answer are both correct, you may award the maximum score even if the solution is **less detailed** than the one in the markscheme.
- If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error is made. If the reasoning remains correct and the error is carried forward, the points for the rest of the solution should be awarded.
- **In the case of a principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information obtained owing to the principal error is carried forward to the next section or in the next part of the problem and used correctly, the maximum score is due for the next part.
- Where the markscheme shows a **unit** in brackets, the solution should be considered complete without that unit as well.
- If there are more than one different approaches to a problem, **assess only one** of them (the one that is worth the largest number of points).
- **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
- **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
- **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I.

1.		
$F\left(-\frac{3}{2}; 1\right)$.	2 points	*1 point should be awarded if only one coordinate is correct.*
Total:	**2 points**	

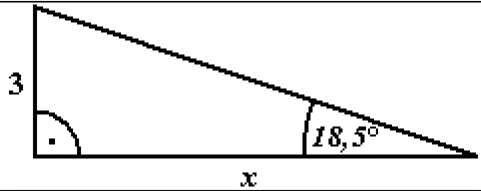
2.		
B.	2 points	
Total:	**2 points**	

3.		
[2; 6] Or: $2 \le y \le 6$.	3 points	*1 point should be subtracted if the left or right end of the interval is wrong or if the interval is open or partly open.*
Total:	**3 points**	

4.		
A: false.	1 point	
B: true.	1 point	
C: false.	1 point	
Total:	**3 points**	

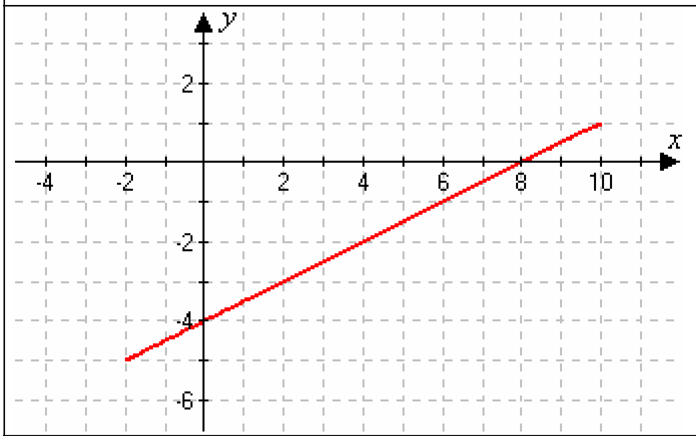
5.		
$(x+3)^2+(y-5)^2=16$. Or: $x^2+y^2+6x-10y+18=0$.	2 points	
Total:	**2 points**	

6.		
$\frac{21}{150}$ or 14% or 0.14.	2 points	*Any form of the answer is acceptable.*
Total:	**2 points**	

7.		
	1 point	*1 point is given for indicating the data in the diagram.*
$\tan 18.5° = \frac{3}{x}$.	1 point	
The other leg is $x \approx 8.966 \approx 9$ (cm).	1 point	*It is also correct without rounding.*
Total:	**3 points**	

8.		
$a_5 = \frac{1}{2}$.	2 points	
Total:	**2 points**	

9.		
The number of edges is 4.	2 points	*1 point should be awarded if there is only a correct sketch.*
Total:	**2 points**	

10.		
	2 points	*If the graph is good but it is not restricted to the given interval, 1 point should be awarded.*
Total:	**2 points**	

11.		
a)		
$\binom{22}{5} = 26\ 334.$	2 points	*The 2 points should also be awarded if the binomial coefficient is not evaluated.*
Total:	**2 points**	
b)		
$5! = 120.$	2 points	*The 2 points should also be awarded if the factorial is not evaluated.*
Total:	**2 points**	

12.		
$V = \frac{4r^3\pi}{3}.$ $V = \frac{4 \cdot 13^3\pi}{3}.$	1 point	
$V \approx 9202.8$ (cm^3).	1 point	
There is ≈ 9.2 litres of air in the ball.	1 point	*The 1 point is for the conversion.*
Total:	**3 points**	

II/A

13.		
$\cos^2 x + 4\cos x = 3\left(1 - \cos^2 x\right)$.	2 points	
Rearranged: $4\cos^2 x + 4\cos x - 3 = 0$.	1 point	
The roots of this equation are $\cos x = \frac{1}{2}$ or	1 point	
$\cos x = -\frac{3}{2}$.	1 point	
If $\cos x = \frac{1}{2}$, then $x_1 = \frac{\pi}{3} + 2k\pi$, or $x_2 = \frac{5\pi}{3} + 2k\pi$,	3 points	
where $k \in Z$.	1 point	
If $\cos x = -\frac{3}{2}$, then there is no solution, since $\cos \geq -1$ for all x.	2 points	
Since the transformations have been equivalent, both sets of roots are solutions of the original equation.	1 point	*The 1 point for checking is also due if periods are not indicated but the two roots obtained are substituted into the equation.*
Total:	**12 points**	

14.		
a)		
$a_2 = 17$ and $a_3 = 21$. $d = 4$.	1 point	*The 1 point is due for the common difference.*
$a_1 = 13$.	1 point	
$a_{150} = 609$.	1 point	*The value of a_{150} is also accepted if it only appears in the summation formula.*
$S_{150} = \frac{13 + 609}{2} \cdot 150$.	1 point	
$S_{150} = 46\,650$.	1 point	
Total:	**5 points**	

b)		
The rule for divisibility by three can be applied.	1 point	*The 2 points are also due if the divisibility rule is not stated, only applied.*
The digits of 25 863 add up to 24, thus it is divisible by three.	1 point	
The sum remains the same for any order, so the statement is true.	1 point	
Total:	**3 points**	

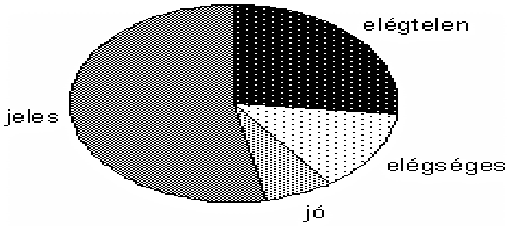
c)		
The rule for divisibility by four can be applied.	1 point	*The 1 point is also due if the rule is not stated but there is evidence of its correct application.*
In this case, the condition is met if the last two digits are 28; 32; 36; 52; 56; 68.	2 points	*If there are only four or five out of the six endings listed, award 1 point instead of 2. If there are fewer than that, award 0 points.*
Thus the digit in the tenths' place may be 2, 3, 5 or 6.	1 point	*This point is only due if all solutions are listed.*
Total:	**4 points**	
If none of the six endings are listed but the rule of divisibility is applied correctly and the answer is also correct, award 4 points. Award 4 points as well if the rule of divisibility is not stated but the endings are listed correctly and the answer is correct, too.		

15.		
a)		
The arithmetic mean is $\frac{3\cdot 100+2\cdot 95+91+2\cdot 80+65+2\cdot 31+2\cdot 17+8+5}{15}=$	2 points	
= 61.	1 point	
Mode: 100.	1 point	
Median: 80.	1 point	
Total:	**5 points**	

b)

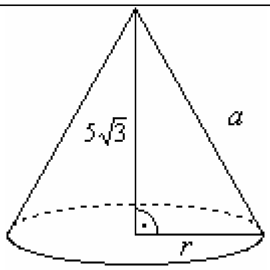
Grade	excellent	good	satisfactory	pass	fail		
Number of students	8	1	0	2	4	2 points	
					Total:	**2 points**	

c)		
Excellent: 192°. Good: 24°. Satisfactory: 48°. Fail: 96°.	2 points	*The calculation of the central angles does not need to be shown but the angles have to be stated.*

fail  excellent pass good	3 points	*Award only 1 point if it is not clear from the pie chart which grade belongs to which sector. The diagram is only acceptable if the marked boundaries of sectors lie between the appropriate ten-degree divisions.*
Total:	**5 points**	

II/B

Out of problems 16 to 18, do not assess the one indicated by the candidate.

16.		
a)		
 $a = 2r$.	2 points*	*The longitudinal section containing the axis is an equilateral triangle.*
From the Pythagorean theorem: $a^2 = r^2 + \left(5\sqrt{3}\right)^2$.	1 point*	
$4r^2 = r^2 + \left(5\sqrt{3}\right)^2$.	2 points*	
$r = 5$ cm.	1 point*	
$a = 10$ cm.	1 point*	
$A = r^2\pi + r\pi a$. $A = 25\pi + 50\pi$.	1 point	
$A = 75\pi$. Or $A \approx 235.6\ \text{cm}^2$.	1 point	
Total:	**9 points**	
** Award the appropriate points as well if these results only appear in the answers to parts b) or c).*		

b)		
$V = \frac{r^2\pi \cdot m}{3}$. $V = \frac{25\pi \cdot 5\sqrt{3}}{3}$.	1 point	
$V \approx 226.7\ \text{cm}^3$.	1 point	
Total:	**2 points**	

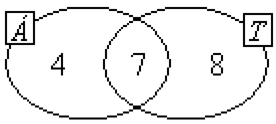
c)		
Solution 1.		
The radius of the sector is *a*.	1 point	
The length of the arc is $a\pi$.	2 points	
$\frac{\alpha}{360°} = \frac{a\pi}{2a\pi}$.	2 points	
The central angle in question is $\alpha = 180°$.	1 point	*The point is also due for correct calculation with an approximate value.*
Total:	**6 points**	

Solution 2.		
The radius of the sector is a.	1 point	
The length of the arc is $a\pi$.	2 points	
The perimeter of the whole circle is $2a\pi$.	1 point	
The arc length is one half of it, i.e. it is a semicircle.	1 point	
Thus $\alpha = 180°$.	1 point	
Total:	**6 points**	

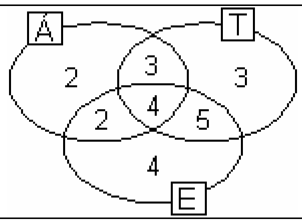
17.		
a)		
Let x denote the price of the magazine.	1 point	*This 1 point is also due if the unknown is not defined but its meaning is made clear by the verbal answer.*
Anna had $0.88x$ forints.	1 point	*Award 4 points altogether for setting up the equation.*
Zsuzsi had $\frac{4}{5}x$ forints.	1 point	
The equation: $0.88x + \frac{4}{5}x - x = 714$.	2 points	
$x = 1050$.	1 point	
$0.88x = 924$ and	1 point	
$\frac{4}{5}x = 840$.	1 point	
The magazine cost 1050 forints. Anna originally had 924 forints and Zsuzsi had 840 forints.	1 point	
Checking:	1 point	
Total:	**10 points**	

b)		
Solution 1.		
Anna receives a share of a forints, and Zsuzsi gets $714 - a$ forints out of the money remaining.	1 point	*This 1 point is also due if the unknown is not defined but its meaning is made clear by the verbal answer.*
$\frac{924}{840} = \frac{a}{714-a}$ or $\frac{0.88}{0.8} = \frac{a}{714-a}$.	2 points	*Either equation is acceptable.*
Hence $a = 374$;	1 point	
$714 - a = 340$.	1 point	
Thus Anna will have 374 forints and Zsuzsi will have 340 forints left after buying the magazine.	1 point	
Checking:	1 point	
Total:	**7 points**	

Solution 2.		
The two of them had 1764 forints.	1 point	
Anna receives $\frac{924}{1764}$ of the money remaining,	1 point	
that is $714 \cdot \frac{924}{1764} =$	1 point	
= 374 forints, and	1 point	
Zsuzsi gets $\frac{840}{1764}$ of it,	1 point	
that is $714 \cdot \frac{840}{1764} =$	1 point	
= 340 forints.	1 point	
Total:	**7 points**	

18.		
a)		
Solution 1.		
	2 points	*If only one or two of the three numbers in the set diagram are correct, award 1 point only.*
The number of differences that at least one of them noticed is 4 + 7 + 8 = 19.	1 point	
Neither of them noticed 23 – 19 = 4 differences.	1 point	
Total:	**4 points**	

Solution 2.		
The number of differences found can also be expressed without a set diagram: 11 + 15 – 7 .	2 points	*Do not give partial credit here.*
Thus the number of differences that at least one of them noticed is 19.	1 point	
Neither of them noticed 23 – 19 = 4 differences.	1 point	
Total:	**4 points**	

b)		
	7 points	*One point for each correct number in the diagram.*
Total:	**7 points**	
c)		
There is a difference that Enikő did not find. OR: Enikő did not find every difference. OR: Enikő did not find all the differences.	2 points	*Do not give partial credit here.*
Total:	**2 points**	

d)		
The number of favourable cases is 14.	1 point	*These points are also due if the diagram in part b) is filled out with errors but those values are carried forward consistently here.*
The number of all cases is 23.	1 point	
The probability in question is $\frac{14}{23}$ or ≈ 0.61 or 61%.	2 points	*The result is acceptable in any form, including values rounded correctly.*
Total:	**4 points**	